68th International Astronautical Congress (IAC), Adelaide, Australia, 25-29 September 2017.

IAC-17,A7,IP,5,x39014

A Study of Uncertainty Analysis for Formation Satellite Detection System in Space Science

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Abstract

A space science mission can be thought as a detection system of its scientific goals. Accuracy of positioning, timing and attitude adjusting, and margins of the payload specification are inevitable uncertainties in parameters which influence the achievement of a space mission goal. Accordingly, mission deviations need to be considered. In the early design phase of a space mission, space engineers are mainly interested whether the requirements of scientific goals are satisfied within specified margins. Thus, a quantitative analysis on how these uncertainties affect the goals would support evaluation and optimization of the mission. As an example, this paper addresses satellite formation missions which play a more and more important role in space science. Such missions provide a unique advantage in detecting high dimensional physical phenomena like the magnetic reconnection in the geo-magnetosphere while single satellites cannot. Merely satellite formation missions can distinguish spatial and temporal variations which are highly coupled parameters. However, this requires differential analysis which is hard to solve. Instead, stochastic-based simulations like the Monte Carlo method are applied where parameter uncertainties are expressed as a distribution formular. The parameter values are randomly generated by these distributions. Then, the simulations are performed multiple times. A massive number of results allows figuring out how the detection system is affected by these uncertainties. But, the resampling process of the Monte Carlo method is complex and time consuming. In this paper, the Weighted Regress Analysis (WRA) method is introduced to speed-up the computation. It approximates the detection system function by a linear function solving a series of coefficients from a set of Monte Carlo results. The drawback of this approach is that the estimation of uncertainties is less precise. To evaluate the availability and accuracy of both methods, we discuss our comparison results in this paper based on a simplified formation detection model.

Keywords: uncertainty analysis, distributed satellite system, Monte Carlo Simulation, Weighted Regression Analysis

Nomenclature

Acronyms/Abbreviations

1. Introduction

A space science mission can be thought as a detection system of its scientific goal. Performance, especially the error of the detection needs to be discussed in the early phases of a mission. Accuracy of positioning, timing and attitude adjusting, the error of the payload itself are inevitable uncertainties in parameters which will have effect on the detection of a space mission. When it comes to distributed satellite detection system, the situation becomes more complex. In the designing phase of such a space mission, engineers concern if the scientific requirement can be met with these engineering uncertainties. Furthermore, the weights of each uncertain parameter on detection should be analysed. A method which can make a quantitative analysis on how these uncertainties affect detection will provide benefits in evaluating and optimizing the mission.

2. State of the Art

Normally, a single spacecraft measures a time series of physical parameters as seen at the spacecraft position.

Spatial and temporal variability cannot be distinguished by this single spacecraft measurement.

2.1 Distributed Satellite detection system

Satellite formation mission is a solution to detect this kind of spatial-temporal physical phenomenon. The Cluster II mission from ESA launched four satellites in 2000 to form a tetrahedron formation. [1] Magnetospheric Multiscale (MMS), a NASA four-spacecraft constellation mission launched on March 12, 2015, will investigate magnetic reconnection in the boundary regions of the Earth's magnetosphere, particularly along its dayside boundary with the solar wind and the neutral sheet in the magnetic tail. [2][3] The SCOPE Mission held by JAXA is made up of five spacecrafts. It will perform formation flying observations which enable data-based study of the key space plasma processes from the cross-scale coupling point of view. [4]

2.2 Analysis in these missions

In Cluster II mission, some indicators are defined for formation analysis. A matrix $\mathbf{R}$ called volumetric tensor is defined, where $R_{jk}=\sum_{\alpha=1}^{N} r_{\alpha j} r_{\alpha k}$ and $\mathbf{r}_{\boldsymbol{\alpha}}=\left(r_{\alpha 1}, r_{\alpha 2}, r_{\alpha 3}\right)$ is the position of each satellite to the formation's geometry centre. Based on the eigenvalues a^2 、b^2 、c^2 of this volumetric tensor, the shape of formation is defined. And error analysis based on these indicators is done. 错误!未找到引用源。

In MMS mission [5] [6], engineers analysis the optimal formation design and sensitivity based on orbit parameters.

