ROBUST TWO DEGREE OF FREEDOM VEHICLE STEERING CONTROL SATISFYING MIXED SENSITIVITY CONSTRAINT

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Abstract

Robust steering control is used here for improving the yaw dynamics of a passenger car. A specific two degree of freedom control structure is adapted to the vehicle yaw dynamics problem and shown to robustly improve performanee. The design study is based on six operating conditions for vehicle speed and the coefficient of friction between the tires and the road representing the operating domain of the vehicle. The relevant design specifications are formulated as attaining Hurwitz stability and a mixed sensitivity frequeney domain bound. Simple, and therefore, easily implementable controller transfer functions with two design parameters are chosen for the two steering eontroller degrees of freedom. Using the parameter space method, the design specifications are mapped into the plane of controller parameters. The effectiveness of the final design is demonstrated using simulations.

1 Introduction

Unexpected yaw disturbances caused by unsymmetrical car dynamics perturbations (unilateral loss of tire pressure or braking on unilaterally icy road for example) may result in dangerous yaw motions of an automobile. The driver who should react quickly and precisely in such dangerous situations is impaired by his natural delay time with the result being instability. Improvement of automobile yaw dynamics by active control is, thus, a subject of active research. The use of a steer-by-wire system, where the steering controller and driver steering commands are superimposed and sent to the steering actuator, is assumed in this paper. The steering control should be robust with respect to large variations in longitudinal speed, payload and road adhesion along with uncertainty due to unmodeled dynamics. Comfortable operation for the driver and passengers should also be sustained.

A steering controllcr structure that has the capability of effectively satisfying the requirements outlined above was successfully applied to automobile yaw dynamics improvement in [4]. The same two degree of freedom steering controller structure is used here. In contrast to the abovementioned reference, however, low speed operation results are also reported here and the design specification is formulated as a mixed sensitivity type frequency domain bound. The final design, thus, satisfies robust performance (the non-standard H_∞ problem). The organization of the paper is as follows. The linearized single track vehicle yaw dynamics model being used for control design and analysis is introduced in Section 2 where the numerical data being used is also given. The steering controller design specifications are presented in Section 3. The two degree of freedom steering control architecture being used is presented in Section 4. The main eontribution of the paper is given in Section 5. Design satisfying a mixed sensitivity frequency domain bound is carried out in controller parameter space. Linear simulation results are also given in this section to demonstrate the effectiveness of the proposed approach. The paper ends with a summary of the main results and recommendations for future work in Section 6.

2 Vehicle model and numerical data

The car model which is used for the investigations in this paper is the classical linearized single track model shown in Figure 1. Its major variables and geometric parameters are

$F_f\,(F_r)$: Lateral wheel force at front (rear) wheel
r : Yaw rate
β : Chassis side slip angle at center of gravity (CG)
v : Magnitude of velocity vector at CG (v>0,dv/dt=0)
$l_f\,(l_r)$: Distance from front (rear) axle to CG
δ_f : Front wheel steering angle
m : Vehicle mass

J : Moment of inertia w.r.t. vertical axis through the CG

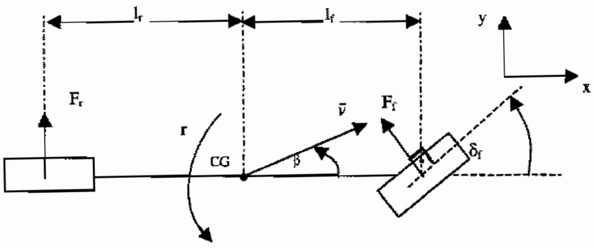

Figure 1: Single track model for car steering.

