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# Benefits of Decomposition Methods to Speed-up Energy System Modelling and Application to Stochastic Optimization

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Manuel Wetzel, Frieder Borggrefe

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DLR

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Deutsches Zentrum  
für Luft- und Raumfahrt  
German Aerospace Center

**Contact:**

[Manuel.Wetzel@dlr.de](mailto:Manuel.Wetzel@dlr.de)



## 1. Introduction

- The problem of uncertainty in Energy System Modelling

## 2. Current implementation of stochastic optimization

- From deterministic to stochastic modelling
- Improving convergence by Enhanced Benders approaches

## 3. Challenges and possible improvements

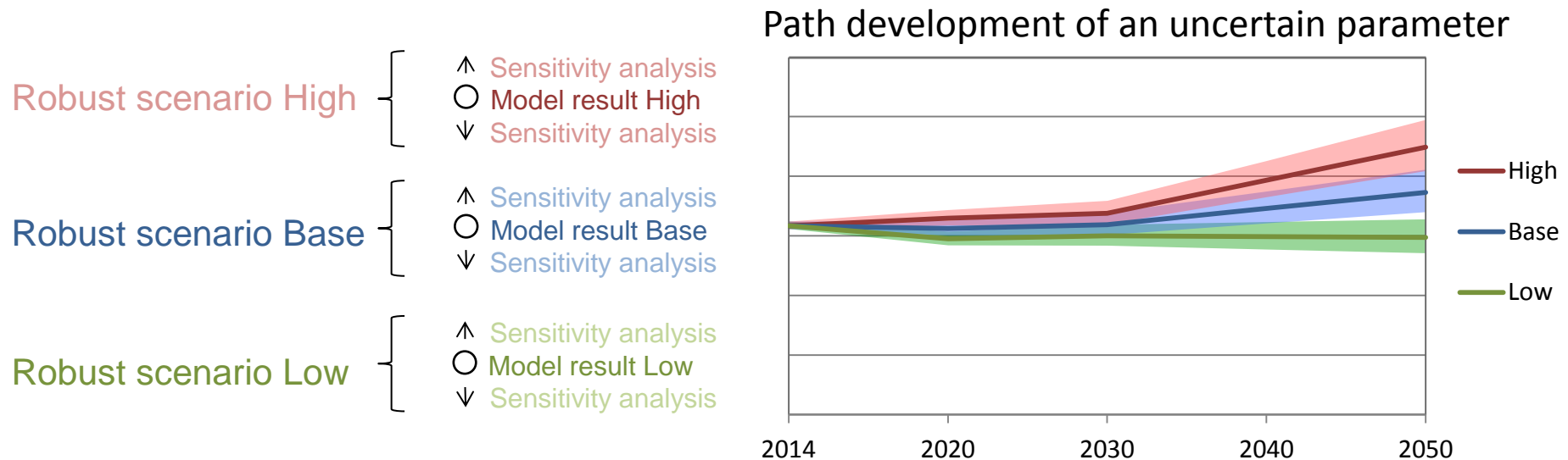
- Current challenges
- Application to path optimization



# The problem of uncertainty in Energy System Modelling

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- Long term capacity expansion planning requires **making decisions now**, which have an impact on the energy system for several decades.
- **Context scenarios** can give a general direction of development, however a large number of **consistent sub-scenarios** are possible. The current approach usually considers only the main scenario and evaluates robustness by sensitivity analysis of sub scenarios.



## Benders decomposition for two-stage stochastic optimization

(L-shaped Method)

[Benders 1962 and Van Slyke 1969]

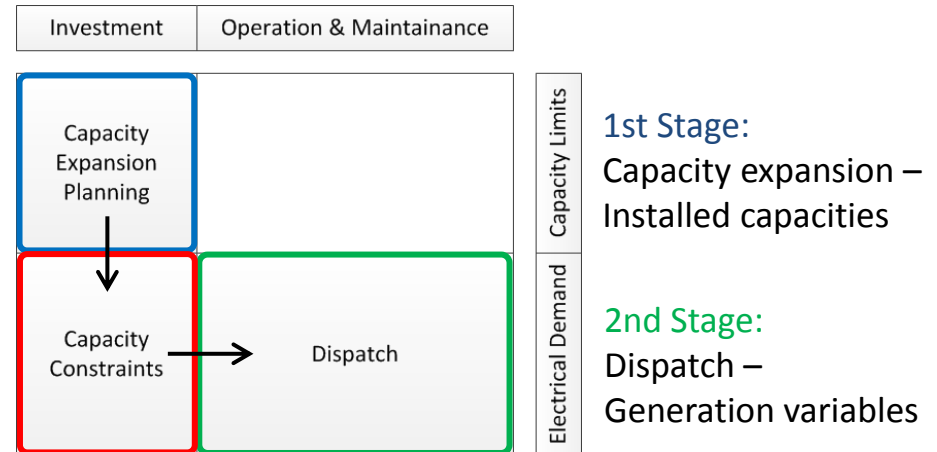
### Master problem:

- **Decide now** which capacities should be expanded

### Sub problem:

- **Decide later** on economic dispatch to satisfy electrical demand with capacities given by master problem

## Mathematical formulation in LP table



### Linking variables:

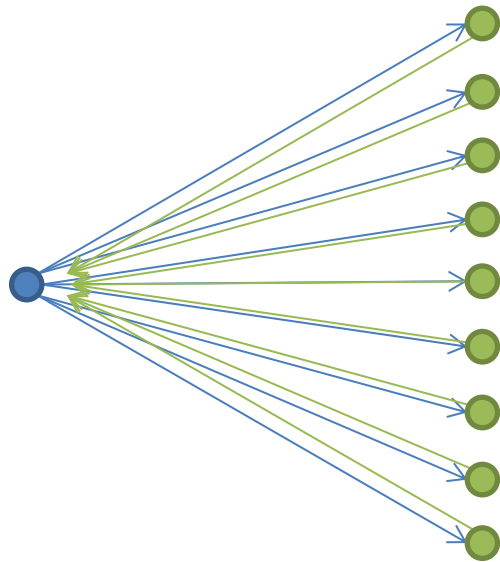
Installed capacities, connecting the capacity expansion problem and the dispatch problem