The sensitivity analysis work of Cluster II and MMS mission are mainly focus on the formation. The relation between uncertainty in engineering parameters and detection result is seldom talked.

In an engineering sense, uncertainty in the design process can come from a variety of sources. Probabilistic method which uses sensitivity factors and contours is useful for formulating direct trades of design margin. [7]

When the uncertainties are in form of random variables, three methods can be used: [9]

1) Differential analysis;
2) Variance-based methods;
3) Sampling methods.

Besides these numerical methods, in structural reliability analysis, approximation methods are been used to reduce the computing costs. [11]

In this paper, we want to build the connection between the uncertainties of the engineering parameters such as positioning, timing and attitude adjusting, the error of the payload and the error of final detection.

3. A typical formation detection use case

One of the most useful data analysis methods for data from formation mission is Timing method. This method is used to detect some structures which are moving in the space. Assume that this structure is a moving boundary and its moving vector and velocity are $(\mathbf{n}, v)$, as shown in Fig. 1.

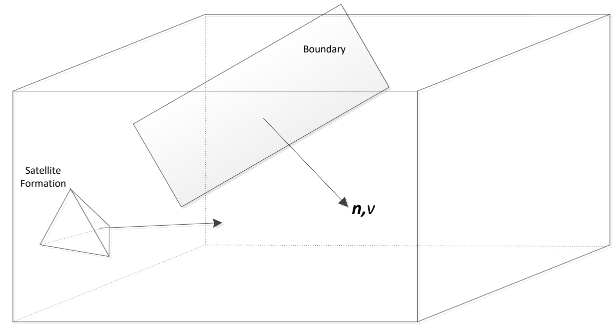

Fig. 1 Demonstration of the typical use case

For a satellite formation which consists of N spacecraft. The position of each satellite to its geometry centre is $\mathbf{r}_{\boldsymbol{\alpha}}=\left(r_{\alpha 1}, r_{\alpha 2}, r_{\alpha 3}\right)$. And the time when each satellite crosses the boundary is t_α while the time when the geometry centre crosses the boundary is t_c.

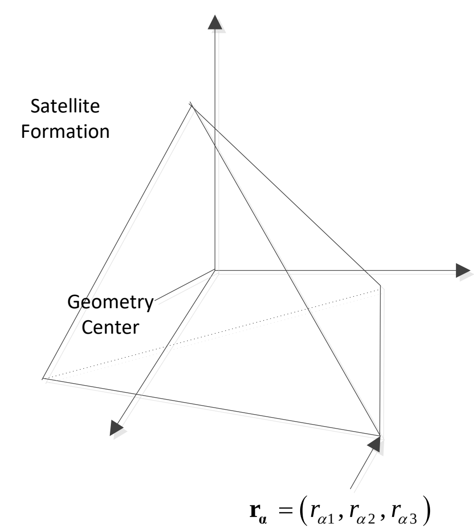

Fig. 2 Demonstration of definition

The velocity multiple the time interval equals the distance along the moving vector. The equation of this geometry problem is $v\left(t_\alpha - t_c\right)=\mathbf{n} \bullet \mathbf{r}_\alpha$.

With N spacecraft, N functions can be listed. This equation set can be solved by least squares method:

$$S = \sum_{\alpha=1}^{N} (\mathbf{n}\bullet\mathbf{r}_{\alpha} - v(t_{\alpha} - t_{c}))^2$$

Since $\mathbf{n}$ is a normal vector, use vector $\mathbf{m}$ to simplify the equation, where $\mathbf{m} = \frac{\mathbf{n}}{v}$.

