Innovation vs Residual KF based GNSS/INS Autonomous Integrity Monitoring in Single Fault Scenario

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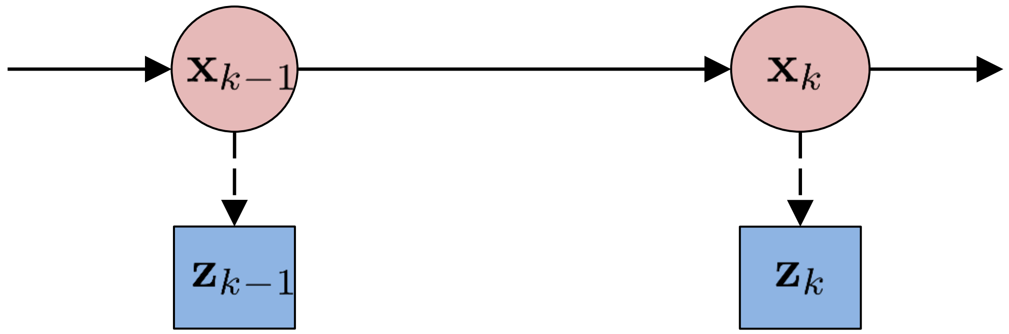
Motivation

- IRS Systems integrated in Civil Aviation
  - Improve accuracy
  - Guarantee continuity
- In GNSS/INS, inertial can potentially also increase the fault detection capability to meet stringent requirements (e.g., CAT III, GBAS,..)
- GNSS/INS is now a baseline solution for new applications that operate in more challenging scenarios (Automotive, UAV)

Importance of **Fault Detection** in GNSS/INS Integration and **assessment of Integrity**

Introduction

- Kalman filtering



Prediction:

$$\mathbf{x}_k^- = \mathbf{f}(\mathbf{x}_{k-1}^+)$$
$$\mathbf{P}_k^- = \mathbf{\Phi}_k \mathbf{P}_{k-1}^+ \mathbf{\Phi}_k^T + \mathbf{Q}_k$$

Update:

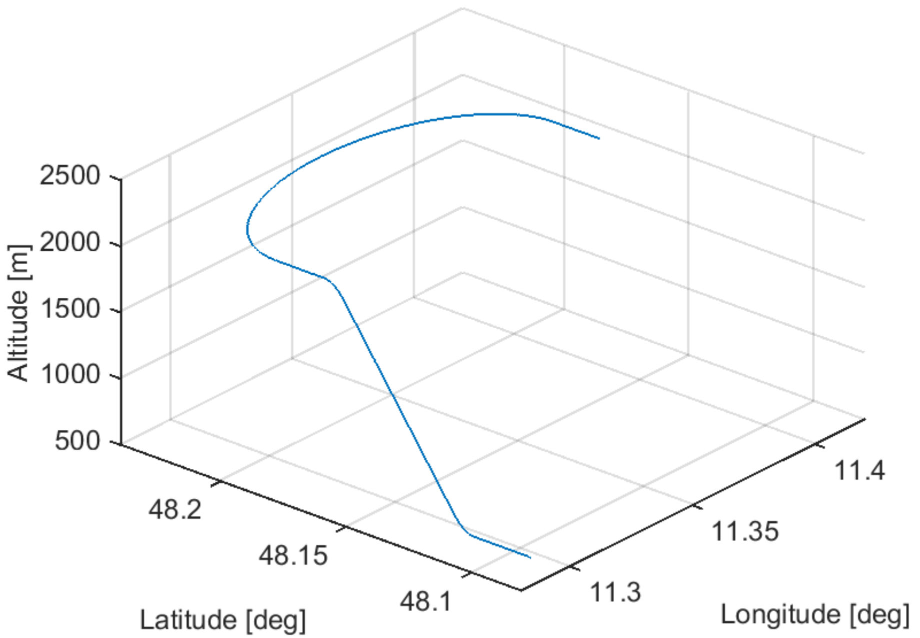
$$\mathbf{x}_k^+ = \mathbf{x}_k^- + \mathbf{K}_k \left(\mathbf{z}_k - \mathbf{h}(\mathbf{x}_k^-)\right)$$
$$\mathbf{P}_k^+ = \left(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k\right) \mathbf{P}_k^-$$
$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k\right)^{-1}$$

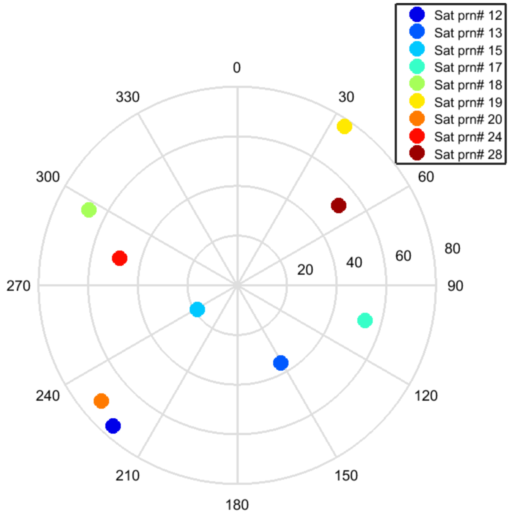
- **Objectives:**
  1. Exploit the residuals in GNSS/INS
  2. Comparison between Innovation and Residual-based IM in single snapshot fault scenario
     - Distribution of Test
     - Minimum Detectable Bias
     - Protection Levels