## 1. Two-stage decision tree

**Stage 1**  
**(Master problem)**  
Investment decision

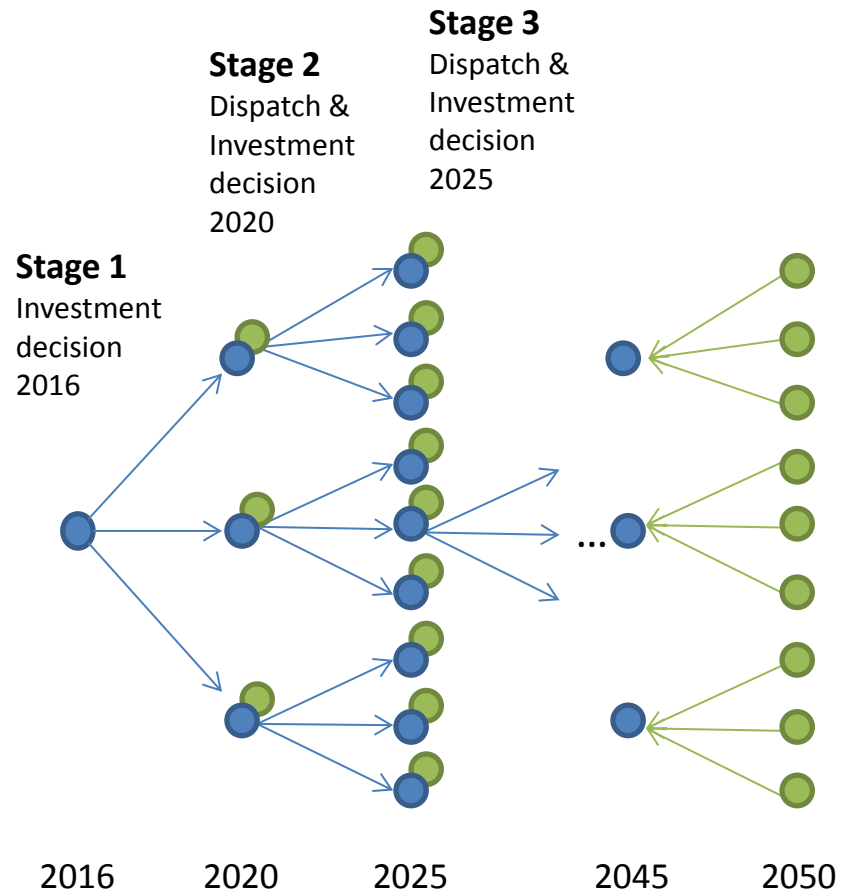
**Stage 2**  
**(Sub problem)**  
Stochastic economic dispatch scenarios



● Investment

● Dispatch

## 2. Multi-stage decision tree





# Current implementation of stochastic programming

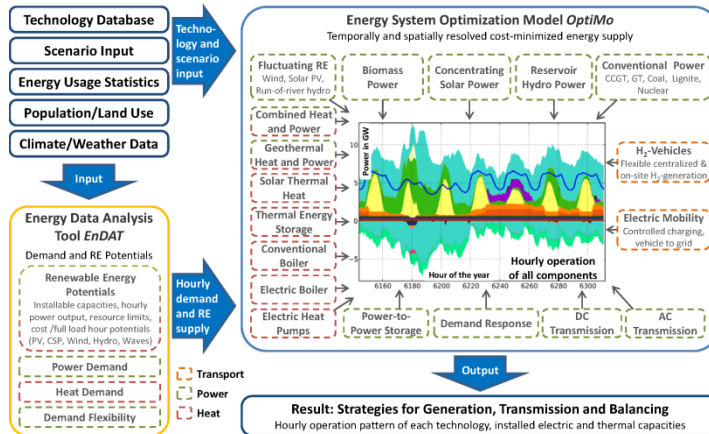
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# The REMix Model



REMix



SIMPLE

```

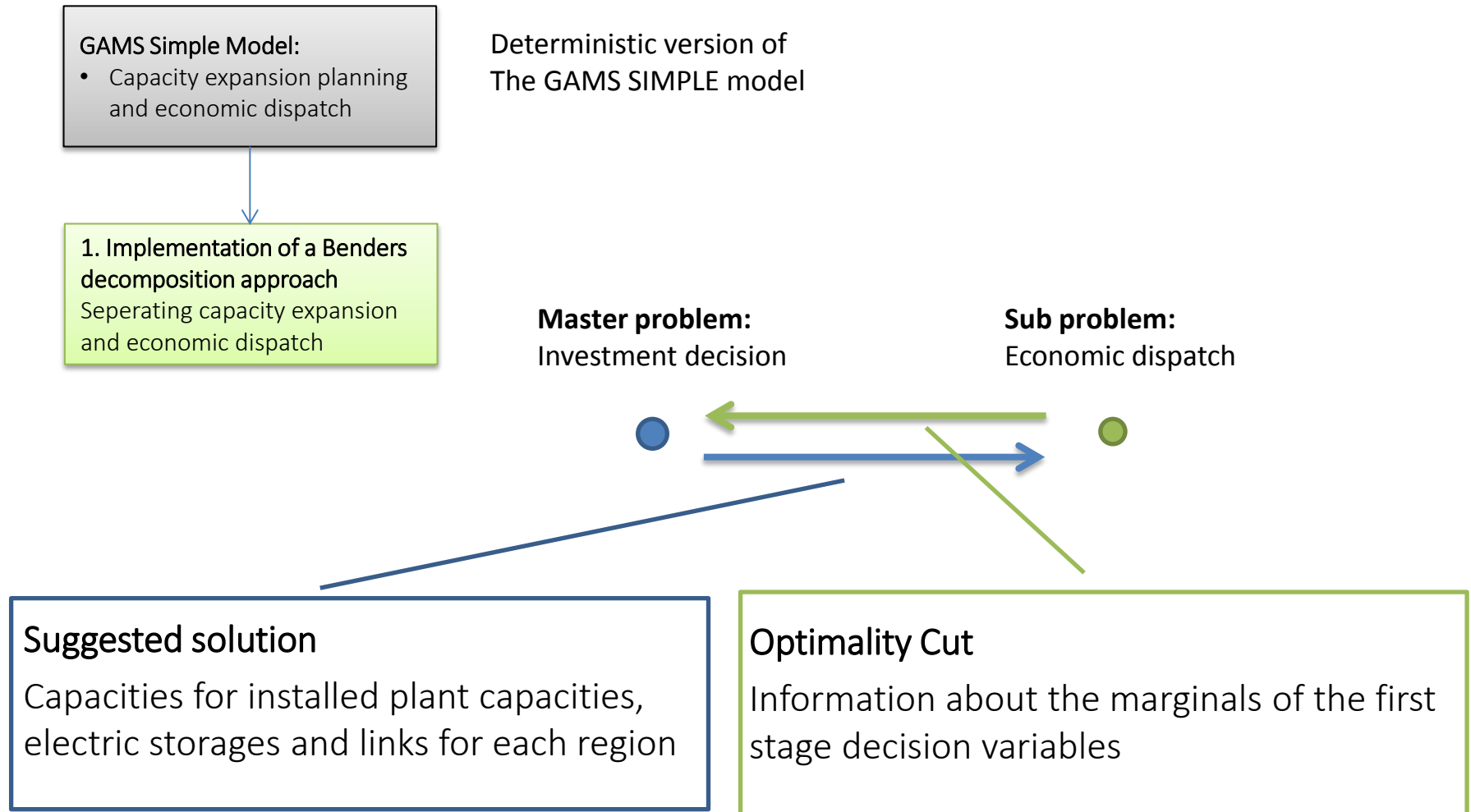
%#_obj ...
obj = sum(totals_type, k);
%#_vars ...
vars = sum(totals_type, k);
%#_constraints ...
constraints = sum(totals_type, k);
%#_model ...
model = sum(totals_type, k);
%#_solve ...
solve model;
%#_display ...
display model;
%#_write ...
write model;
%#_read ...
read model;
%#_close ...
close model;