Solve the equation by minimizing the value of S:

$$\frac{\partial S}{\partial t_c} = 2\sum_{\alpha=1}^{N} (\mathbf{m}\bullet\mathbf{r}_{\alpha} - (t_{\alpha} - t_{c})) = 0$$

$$\frac{\partial S}{\partial m_i} = 2\sum_{\alpha=1}^{N} r_{\alpha i} (\sum_{j=1}^{3} m_j \bullet r_{\alpha j} - (t_{\alpha} - t_{c})) = 0$$

Result $\mathbf{m} = \mathbf{Y}\mathbf{R}^{-1}$, where $\mathbf{R}$ is the volume tensor of the geometry formation information.[8]

And Y is a vector:

$$\mathbf{Y} = (y_1, y_2, y_3)^{\mathrm{T}} = \left(\sum_{\alpha=1}^{N} r_{\alpha 1}(t_{\alpha} - t_c), \sum_{\alpha=1}^{N} r_{\alpha 2}(t_{\alpha} - t_c), \sum_{\alpha=1}^{N} r_{\alpha 3}(t_{\alpha} - t_c) \right)^{\mathrm{T}}$$

In this timing method, obviously, the scientific goal is to detect the motion velocity and vector $(\mathbf{n}, v)$ of the structure which scientists are interested. The position and time are two key factors in calculation. Thus, the accuracy of timing and positioning do effect on the detection. However, how these accuracy affect is hard to distinguish since the equation is quite complex.

4. Methods

As mentioned before the basic three ways to do this kind of analysis are differential analysis, variance-based methods and sampling methods.

4.1 Differential analysis

Differential analysis calculates the rate of change in output caused by change in input by using the differential system function.

Extend the result equation in chapter3 :

$$\begin{cases} m_1 = \dfrac{R_{23}^2 y_1 - R_{12}R_{23}y_3 + R_{13}R_{22}y_3 - R_{13}R_{23}y_2 + R_{12}R_{33}y_2 -}{R_{33}R_{12}^2 - 2R_{12}R_{13}R_{23} + R_{22}R_{13}^2 + R_{11}R_{23}^2 - R_{11}R_{22}R_{33}} \\ m_2 = \dfrac{R_{13}^2 y_2 - R_{12}R_{13}y_3 + R_{11}R_{23}y_3 - R_{13}R_{23}y_1 - R_{11}R_{33}y_2 + R_{12}R_{33}y_1}{R_{33}R_{12}^2 - 2R_{12}R_{13}R_{23} + R_{22}R_{13}^2 + R_{11}R_{23}^2 - R_{11}R_{22}R_{33}} \\ m_3 = \dfrac{R_{12}^2 y_3 - R_{12}R_{13}y_2 - R_{11}R_{22}y_3 + R_{11}R_{23}y_2 - R_{12}R_{23}y_1 + R_{13}R_{22}y_1}{R_{33}R_{12}^2 - 2R_{12}R_{13}R_{23} + R_{22}R_{13}^2 + R_{11}R_{23}^2 - R_{11}R_{22}R_{33}} \end{cases}$$

Solve the differential part to the input parameter $r_{\alpha 1}$:

$$\begin{cases} \dfrac{\partial m_1}{\partial r_{\alpha 1}} = \dfrac{\mathrm{G}(\mathbf{R}) \dfrac{\partial \mathrm{F}_1(\mathbf{R})}{\partial r_{\alpha 1}} - \mathrm{F}_1(\mathbf{R}) \dfrac{\partial \mathrm{G}(\mathbf{R})}{\partial r_{\alpha 1}}}{(\mathrm{G}(\mathbf{R}))^2} \\ \dfrac{\partial m_2}{\partial r_{\alpha 1}} = \dfrac{\mathrm{G}(\mathbf{R}) \dfrac{\partial \mathrm{F}_2(\mathbf{R})}{\partial r_{\alpha 1}} - \mathrm{F}_2(\mathbf{R}) \dfrac{\partial \mathrm{G}(\mathbf{R})}{\partial r_{\alpha 1}}}{(\mathrm{G}(\mathbf{R}))^2} \\ \dfrac{\partial m_3}{\partial r_{\alpha 1}} = \dfrac{\mathrm{G}(\mathbf{R}) \dfrac{\partial \mathrm{F}_3(\mathbf{R})}{\partial r_{\alpha 1}} - \mathrm{F}_3(\mathbf{R}) \dfrac{\partial \mathrm{G}(\mathbf{R})}{\partial r_{\alpha 1}}}{(\mathrm{G}(\mathbf{R}))^2} \end{cases},$$

where:

$$\mathrm{G}(\mathbf{R}) = \left(R_{33}R_{12}^2 - 2R_{12}R_{13}R_{23} + R_{22}R_{13}^2 + R_{11}R_{23}^2 - R_{11}R_{22}R_{33} \right)$$