For small steering angle δ_f and small side slip angle β, the linearized equations of motion are (see [1])

$$\begin{bmatrix} mv(d\beta/dt + r) \\ ml_f l_r \, dr/dt \end{bmatrix} = \begin{bmatrix} F_f + F_r \\ F_f l_f - F_r l_r \end{bmatrix} \qquad (1)$$

The tire force characteristics are linearized as

$$F_f(\alpha_f) = \mu c_{f0} \alpha_f \, , \quad F_r(\alpha_r) = \mu c_{r0} \alpha_r \qquad (2)$$

where c_{f0}, c_{r0} are the tire cornering stiffnesses, μ is the road adhesion factor and α_f and α_r are the tire side slip angles given by

$$\alpha_f = \delta_f - \left(\beta + \frac{l_f}{v} r \right) \qquad (3)$$

$$\alpha_r = -\left(\beta - \frac{l_r}{v} r \right) \qquad (4)$$

The transfer function from the front wheel steering angle δ_f to the yaw rate r can be computed from (1)-(4) as

$$G(s) = \frac{r(s)}{\delta_f(s)} = \frac{b_0 + b_1 s}{a_0 + a_1 s + a_2 s^2} \qquad (5)$$

with

$b_0 = c_f c_r (l_f + l_r) v$

$b_1 = c_f l_f m v^2$

$a_0 = c_f c_r (l_f + l_r)^2 + (c_r l_r - c_f l_f) m v^2$

$a_1 = (cf(J + l_f^2 m) + c_r (J + l_r^2 m)) v$

$a_2 = J m v^2$

The steady state gain of the nominal single track model is

$$K_n(v) = \lim_{s \to 0} G(s) \Big|_{\mu = 1} \qquad (6)$$

at the chosen longitudinal speed v and at nominal friction coefficient which is taken as $\mu = 1$ for dry road conditions here.

The vehicle model data used here corresponds to a mid-sized passenger car. The nominal values of the variables in the linearized single track model are l_f=1.25 m, l_r=1.32 m, m=1296 kg, J=1750 kgm^2, c_{f0}=84243 N/rad and c_{r0}=95707 N/rad. Uncertainty in these parameters enters the design process in this paper indirectly through the choice of the low frequency part of thc weight for complementary sensitivity.

3 Design specifications

The two variables exhibiting the largest variation during operation are the longitudinal speed v and the mass m of the vehicle. The tire cornering stiffnesses c_f and c_r can also exhibit large variations due to variations in friction coefficient μ between the road and the tires. The additional uncertainty in the cornering stiffnesses due to uncertain parameters like normal force, longitudinal aceeleration, tire pressure and temperature are captured in uncertainty in c_{f0} and c_{r0} here.

The longitudinal velocity v is treated as a varying parameter here rather than an uncertain one as it can be easily measured and used for gain scheduling. It is assumed to vary between a minimum value of 10 m/s and a maximum value of 50 m/s during operation. Gain scheduling will be necessary to softly switch on the controller with increasing speed. The speeds of v=10 m/s, v=30 m/s and v=50 m/s are investigated in this paper. The assumed operating domain in terms of μ and v is displayed in Figure 2. The maximum value of μ is unity (dry road) at all speeds in this figure while its minimum value rises linearly from 0.2 (icy road) at low speeds to 0.8 (wet road) at high speeds. The control is switched on only above v=10 m/s as the driver can easily take care of yaw disturbances occuring at lower speeds. The maximum speed of the vehicle is v=50 m/s.

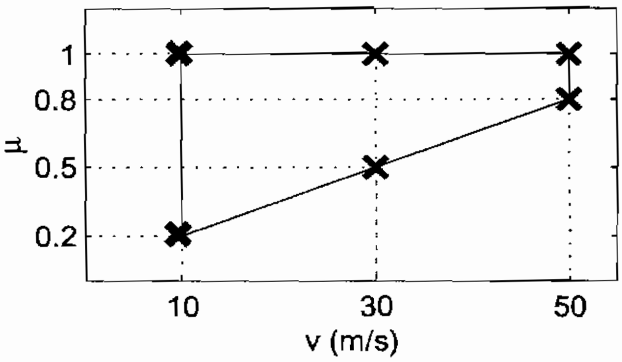

Figure 2: Operating domain.

The six operating conditions considered in design are all at the boundary of the operating domain and are marked with crosses in Figure 2. The aim in steering controller design is to make sure that first stable operation and then improved yaw dynamics is achieved for all operating eonditions and all possible values of the uncertain parameters. The improved yaw dynamics corresponds to good disturbance rejection properties where the possible disturbances include the effect of yaw disturbance torques induced by side wind forces and μ-split braking. In the following sections, a two degree of freedom steering controller is designed and shown to effectively achieve the desired goals.