```

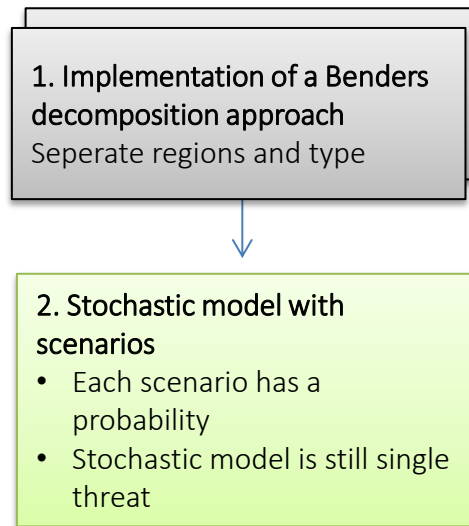
- Capacity expansion planning and economic dispatch
- **High** spatial, temporal and technological **resolution**
- **Modular approach** to include heat sector, electromobility, DSM and others
- **Simplified version** of REMix, used as a development platform
- **Scalability** of model dimensions by generic data generation
- Modified for demonstrating implementation approaches for stochastic optimization



# Implementation of Enhanced Benders decomposition approaches

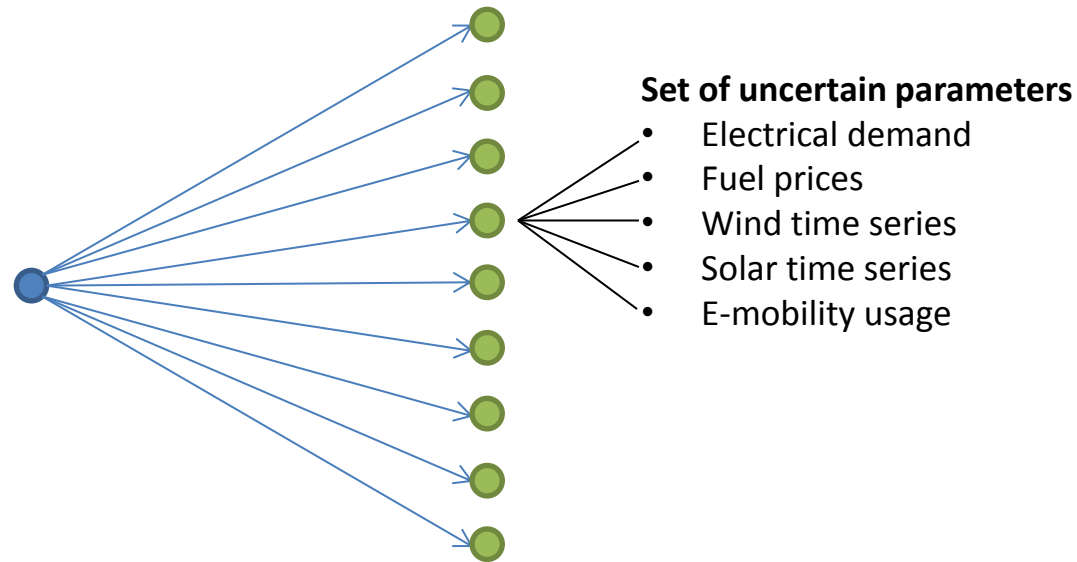






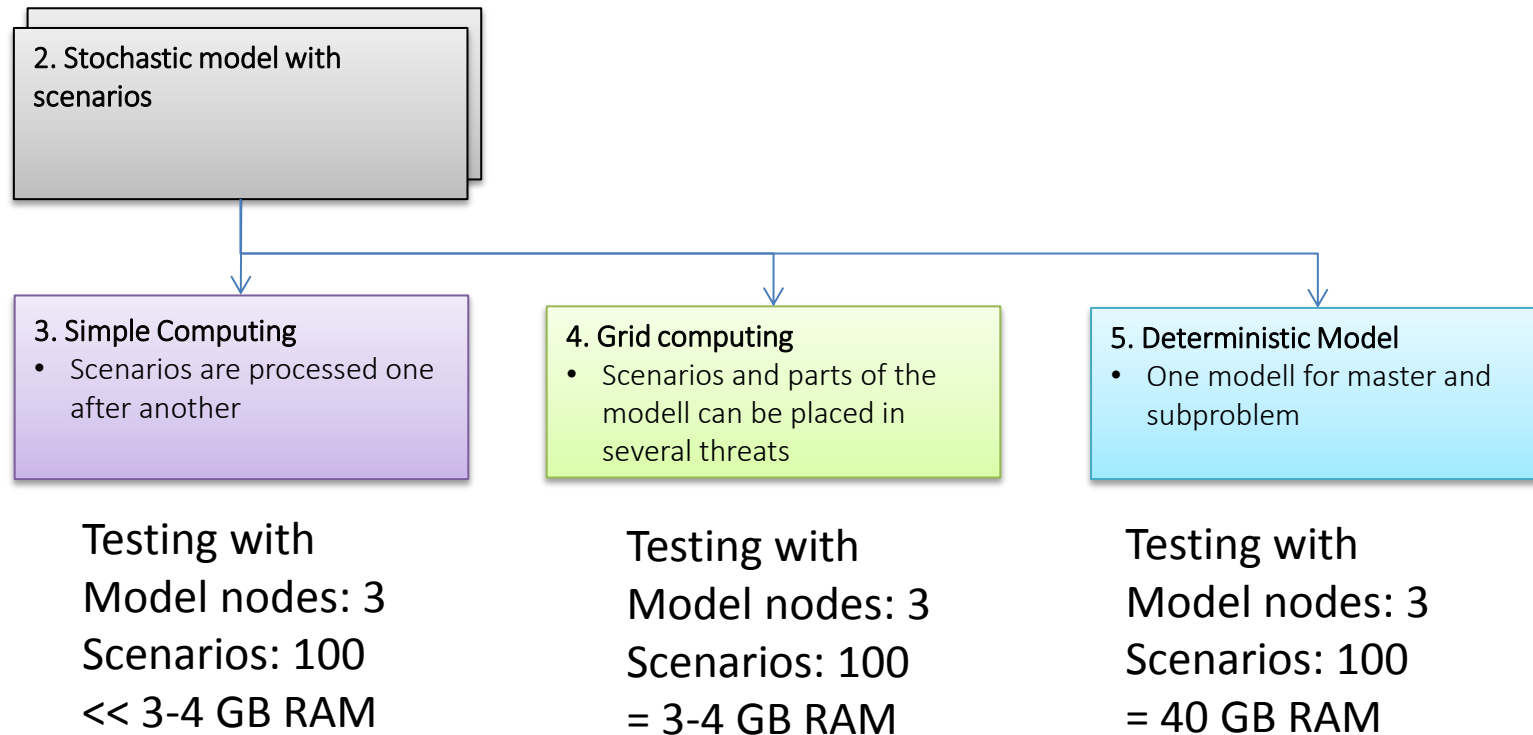
**Master problem:**  
Investment decision

**Sub problem:**  
Economic dispatch scenarios