$$\begin{cases} \mathrm{F}_1(\mathbf{R}) = R_{23}^2 y_1 - R_{12}R_{23}y_3 + R_{13}R_{22}y_3 - R_{13}R_{23}y_2 + R_{12}R_{33}y_2 - R_{22}R_{33}y_1 \\ \mathrm{F}_2(\mathbf{R}) = R_{13}^2 y_2 - R_{12}R_{13}y_3 + R_{11}R_{23}y_3 - R_{13}R_{23}y_1 - R_{11}R_{33}y_2 + R_{12}R_{33}y_1 \\ \mathrm{F}_3(\mathbf{R}) = R_{12}^2 y_3 - R_{12}R_{13}y_2 - R_{11}R_{22}y_3 + R_{11}R_{23}y_2 - R_{12}R_{23}y_1 + R_{13}R_{22}y_1 \end{cases}$$

and

$$\frac{\partial \mathrm{G}(\mathbf{R})}{\partial r_{\alpha 1}} = \left(2R_{33}R_{12}r_{\alpha 2} - 2r_{\alpha 2}R_{13}R_{23} + 2R_{22}R_{13}r_{\alpha 3} + 2r_{\alpha 1}R_{23}^2 - 2r_{\alpha 1}R_{22}R_{33} \right)$$

$$\begin{cases} \dfrac{\partial \mathrm{F}_1(\mathbf{R})}{\partial r_{\alpha 1}} = \left(R_{23}^2(t_{\alpha} - t_c) - r_{\alpha 2}R_{23}y_3 + r_{\alpha 3}R_{22}y_3 - r_{\alpha 3}R_{23}y_2 + r_{\alpha 2}R_{33}y_2 - R_{22}R_{33}(t_{\alpha} - t_c) \right) \\ \dfrac{\partial \mathrm{F}_2(\mathbf{R})}{\partial r_{\alpha 1}} = \begin{pmatrix} 2R_{13}y_2 r_{\alpha 3} - (R_{13}r_{\alpha 2} + R_{12}r_{\alpha 3})y_3 + 2r_{\alpha 1}R_{23}y_3 - (r_{\alpha 3}y_1 + (t_{\alpha} - t_c)R_{13})R_{23} \\ + r_{\alpha 2}R_{33}y_2 - R_{22}R_{33}(t_{\alpha} - t_c) - 2r_{\alpha 1}R_{33}y_2 + (r_{\alpha 2}y_1 + (t_{\alpha} - t_c)R_{12})R_{33} \end{pmatrix} \\ \dfrac{\partial \mathrm{F}_3(\mathbf{R})}{\partial r_{\alpha 1}} = \begin{pmatrix} 2r_{\alpha 2}R_{12}y_3 - (R_{13}r_{\alpha 2} + R_{12}r_{\alpha 3})y_2 - 2r_{\alpha 1}R_{22}y_3 + 2r_{\alpha 1}R_{23}y_2 \\ - (r_{\alpha 2}y_1 + (t_{\alpha} - t_c)R_{12})R_{23} + (r_{\alpha 3}y_1 + (t_{\alpha} - t_c)R_{13})R_{22} \end{pmatrix} \end{cases}$$

It can be figured out that the differential result of $r_{\alpha 1}$ is related to the formation matrix $\mathbf{R}$ and detect time point t_{α}. Thus, this result is non-linear function and hard to solve which means it cannot help in practical engineering.

4.2 Monte Carlo me

Monte Carlo method is one of the sampling methods. It involves three steps:

1) Generating uncertain input as sample set;
2) Calculating the output with each sample input;
3) Analysing the statistical result of all output.

4.3 Linear approximated method

There is no doubt that traditional methods can draw a specific result of this problem. However, in the early design phases, what may more important is the probability that this mission may fail with these uncertainties in engineering parameters. Furthermore, how different uncertainty weights in the system.

Approximation methods provide advantage in time cost and used in structural design. One of them is Weighed Regression Analysis (WRA) method.

In this method, assume that the function of formation detection system can be approximated to a linear one:

$$G_L(\mathbf{X}) = a_0 + \sum_{j=1}^{n} a_j X_j$$

where $\mathbf{X} = [X_1, X_2, ..., X_j]$ are the input parameters of the system.