4 Two degree of freedom steering control

The two degree of freedom control architecture used here is based on the disturbance observer method which is used to achieve the two aims of being insensitive to modeling error and the rejection of disturbances [10], [12]. This method has been used successfully in a variety of motion control applications including high speed direct drive positioning in [8], friction compensation in [5] and automated vehicle path following in [3]. The two degree of freedom steering controller is shown in Figure 3. The steering controller design comprises of designing the two filters $G_n(s)$ and $Q(s)$. $G_n(s)$ is the desired yaw dynamics model whose input–output behavior is desired to be followed and $Q(s)$ is a unity d.c. gain low pass filter. The steering controller of Figure 3 forces the yaw dynamics seen between input δ_s and output r to follow G_n within the bandwidth of the filter Q. This is achieved, in essence, by passing the error between the actual input and the "should be" input (based on the output and our knowledge of the desired model) through a positive feedback loop.

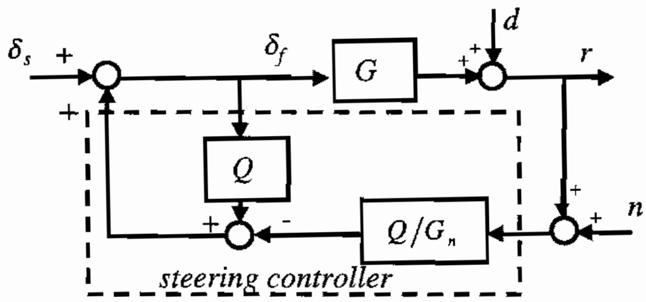

Figure 3: Two degree of freedom steering controller.

The relative degree of the unity d.c. gain low pass filter Q is ehosen to be at least equal to the relative degree of G_n for causality of Q/G_n. The loop gain of the steering controlled system in Figure 3 is

$$L = \frac{GQ}{G_n(1-Q)} \tag{7}$$

The model regulation, disturbance rejection (i.e. sensitivity S) and sensor noise rejection (i.e. complementary sensitivity T) transfer functions are given by

$$\begin{aligned} \frac{r}{\delta_s} &= \frac{G_n G}{G_n(1-Q)+GQ} \\ S \equiv \frac{r}{d} &= \frac{1}{1+L} = \frac{G_n(1-Q)}{G_n(1-Q)+GQ} \\ T \equiv -\frac{r}{n} &= \frac{L}{1+L} = \frac{GQ}{G_n(1-Q)+GQ} \end{aligned} \tag{8}$$

It is obvious from (8) that Q must be a unity gain low pass filter for achieving the abovementioned characteristics. This choice will result in $r/\delta_s \rightarrow G_n$, $r/d \rightarrow 0$ at low frequencies where $Q \rightarrow 1$ and $r/n \rightarrow 0$ at high frequencies where $Q \rightarrow 0$ as is desired. Disturbance observer design is thus shaping of the filter Q to satisfy the design objectives. There is a limitation on the bandwidth of Q, however, due to sensor noise rejection at sensor noise frequencies and due to stability robustness in the presence of unstructured uncertainty. The application of the Nyquist stability criterion results in the stability robustness requirement

$$|Q| < \left|\frac{1}{\Delta_m}\right|, \quad for\ \forall \omega \tag{9}$$

in the presence of unstructured multiplicative model uncertainty Δ_m (Kempf and Kobayashi, 1999). This stability robustness constraint will be posed as a bound on weighted complementary sensitivity in the next section on design. The disturbance observer design requirements specified in terms of the filter Q are summarized in Figure 4.