- Each subproblem is weighted by a probability and represents a specific set of assumptions for uncertain parameters

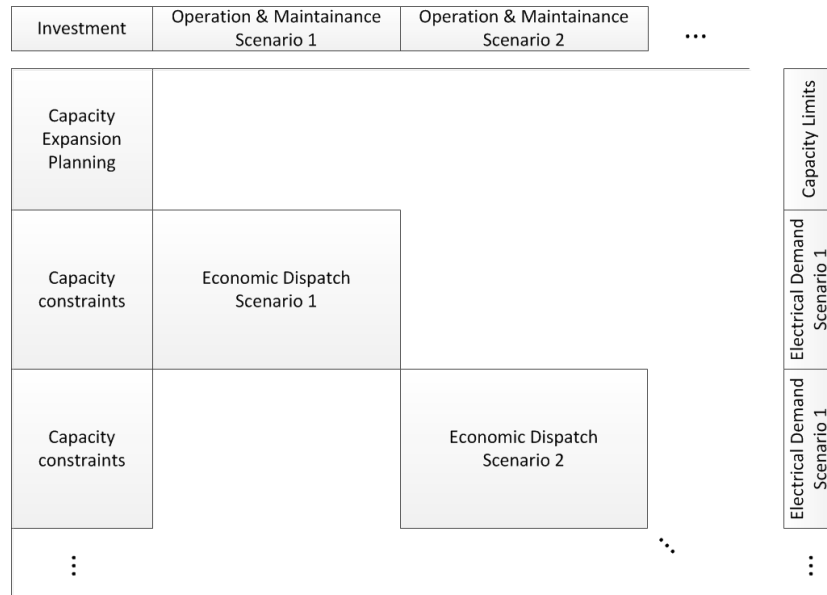




- **Small ESM** with 3 power plants, 3 storages, 2 node links and 8760h
- Formulation as Deterministic Equivalent for small problems faster than Benders decomposition, but leads to **memory restrictions** in large models



## Deterministic Equivalent

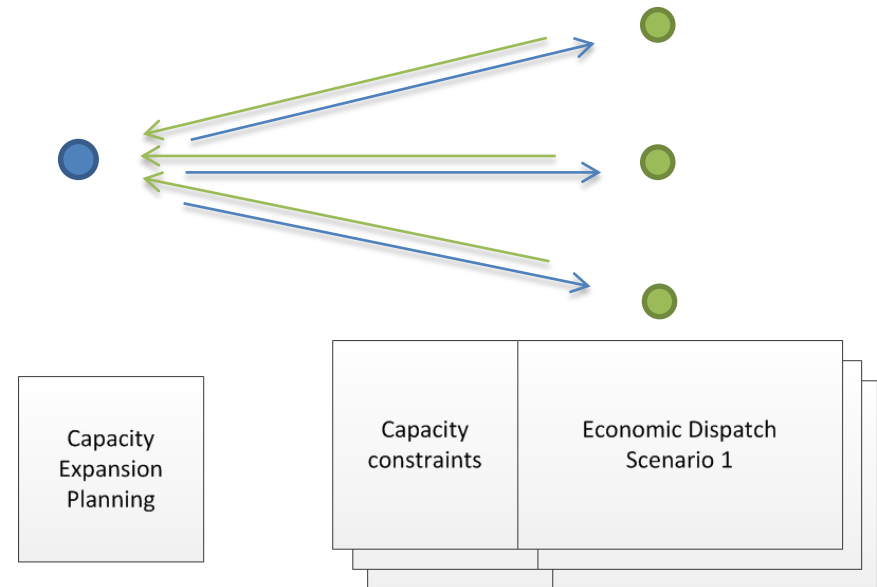


- Size of LP increases with number of scenarios, solved by SIMPLEX / Barrier
- **Out of memory** for typical REMix problems with scenario dimension

## Benders Decomposition

Master

Subproblem



- Subproblems can be **solved in parallel**
- Memory demand **scales** with number of parallel solve processes