In the Monte Carlo method result, $X_D = \begin{bmatrix} X_1(1)X_2(1)...X_j(1) \\ X_1(2)X_2(2)...X_j(2) \\ .... \\ X_1(m)X_2(m)...X_j(m) \end{bmatrix}$ are the input, $\mathbf{G} = [G_1, G_2, ..., G_m]$ are the outputs.

The coefficients can be solved. $\mathbf{a} = (\mathbf{X}_D^T \mathbf{X}_D)^{-1} \mathbf{X}_D^T \mathbf{G}$

Considering the sampling results have different weighted. The weighted matrix is built as

$$\mathbf{w}(\mathbf{X}) = \begin{bmatrix} w(\mathbf{X}_1) & 0 & .... & 0 \\ 0 & w(\mathbf{X}_2) & .... & 0 \\ ..... & & & \\ 0 & 0 & .... & w(\mathbf{X}m) \end{bmatrix},$$ where

$$w(\mathbf{X}_i) = \exp[-\frac{|G(\mathbf{X}_i)| - g_{worst}}{g_{worst}}]$$. [11]

$$g_{worst} = \max |G(\mathbf{X}_i)|, i = 1, 2, ..., m$$

By this linear approximation, the relationship of input uncertainty and output detection error can be built.

5. Simulation model and parameters

Use the typical use case mentioned in chapter 3. The basic simulation model is built. And the parameters in Monte Carlo Simulation method is also been defined.

5.1 Basic simulation Model

The distribution satellite system is set as a regular tetrahedron formation. The position of each satellite to its geometry centre is shown in Table 2 .

The boundary which is assumed to be detected is moving along the vector $\mathbf{n}$ in velocity v .

Table 1. Parameter of assumed moving boundary

$\mathbf{n}$ x	y(km)	z(km)	v(km/s)
0.455842	0.569803	0.683763	0.01

Table 2. The position of each satellite in the formation

	Position to the geometry centre x(km)	y(km)	z(km)
Satellite1	1	0	$-\frac{1}{\sqrt{2}}$
Satellite2	-1	0	$-\frac{1}{\sqrt{2}}$
Satellite3	0	1	$\frac{1}{\sqrt{2}}$
Satellite4	0	-1	$\frac{1}{\sqrt{2}}$

The sampling frequency of the payload is set to be 1000 times/sec.

Based on these assumptions, without considering any uncertainty from the input parameters, the simulated result from this system is shown in Table 3.

Table 3. Calculated result without any uncertain input

Calculated $\mathbf{n}$ x	y(km)	z(km)	Calculated v(km/s)
0.455841	0.569801	0.683766	0.010000

5.2 Monte Carlo Simulation parameters

To analysis the influence of the uncertainties in input, some uncertain errors are added to parameters as input for Monte Carlo sampling simulation. The details are shown in Table 4. Parameters T_1, T_2, T_3, and T_4 are the timing accuracy for satellite 1-4 while parameters P_1, P_2, P_3, and P_4 are the positioning accuracy for satellite 1-4. According to the definition of normal distribution, these uncertainty parameters mean that the timing accuracy of each satellite is less than 0.01s in 99.7% situations and the positioning accuracy of each satellite is less than 0.001km in 99.7% situations.

In each sampling progress of Monte Carlo Method, the random generator will generate a series of error which is obeyed the distribution mentioned above. The deviation between detected velocity v' and the actual velocity v is chosen as the result indicator of Monte Carlo Method.

Table 4. Parameters of Monte Carlo Simulation input

Input Parameter	Distribution Type	Mean Value	σ (Standard Variation)
T_1	Normal	0	0.01
T_2	Normal	0	0.01
T_3	Normal	0	0.01
T_4	Normal	0	0.01
P_1	Normal	0	0.001
P_2	Normal	0	0.001
P_3	Normal	0	0.001
P_4	Normal	0	0.001

6. Result

The results of Monte Carlo method and WRA method are shown in this chapter. The validation and comparison are also shown.

6.1 Monte Carlo result

The statistical analysis of this sampling result is shown in Fig. 3.