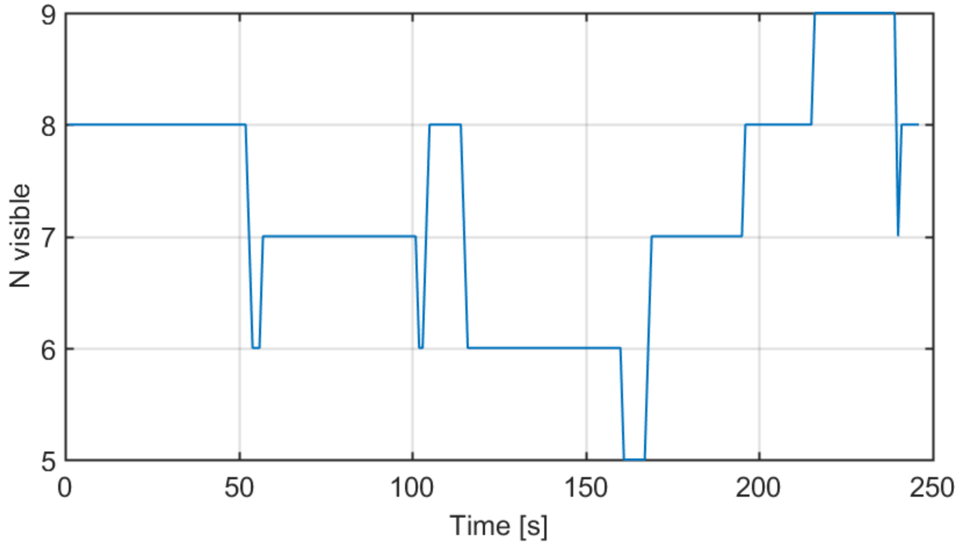
Innovation	Residual
$\gamma_k = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_k^-$	$\mathbf{r}_k = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_k^+$

Simulation Setup

- EKF with 17-states: $\mathbf{x}_{\text{EKF}} = \begin{pmatrix} \delta\boldsymbol{\psi} & \delta\mathbf{v} & \delta\mathbf{p} & \mathbf{b}_a & \mathbf{b}_g & \mathbf{b}_{\text{clk}} \end{pmatrix}^T$
- Tactical Grade IMU
- GPS code sigma = 5 m







Test Statistics

KF Innovation-based

- KF Innovation vector:

$$\boldsymbol{\gamma}_k = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_k^-$$

- Test Statistic:

$$q_{\gamma,k} = \boldsymbol{\gamma}_k^T \mathbf{S}_k^{-1} \boldsymbol{\gamma}_k$$

- Test statistic distribution:
  - H0: $q_{\gamma,k}, \sim \chi^2 \left(0, n_k\right)$
  - H1: $q_{\gamma,k}, \sim \chi^2 \left(\lambda^2, n_k\right)$

KF Residual-based

- KF residual vector:

$$\mathbf{r}_k = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_k^+$$

- Test Statistic:

$$q_{\mathrm{R},k} = \mathbf{r}_k^T \mathbf{R}_k^{-1} \mathbf{r}_k$$

- Test statistic distribution follows a Generalized Chi-Square*:

$$q_{\mathrm{R},k} = \sum_i^{p_B} \alpha_i^2 \chi_i^2$$

- CDF, PDF and Threshold computed numerically.

RAIM

- LS residuals:

$$\mathbf{r}_{\mathrm{LS},k} = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{\mathrm{LS},k}$$

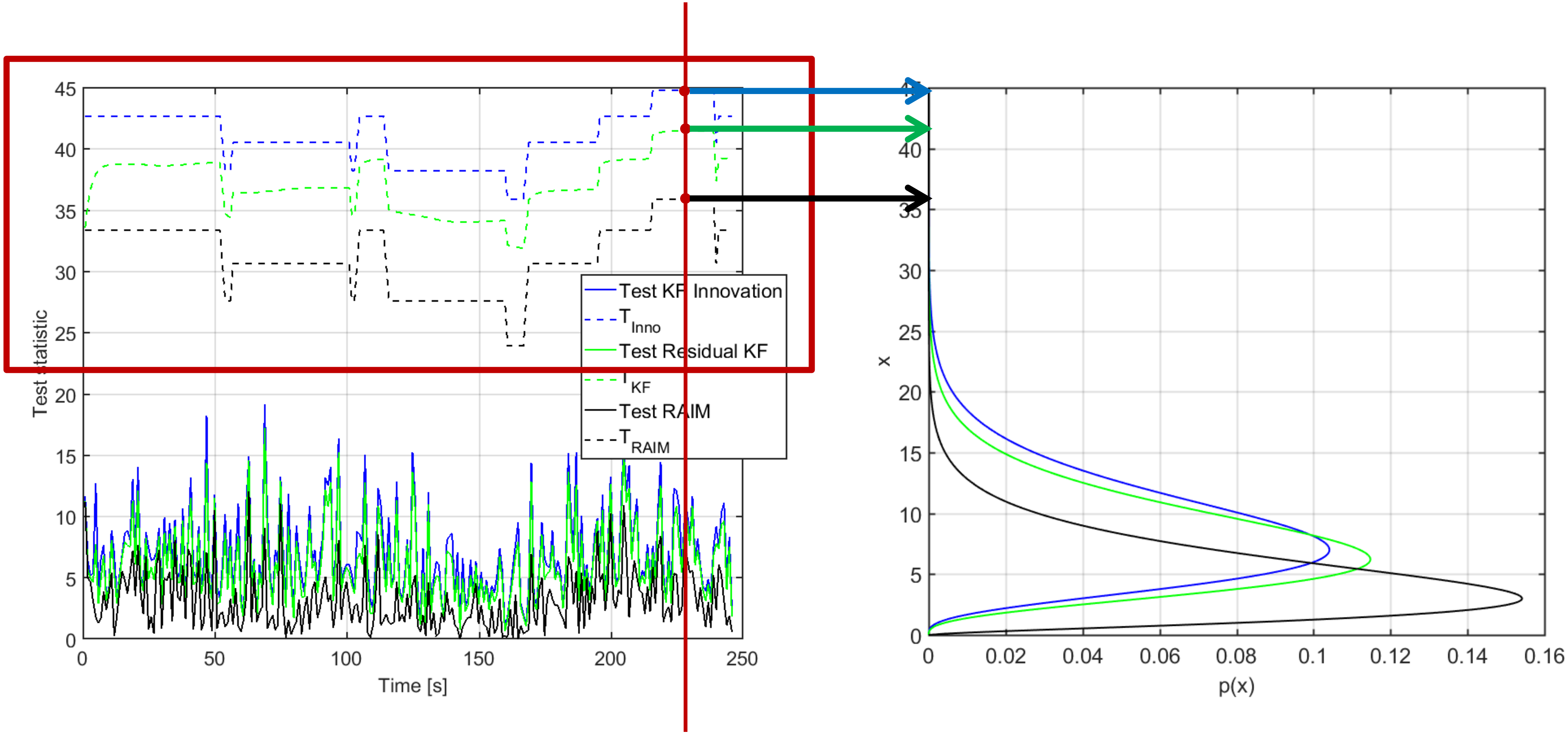
- Test Statistic:

$$q_{\mathrm{LS},k} = \mathbf{r}_{\mathrm{LS},k}^T \mathbf{R}_k^{-1} \mathbf{r}_{\mathrm{LS},k}$$

- Test statistic distribution:
  - H0: $q_{\mathrm{LS},k}, \sim \chi^2 \left(0, n_k - 4\right)$
  - H1: $q_{\mathrm{LS},k}, \sim \chi^2 \left(\lambda^2, n_k - 4\right)$

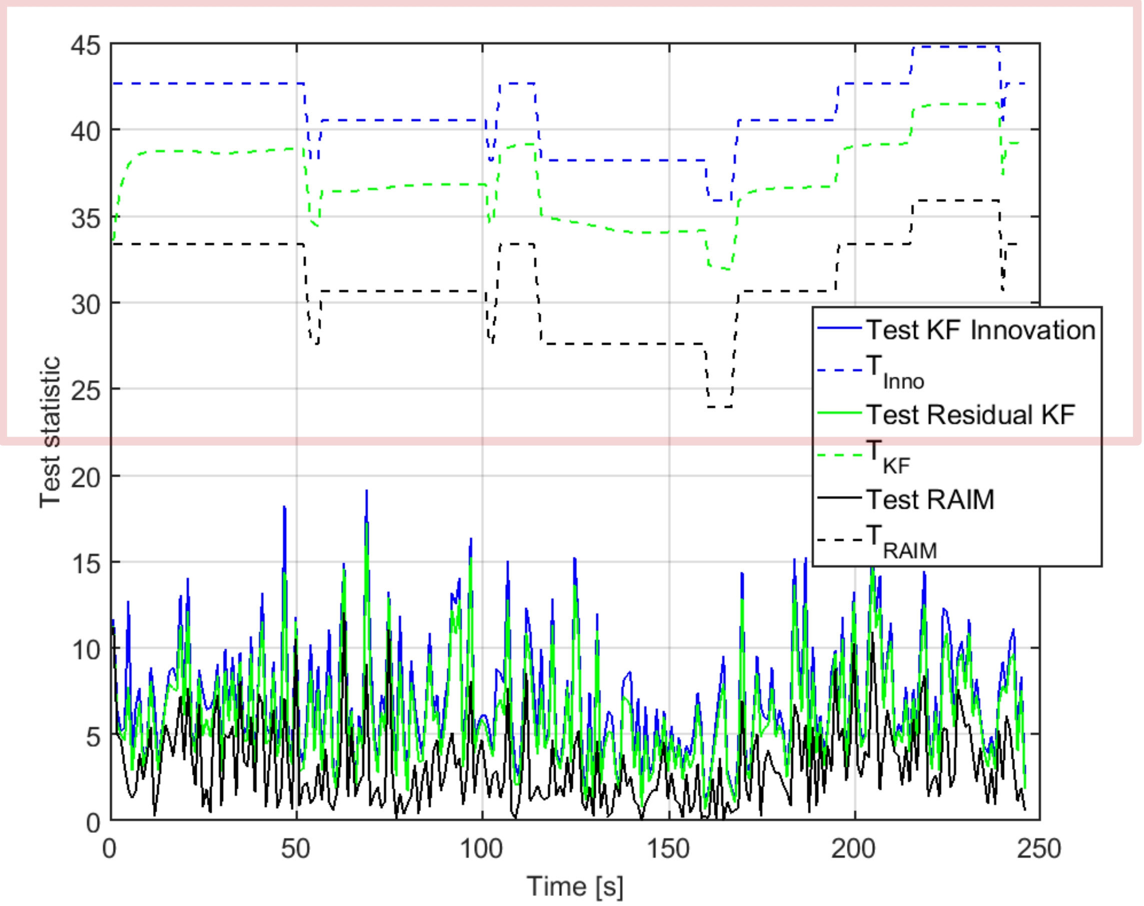
*M. Joerger and B. Pervan, "Kalman Filter-based Integrity Monitoring Against Sensor Faults," Journal of Guidance, Control, and Dynamics, vol. 36, no. 2, mar-apr 2013.