# Improvement by optimality cut type

## Singlecut

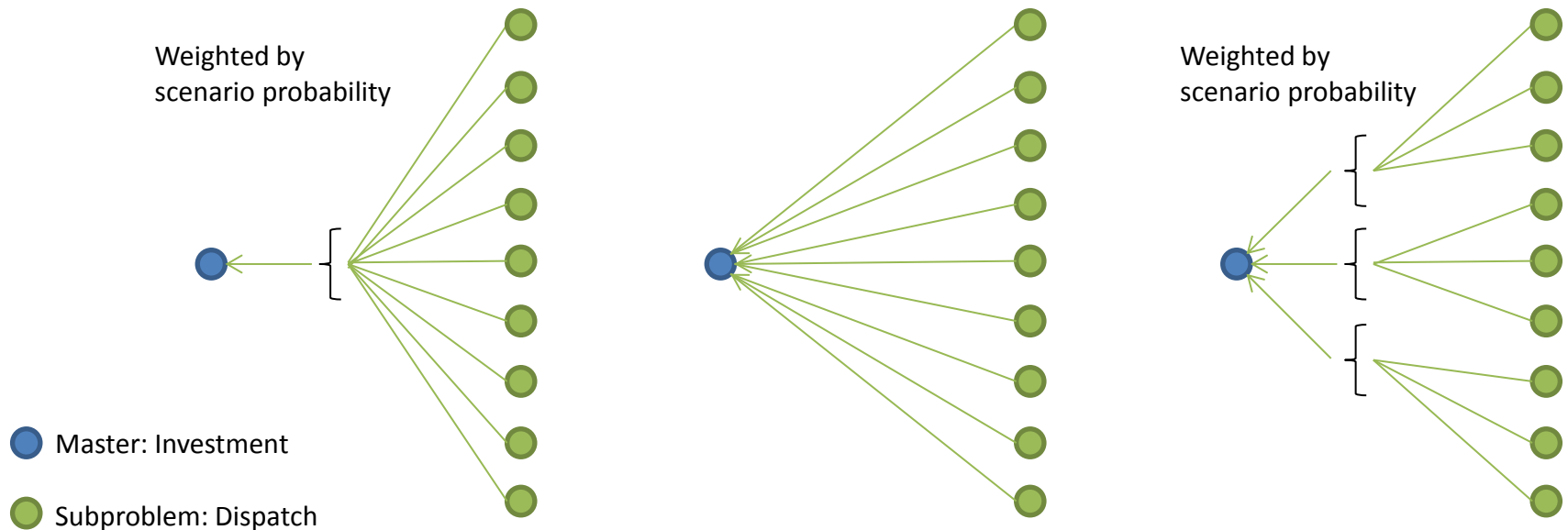
- One optimality cut per iteration is submitted to master problem

## Multicut

- Each subscenario submits an optimality cut each iteration

## Clustercut

- Each cluster submits an optimality cut each iteration

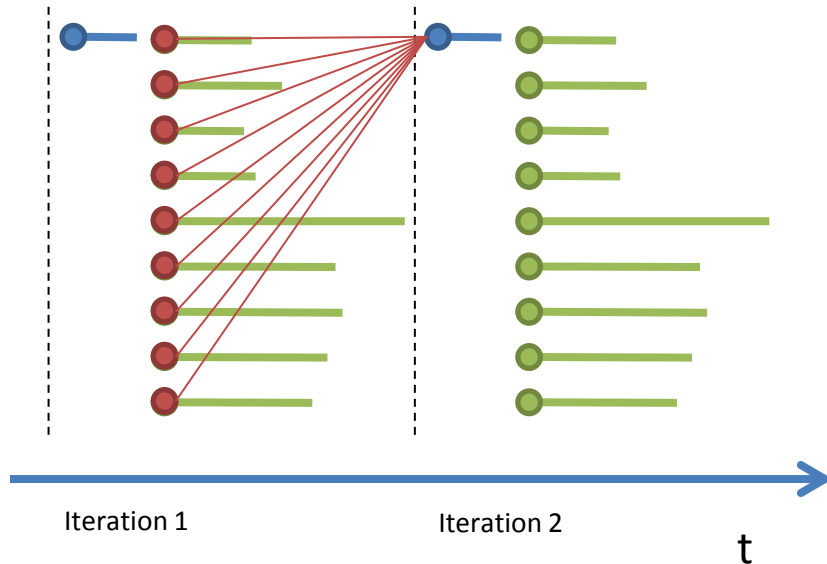


- Multicuts give **detailed information** but increase complexity [Birge 1988]
- Singlecuts **aggregate information** therefore more iterations are necessary
- Dynamic switching between cut-types is possible [Skar 2014]