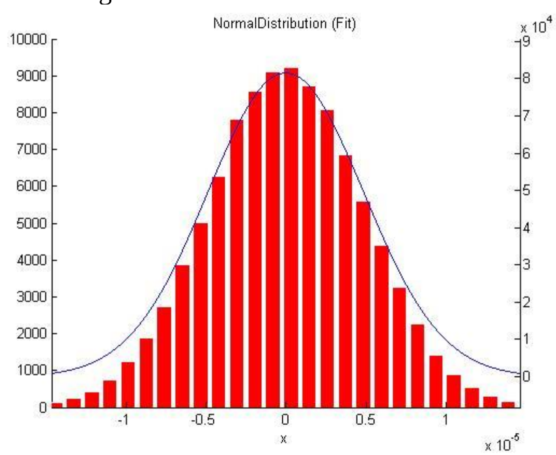

Fig. 3 Monte Carlo Method result

The red bars are the frequency histogram statistical result. The Blue curve is the normal fit of the result data.

6.2 Result from Weighted Regressed Analysis method

Based on the WRA method mentioned before and formal Monte Carlo Method result, the arguments of approximated linear function of this system is calculated, shown in Table 5.

Table 5. Coefficients from WRA Method

Coefficient for each parameter	Value
A_{t1}	1.37948 e-06
A_{t2}	4.69637 e-05
A_{t3}	-5.26655 e-05
A_{t4}	4.31453 e-06
A_{p1}	9.45529 e-05
A_{p2}	3.21143 e-03
A_{p3}	-3.60108 e-03
A_{p4}	2.95046 e-04

The linear approximated linear function is:

$$G(\mathbf{X}) = A_{t1}T_1 + A_{t2}T_2 + \ldots + A_{p4}P_4$$

By this analysis, it can be figured out that the uncertainty of different parameter has different contribution to the uncertainty of final measurement. In this situation, the positioning accuracy of satellite 2 and 3 in the formation has more weighted in the progress.

6.3 Comparison with Monte Carlo Simulation

Monte Carlo method is a numerical sampling method of which the result can be used to evaluate the approximated method. To evaluate the WRA method coefficients, another set of input is set, as shown in Table 6.

Table 6. Validation input parameters

Input Parameter	Distribution Type	Mean Value	Variation Value
T_1	Normal	0	0.01
T_2	Normal	0	0.02
T_3	Normal	0	0.03
T_4	Normal	0	0.04
P_1	Normal	0	0.004
P_2	Normal	0	0.003
P_3	Normal	0	0.002
P_4	Normal	0	0.001

From the approximated function, the mean value and variation of result are in Table 7.

Table 7. Comparison of different methods

Result from Weighted Regression Method		Statistic Result from Monte Carlo Method	
Mean Value	Variation	Mean Value	Variation
0	1.218 e-05	1.732 e-08	1.217 e-05

The Fig. 4 is the comparison of the result from Weighted Regression method and Monte Carlo Simulation method (10^6 samples). The red bars are the frequency histogram of Monte Carlo method result. And the Blue curve is the probability density function of weighted regression method.

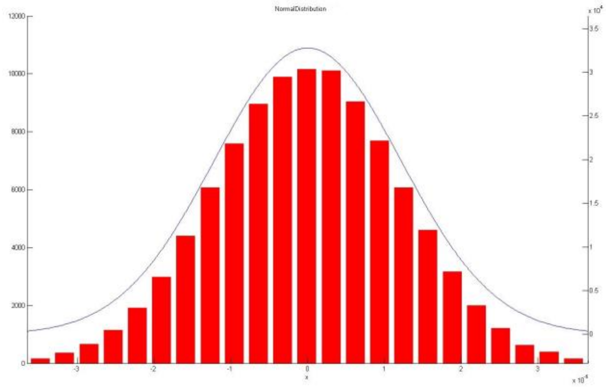

Fig. 4. Frequency histogram from Monte Carlo Method and normal distribution fit.

6.4 Further result on WRA method

In the former result from WRA method, the positioning accuracy of satellite 2 and 3 play more weighted role. To verify this conclusion, two more set of input are set as Table 8 and Table 9.