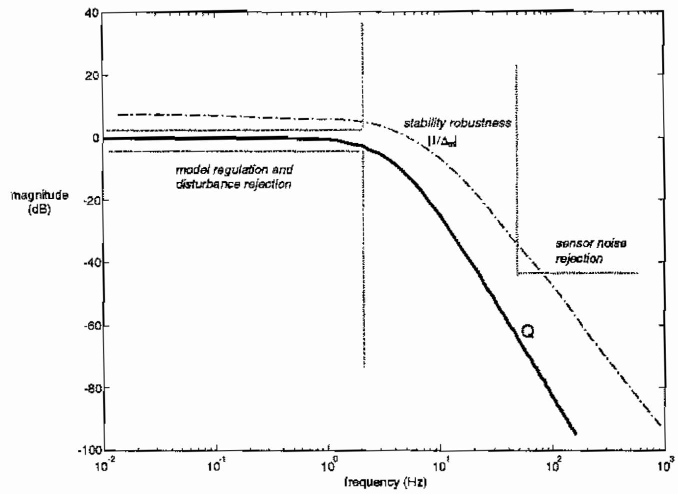

Figure 4: Q filter design specifications.

5 Parameter space design and simulations

The desired yaw dynamics model is chosen as a first order system here given by

$$G_n(s) = \frac{K_n(v)}{\tau_n s + 1} \tag{10}$$

where $K_n(v)$ is according to (6). Note that $G_n(s)$ or the parameters in $G_n(s)$ can be easily adjusted for other forms of desired yaw dynamics, to also satisfy desired handling and ride dynamics for example. The Q filter is chosen to be of the form

$$Q(s) = \frac{1}{\tau_Q s + 1} \tag{11}$$

The two free controller parameters τ_n and τ_Q are tuned in the design effort to meet the mixed sensitivity requirement

$$|W_S S| + |W_T T| < 1 \quad for\ \ \forall \omega \tag{12}$$

The sensitivity function weight W_S and the complementary sensitivity function weight W_T are chosen as in reference [4] where the nominal performance (weighted sensitivity specification) and robust stability (weighted complementary sensitivity specification) problems were treated separately. This, of course, leads to a conservative design when the aim is meeting the robust performance criterion of (12), also called the *nonstandard problem* in H_∞ optimization. One distinction is that a controller that satisfies (12) is searched for here rather than a controller that minimizes the left hand side of (12) as in the conventional H_∞ approach. The original weights in reference [4] were scaled down by a factor of 1.5 to make problem (12) solvable for the combination of all six operating conditions considered here. Note that even after this down-scaling, problem (12) here is generally more restrictive than the separate specifications treated in the abovementioned reference. It is also well known that the solutions of the nonstandard problem $|W_S S|+|W_T T|<1$ (the standard problem $|W_S S|^2+|W_T T|^2<1/2$) and the separate, simultaneous minimization $|W_S S|<1 \cup |W_T T|<1$ can be significantly apart from each other (see [11]).

The weights used in design are

$$W_S(s) = \frac{0.6667s+8.4}{1.8s+1.26} \tag{13}$$

$$W_T(\omega) = \max\left\{ \left|\frac{3.333s+12.5667}{s+188.5}\right|_{s=j\omega},\ \left|\frac{0.0854s^2+3.7954s+1.8144}{s^2+9.006s+17.6494}\right|_{s=j\omega} \right\} \tag{14}$$

The complementary sensitivity weight in (14) is designed to penalize parametric uncertainty at low frequencies and unmodeled dynamics uncertainty at high frequencies.

The design approach is based on mapping the frequency domain constraint (12) with weights given in (13) and (14) into the plane of controller parameters τ_n and τ_Q. More detailed information on the solution procedure used can be found in references [9] and [6-7]. This procedure is repeated for all six of the marked operating conditions in Figure 2. The individual solution regions and the final solution region obtained by intersection in controller parameter plane of all regions and the region for Hurwitz stability are shown in Figures 5 and 6, respectively. The final design point satisfying (12) for all six operating conditions is chosen as τ_n=0.16 sec and τ_Q=0.07 sec and is marked with a cross in Figure 6. The $|W_S S|+|W_T T|$ frequency domain plot shown in Figure 7 for this controller at all six operating conditions shows that constraint (12) is satisfied by this design.