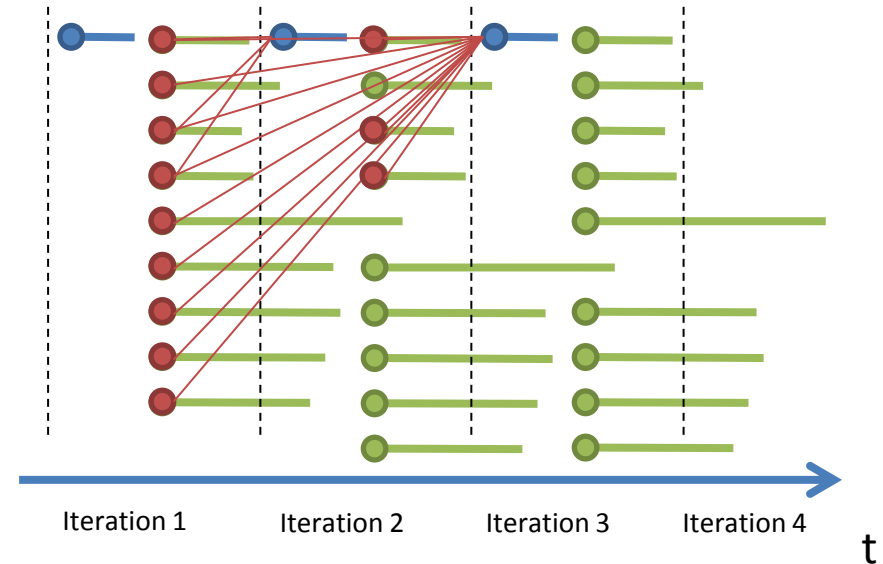


# Improvement to achieve asynchronity

Synchronous Model



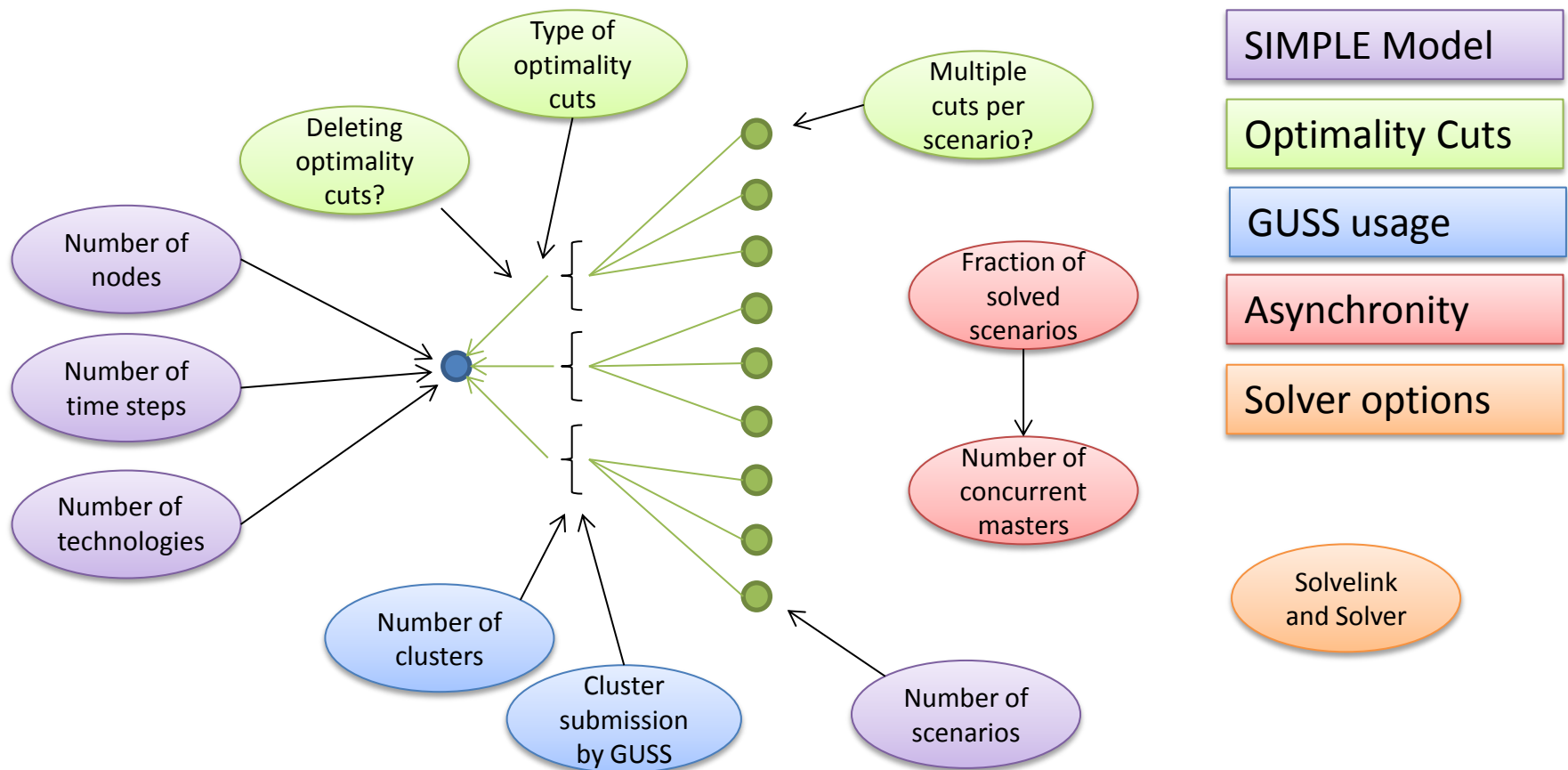
Asynchronous Model



- New Master can be started as soon as a **fraction of scenarios are solved**, new iteration has only **partial information** therefore the number of iterations increases [Linderoth 2003]
- Trade-off between more iterations and parallelisation



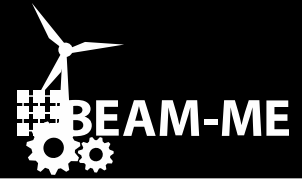
# Algorithm parameters



- Large number of parameters to optimize performance of Enhanced Benders Algorithm making **systematic testing necessary**



# Computational test results



Method	Clustercut	Multicut	Multicut + TR	MC + TR + GUSS
Iterations	138	77	54	61
Time to solve	5:54 h	4:42 h	3:14 h	2:51 h
CPU load max	300 %	449 %	466 %	134 %
CPU load avg.	66.9 %	70.8 %	56.2 %	18.6 %
RAM usage max	0.58 GB	0.58 GB	0.48 GB	0.67 GB
RAM usage avg.	0.11 GB	0.16 GB	0.11 GB	0.26 GB

- Model building time **limiting constraint**, models are solved faster than generated therefore low average CPU load ( 100 % equals 1 core out of 32)
- Exchange of information between GAMS and solver can be accelerated by using **shared memory** and running GAMS and CPLEX in **different threads**, this comes at the cost of increased memory demand