Table 8. Validation input parameters set A

Input Parameter	Distribution Type	Mean Value	Variation Value
T_1	Normal	0	0.01
T_2	Normal	0	0.02
T_3	Normal	0	0.03
T_4	Normal	0	0.04
P_1	Normal	0	0.001
P_2	Normal	0	0.003
P_3	Normal	0	0.002
P_4	Normal	0	0.001

Table 9. Validation input parameters set B

Input Parameter	Distribution Type	Mean Value	Variation Value
T_1	Normal	0	0.01
T_2	Normal	0	0.02
T_3	Normal	0	0.03
T_4	Normal	0	0.04
P_1	Normal	0	0.004
P_2	Normal	0	0.001
P_3	Normal	0	0.002
P_4	Normal	0	0.001

In set A, the positioning accuracy of satellite 1 is improved; its variation value is decreased to 0.001. While in set B, the positioning accuracy of satellite 2 is improved; its variation value is decreased to 0.001.

The result of these two set of input are shown in .

Table 10. Result from input set A and B

Result from set A		Result from set B	
Mean Value	Variation	Mean Value	Variation
0	1.217 e-05	0	8.113 e-06

7. Discussions

7.1 Comparison of time efficiency

As mentioned before, Monte Carlo method is a time consuming method. In this simulation, for 10^6 samples, the Monte Carlo method costs more than two seconds while WRA method costs only 0.6 second for its initialization. After initialized, the WRA method cost only 44718 ns.

Table 11 Time efficiency comparison

Time of Monte Carlo Method	Time of WRA method initialized	Time of WRA after initialized
2194745130 ns	639984722 ns	44718 ns

The computer configuration is listed in **错误!书签自引用无效。**.

Table 12 Computer configuration

Device name	Type
CPU	Intel(R) Core(TM)i7-2640M @2.8GHz
RAM	8.00GB
OS	64-bit Windows 7
IDE	Eclipse neon

It can be figured out that WRA method provides advantage in time consuming.

7.2 Limitation of WRA method

When it comes to evaluate the accuracy of the detected vector, the limitation is normally the angle between detected one and the true one. The result is shown in Fig. 5. The WRA result is the blue curve in the left.

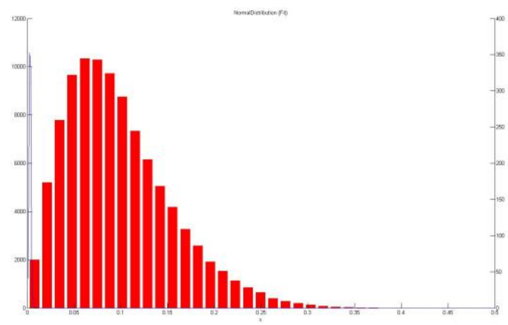

Fig. 5. Angle accuracy result

The WRA method is not suitable in this situation. The reason is that the angle is a scalar of which the minimum value is 0 and distribution is asymmetric. The WRA try to fit the result by normal distribution which is symmetrical to its mean value.

For further study, we try to extend the WRA method to folded normal distribution which may be used for this situation.

8. Conclusions

Distributed satellite system has advantage in detecting space phenomenon which is varying temporal and spatial. It brings more challenges for engineering design. How to evaluate and optimize the performance of a distributed satellite system with kinds of uncertainties in engineering parameters needs to be talked in early design phase.

The traditional numerical method cannot fit this requirement. Monte Carlo method is time consuming. And it has disadvantage in building the direct relationship between uncertainty in parameters and detection performance.

Thus, Weighted Regression Analysis which is a method used in structural design is introduced to solve this problem. By calculating the coefficients of a linear function, the system is approximated expressed. And the weight of each uncertainty is clearly shown.

From the example in this paper, this WRA method has advantage in time efficiency. The initialization time is 3 times faster than Monte Carlo method. After initialization, the time consuming can be ignored compared to Monte Carlo method. This method can help engineers to find out the key uncertainty of parameter in this mission and optimize these uncertainties much easier.

The limitation of this method is that it cannot fit the problem when the performance is a scalar value. The improvement of this will be the next work.

Acknowledgements

The financial support provided by the UCAS (UCAS[2015]37) Joint PhD Training Program is acknowledged.

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