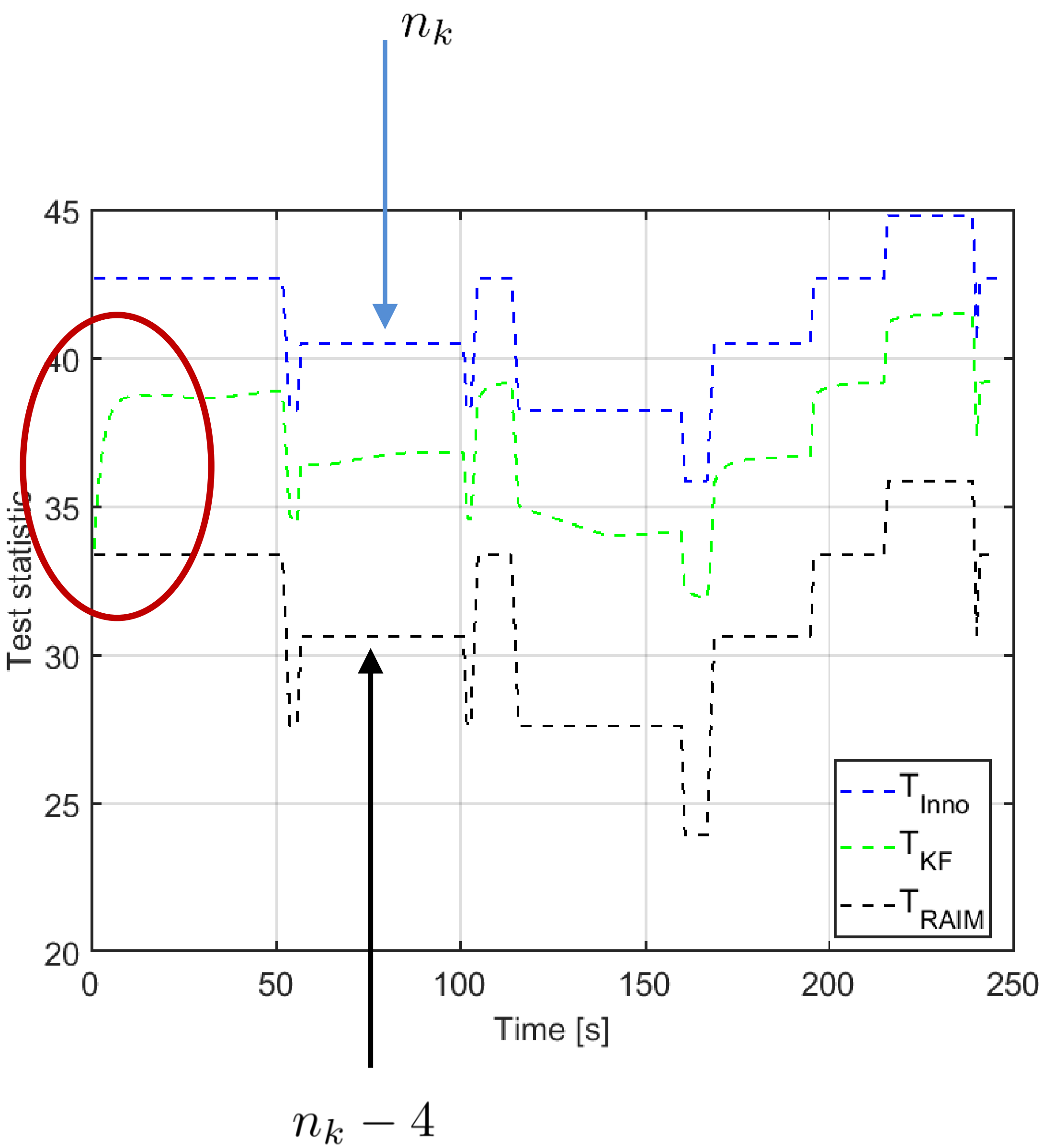
Evaluations: Test Statistics



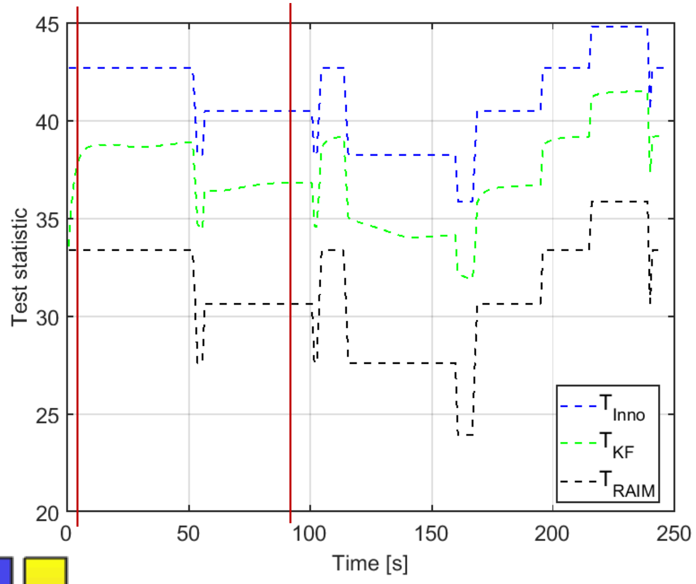
$$P_{\text{fa}} = 1e^{-6}$$

Evaluations: Test Statistics

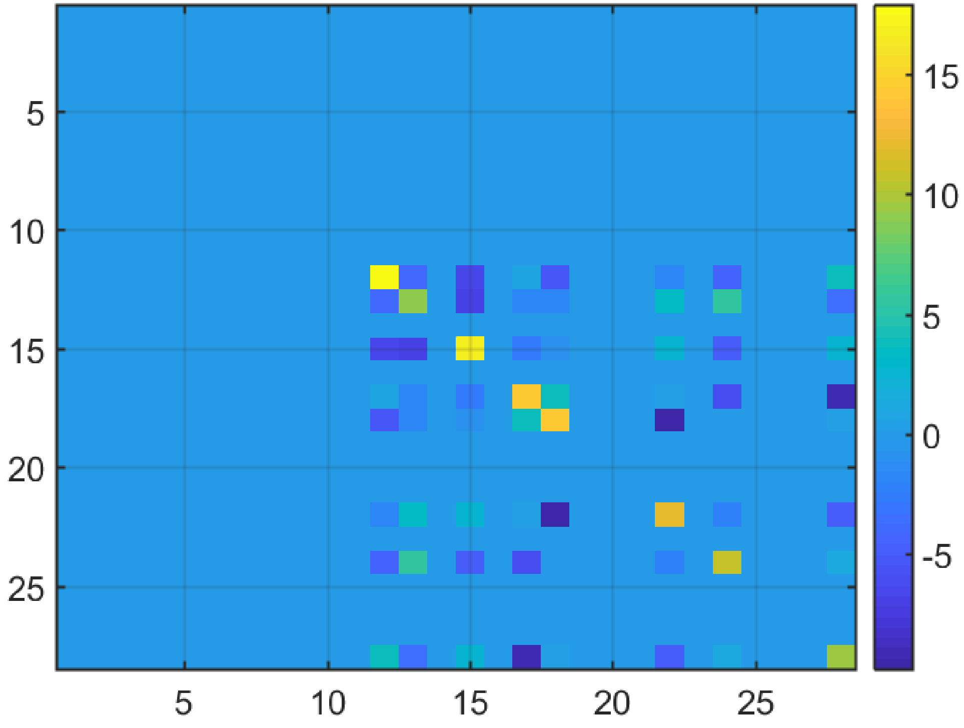




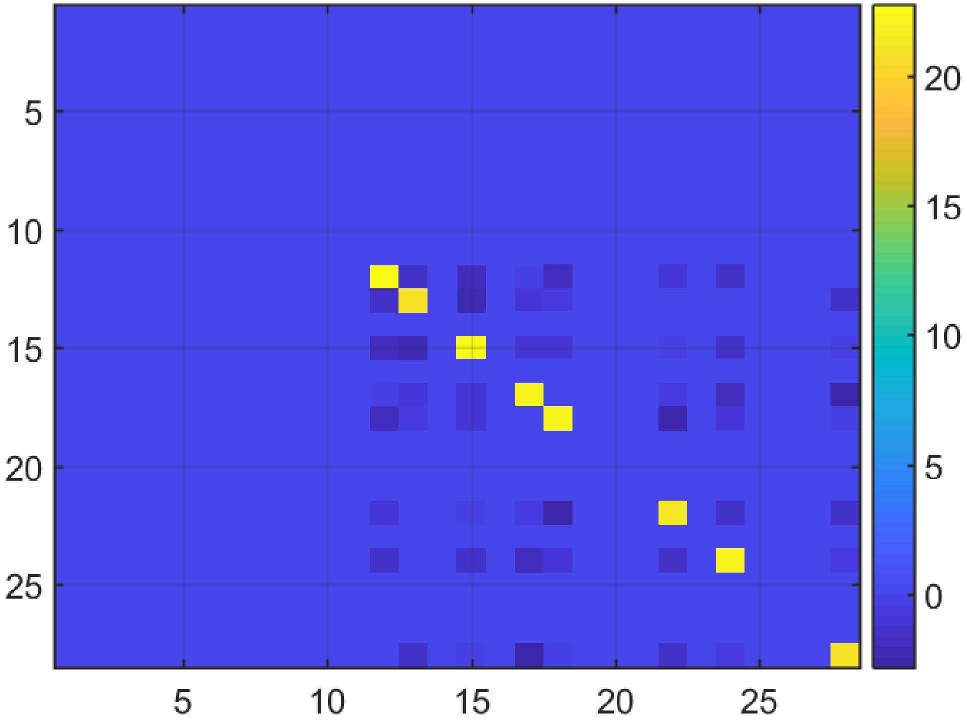
Evaluations: Residuals Correlation



Converging phase



After Converge phase



Less Correlation

Less linear dependency

More degrees of freedom

Minimum Detectable Bias

- **Innovation-based**: $\boldsymbol{\gamma}_k = (\mathbf{z}_k + \mathbf{f}_k) - \mathbf{H}_k \mathbf{x}_k^-$
- Minimum Detectable Bias (MDB):

$$MDB_i = \frac{\lambda_0}{\sqrt{\mathbf{e}_i^T \mathbf{S}^{-1} \mathbf{e}_i}}.$$