A linear simulation study is performed next to assess the time domain performance that is achieved. Steering wheel and yaw disturbance step inputs are the two simulation maneuvers that were investigated. The steering wheel step input is normalized by the gain $K_n(v)$ for dry road (μ=1) in the simulations for easier comparison of the results. The simulation results shown in Figures 8-10 demonstrate the achievement of excellent disturbance rejection and good steering command tracking at all six operating points.

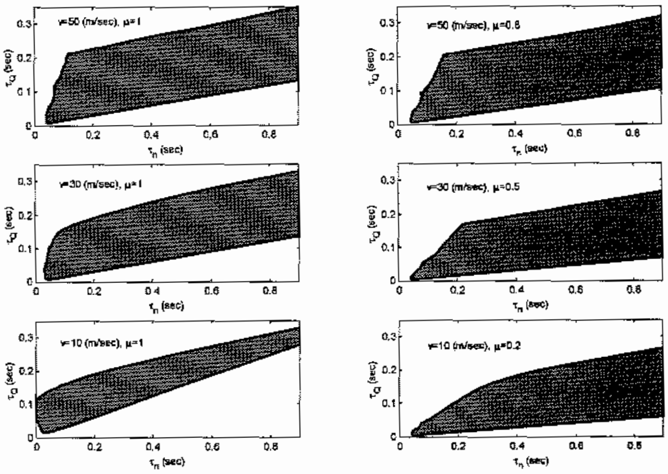

Figure 5: Individual solution regions at the six operating conditions.

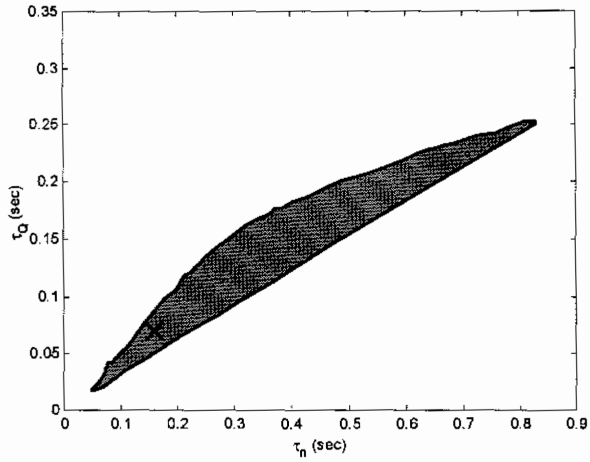

Figure 6: Solution region satisfying constraints at all six operating points.

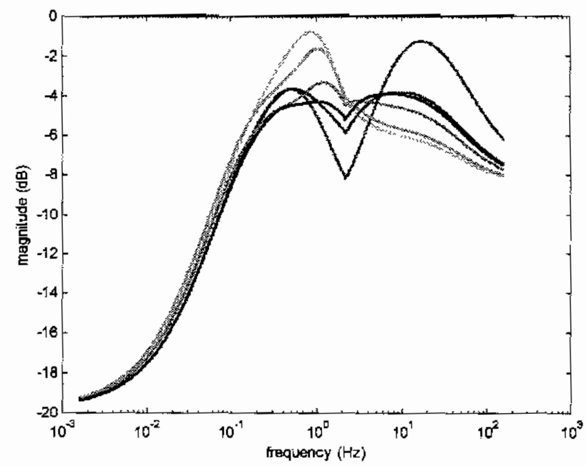

Figure 7: $|W_S S|+|W_T T|$ for final design at all six operating conditions.

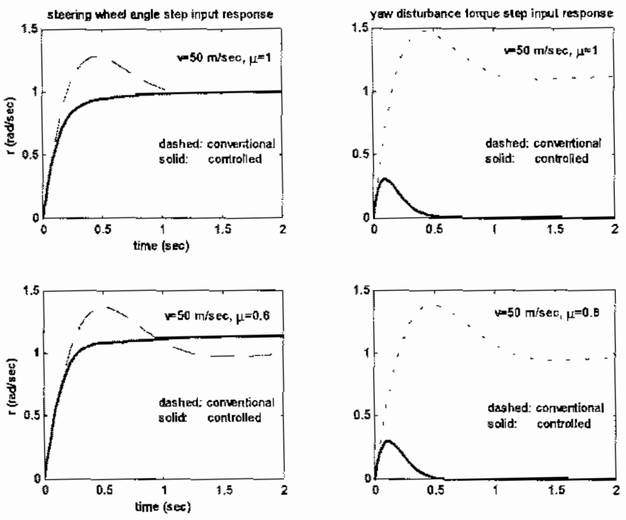

Figure 8: Simulation results at v=50 m/sec.