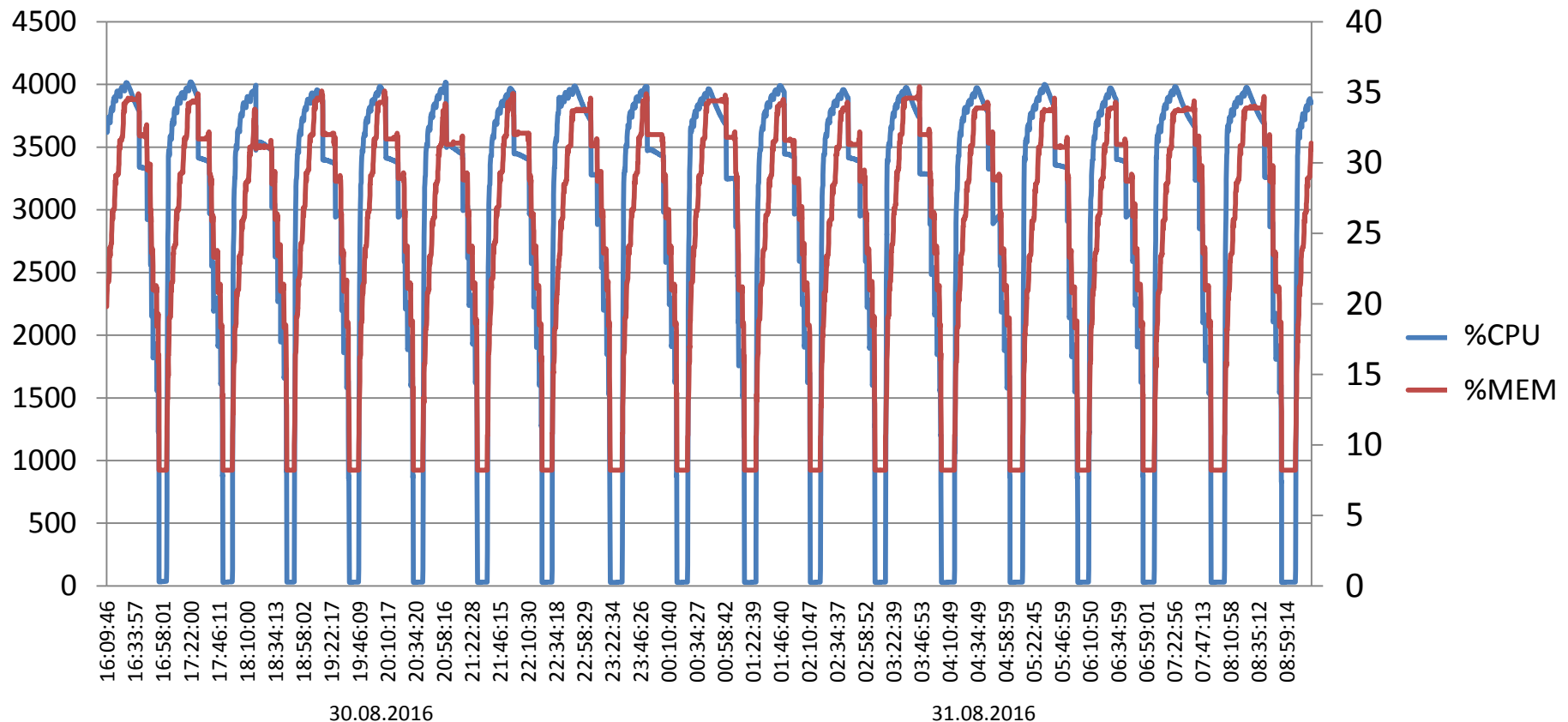
# Challenges and possible improvements

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- **Reducing computational overhead** by using methods which simplify or reduce model building time (Usage of GUSS, externalizing model building)
- **Balancing CPU load** for migration to high performance computing
- **Generating better cuts** - insufficient electrical generation will cause all plant types in a region to be built
- **Deleting unused cuts** in order to keep the masterproblem as small as possible while retaining important information
- **Improving the starting point** for the Trust-Region approach
- Combining stochastic optimization with **decomposition on the scenario level** (decomposition in model nodes or timesteps) in a Nested Benders approach

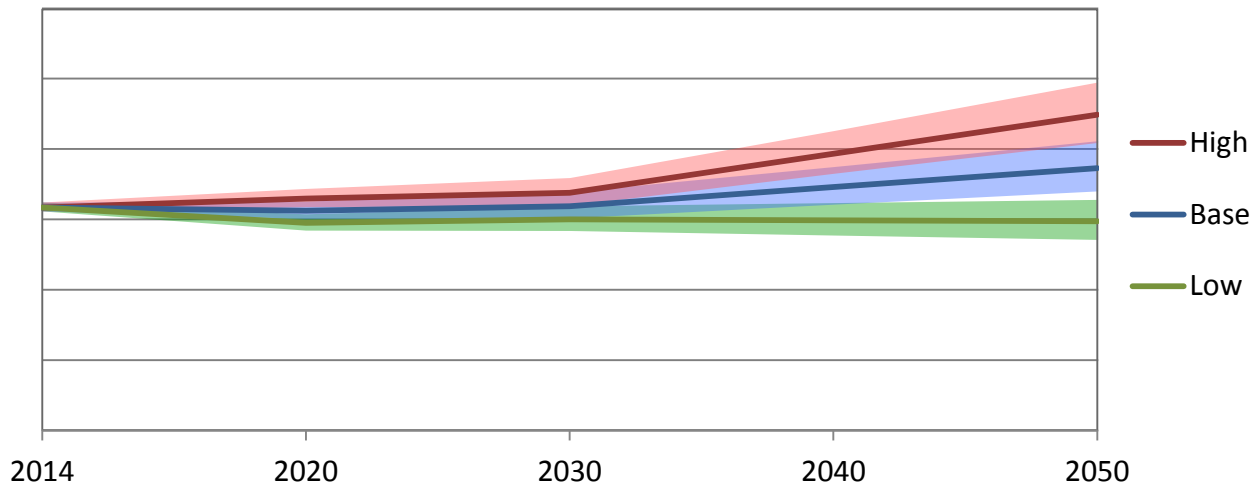




- Large scale scenario with typical **synchronous behavior**
- Gaps indicate solving master while peaks represent parallel subproblems
- High correlation between CPU and memory load indicating scalability with number of parallel processes