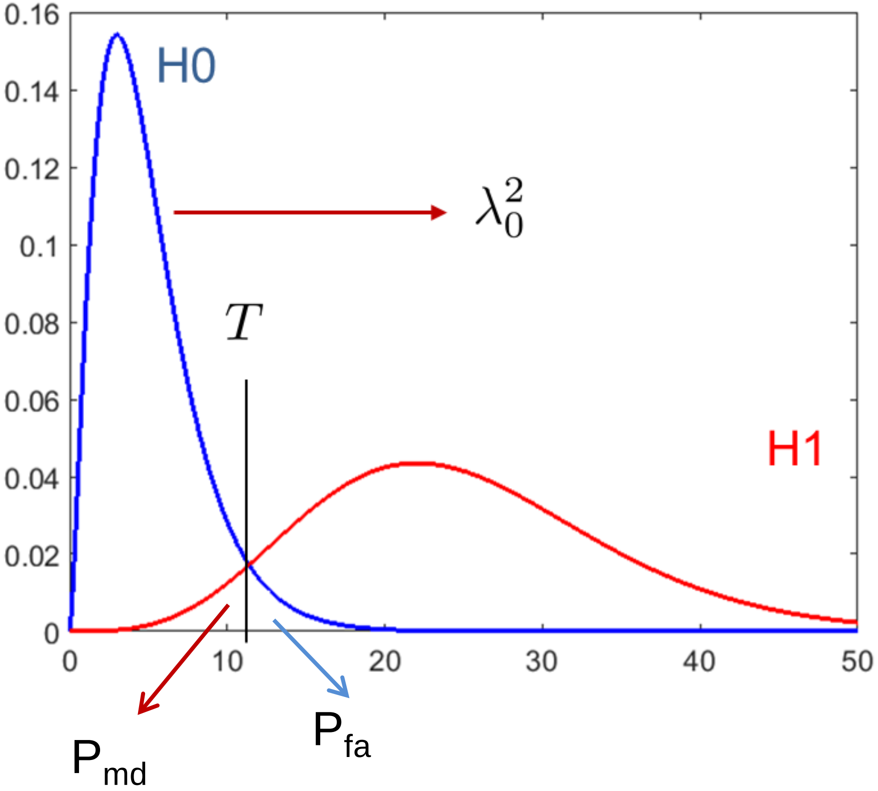
- **Residual-based**:

$$q_{\mathrm{R},k} = \sum_i^{p_B} \alpha_i^2 y_i^2$$

$$\mathbf{B} = \mathbf{R}_k^{-1/2} \mathbf{P}_{r,k}^{-1/2} = \mathbf{U} \Sigma \mathbf{V}^T$$

$$y_i \sim \mathcal{N}\left(\alpha_i^{-1} \mathbf{T}_i^T \mathbf{V}^T \mathbf{P}_{r,k}^{-1/2} E[\mathbf{r}_k], 1\right)$$

$$E[\mathbf{r}_k] = (\mathbf{I} - \mathbf{H}_k \mathbf{K}_k) \underbrace{\mathbf{e}_i MDB_i}_{\mathbf{f}_i}$$



$$\hat{MDB}_i = \min_{\boldsymbol{\mu}_i(MDB_i)} \left\| P_{md} - [Pr\left(q_{\mathrm{R},k}\left(p_B, \boldsymbol{\alpha}, \boldsymbol{\mu}_i\right) \leq T_{\mathrm{R},k}\right)] \right\|^2$$