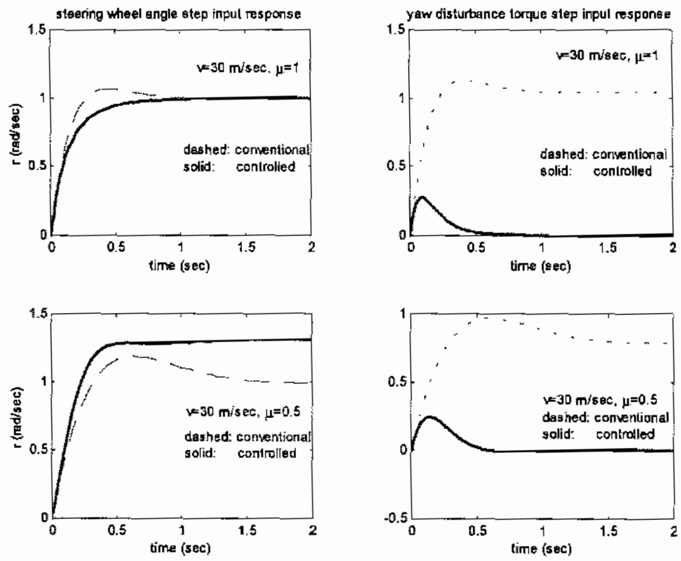

Figure 9: Simulation results at v=30 m/sec.

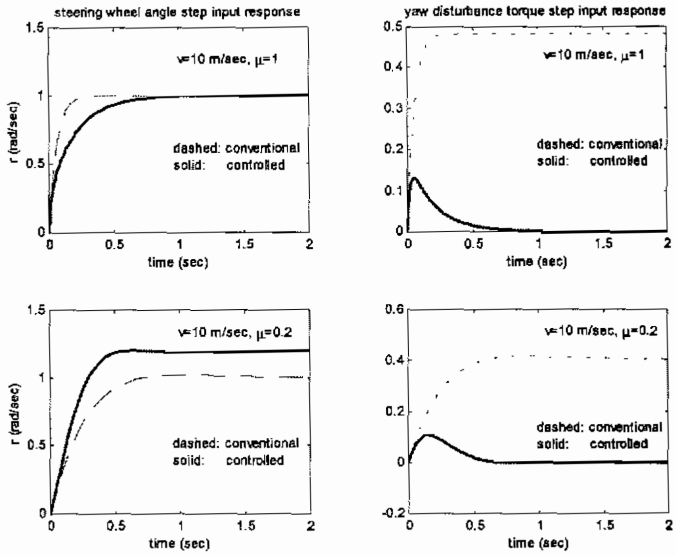

Figure 10: Simulation results at v=10 m/sec.

6 Summary and future work

A two degree of freedom steering controller based on the disturbance observer was used here for vehicle yaw dynamics improvement. Steering control design was carried out in controller parameter space to satisfy a mixed sensitivity frequency domain bound, thus solving a robust performance problem. This robust performance problem is more restrictive than an earlier design with the same control architecture available in the literature. Unlike in earlier work with the same control structure, low speed operation was also reported here. Linear simulation results were used to demonstrate the effectiveness of the method. Although not reported here, the steering controller of this paper has been successfully tested using a higher order, more realistic, nonlinear simulation model with a gain scheduling implementation for switching on the control smoothly with speed. The important issuc of automatic steering controller and driver interaction was not considered here but a solution that effectively bypasses this problem by intervening only outside the bandwidth of the driver can be found [2]. Future work should concentrate on implementing an individual wheel braking (steering by braking) version of the steering controller reported here and coordinating/combining its use with the steering implementation of this paper.

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