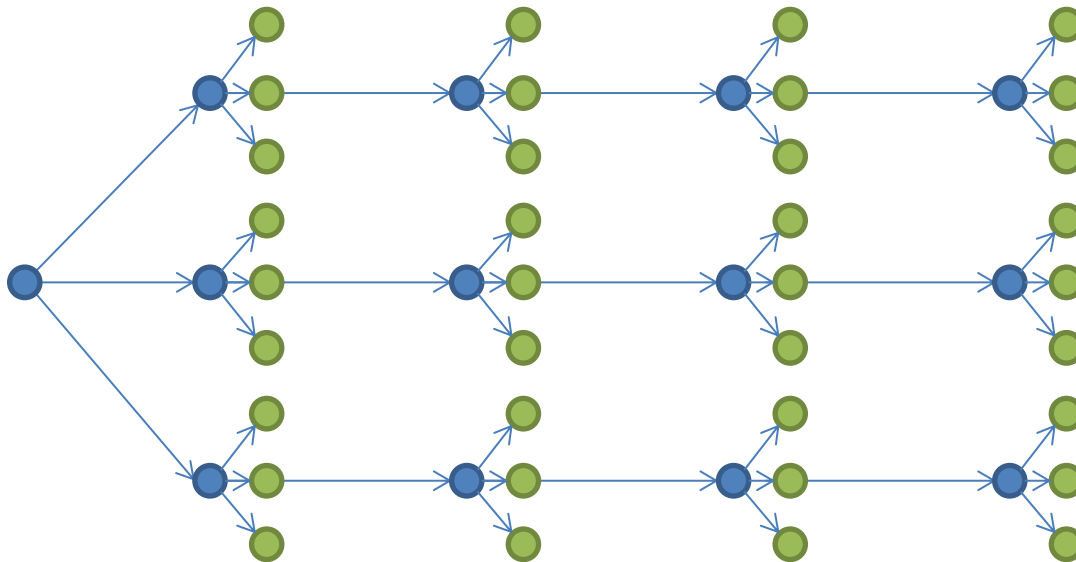
# Application to path optimization



*Motivating example  
from introduction*

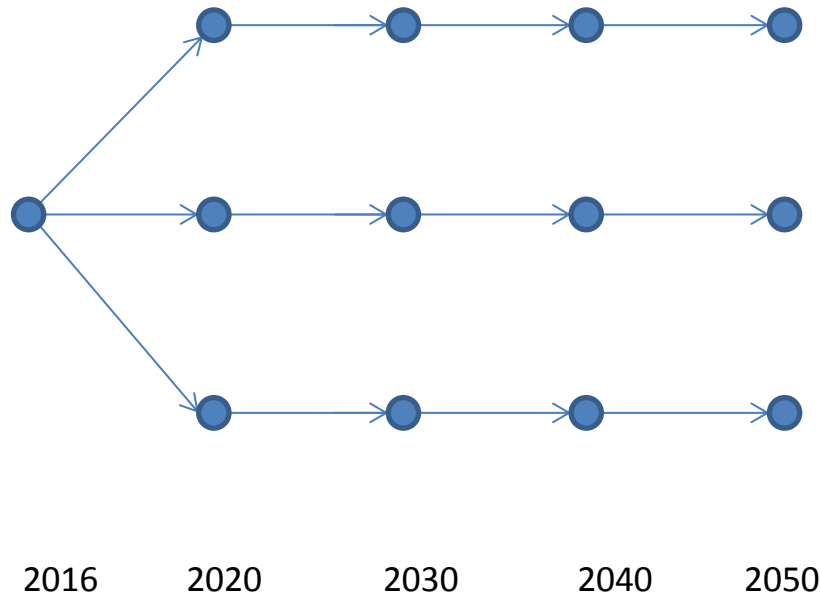
## Result:

3 Context scenario  
pathways including  
3 subscenarios each

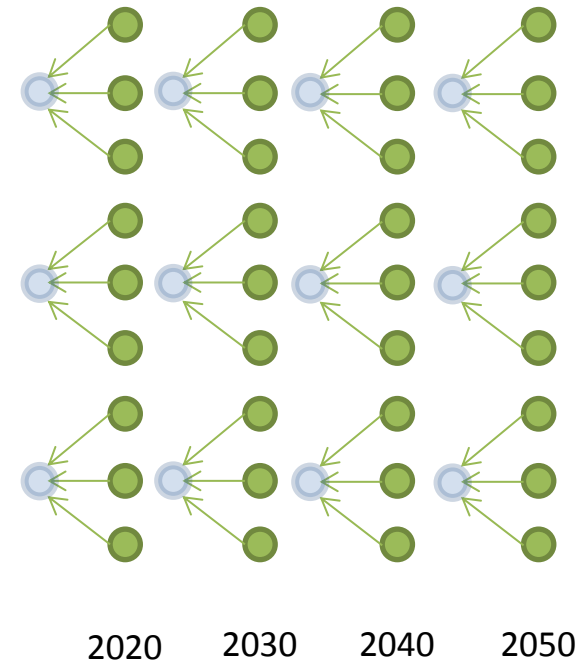




Masterproblem



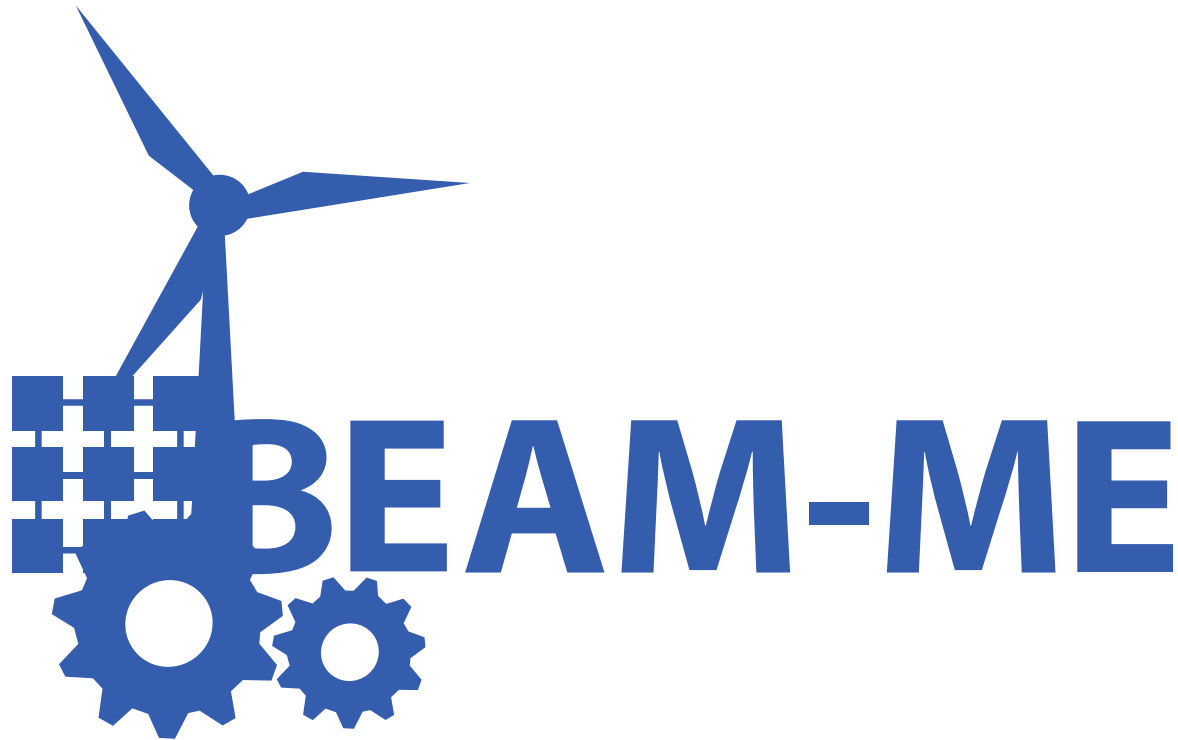
Subproblems



- Application of Benders decomposition in capacity expansion planning and economic dispatch leads to a **large number of subproblems** which can be solved in parallel



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