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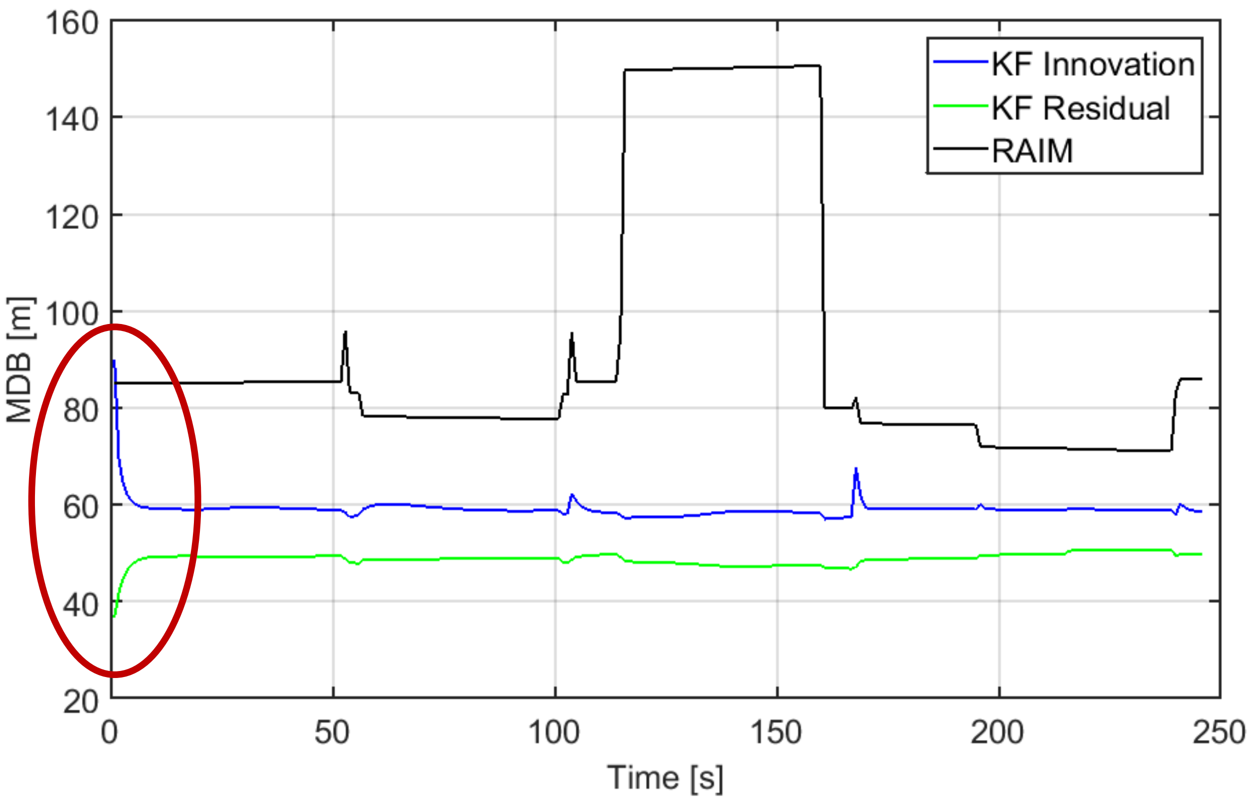
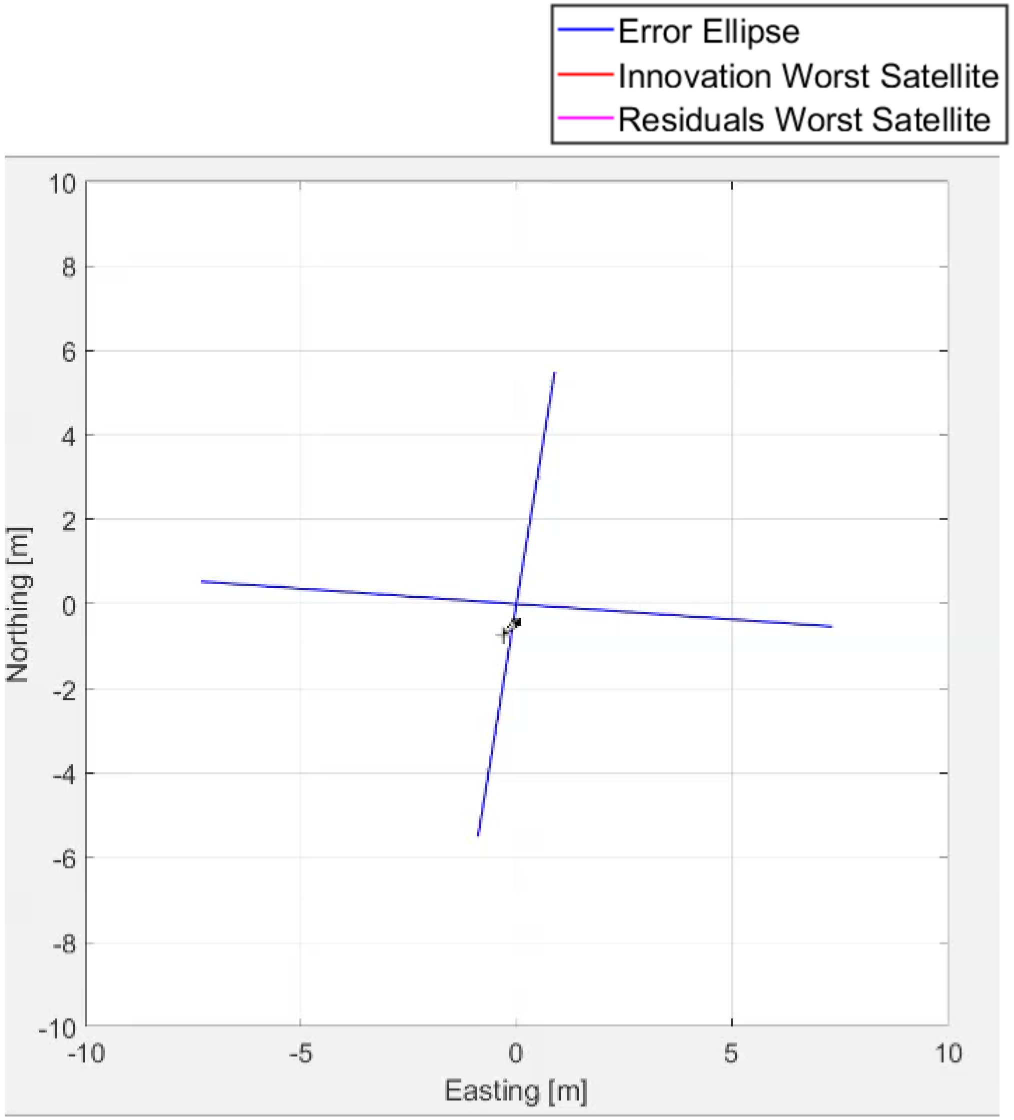
Evaluations: Minimum Detectable Bias





Protection levels

$$VPL = Vslope_{max}\sqrt{T(P_{fa})} + K_v\sqrt{P_{uu}}.$$

- Innovation based:

$$Vslope_{\gamma,k,i} = \sqrt{\frac{\mathbf{e}_i^T\mathbf{K}_k^T\boldsymbol{\epsilon}_v^T\boldsymbol{\epsilon}_v\mathbf{K}_k\mathbf{e}_i}{\mathbf{e}_i^T\mathbf{S}_k^{-1}\mathbf{e}_i}}$$

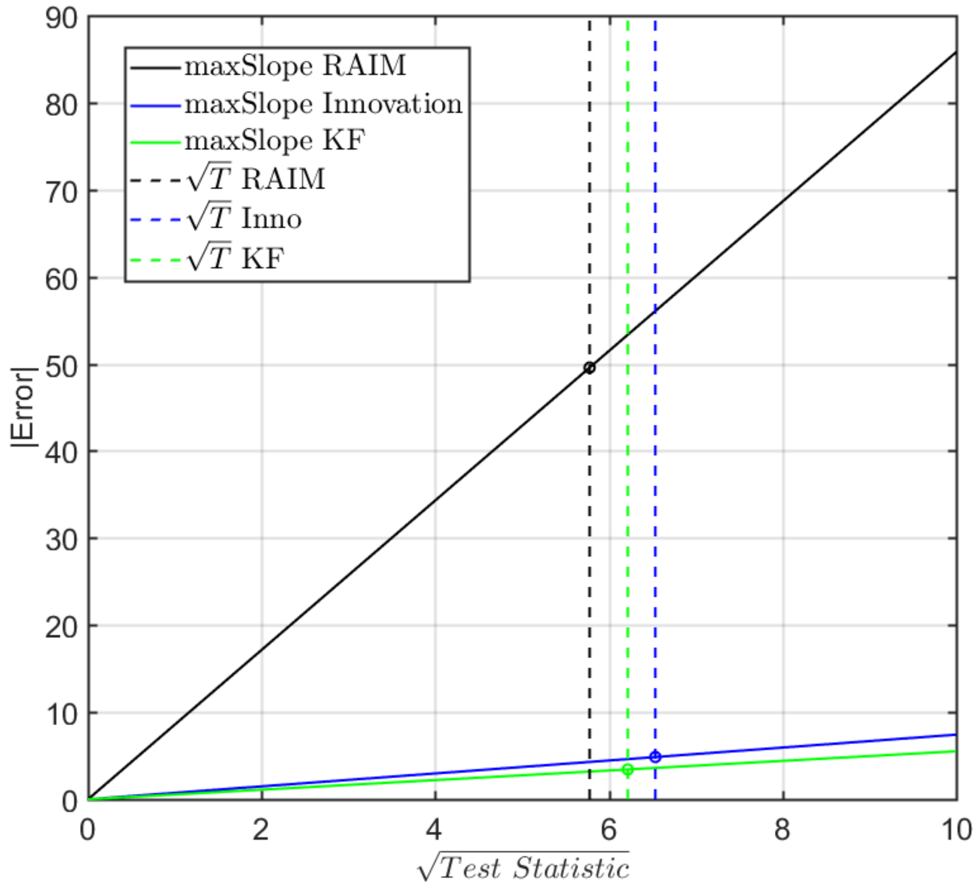
Bias propagation to Error (numerator)

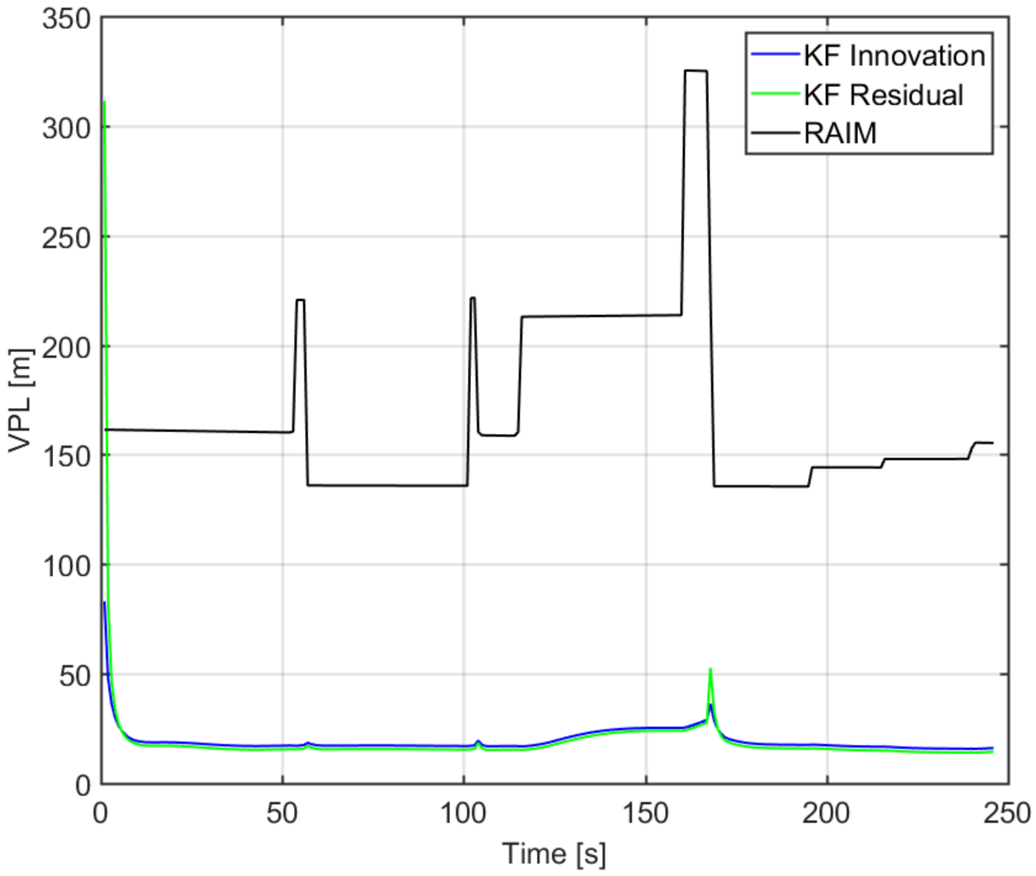
Bias propagation to non-central parameter (denominator)

- Residual based:

$$Vslope_{\mathrm{R},k,i} = \sqrt{\frac{\mathbf{e}_i^T\mathbf{K}_k^T\boldsymbol{\epsilon}_v^T\boldsymbol{\epsilon}_v\mathbf{K}_k\mathbf{e}_i}{\sum_j^{p_B}\left(\mathbf{T}_j^T\mathbf{V}^T\mathbf{P}_{r,k}^{-1/2}\left(\mathbf{I}-\mathbf{H}_k\mathbf{K}_k\right)\mathbf{e}_{i,k}\right)^2}}$$

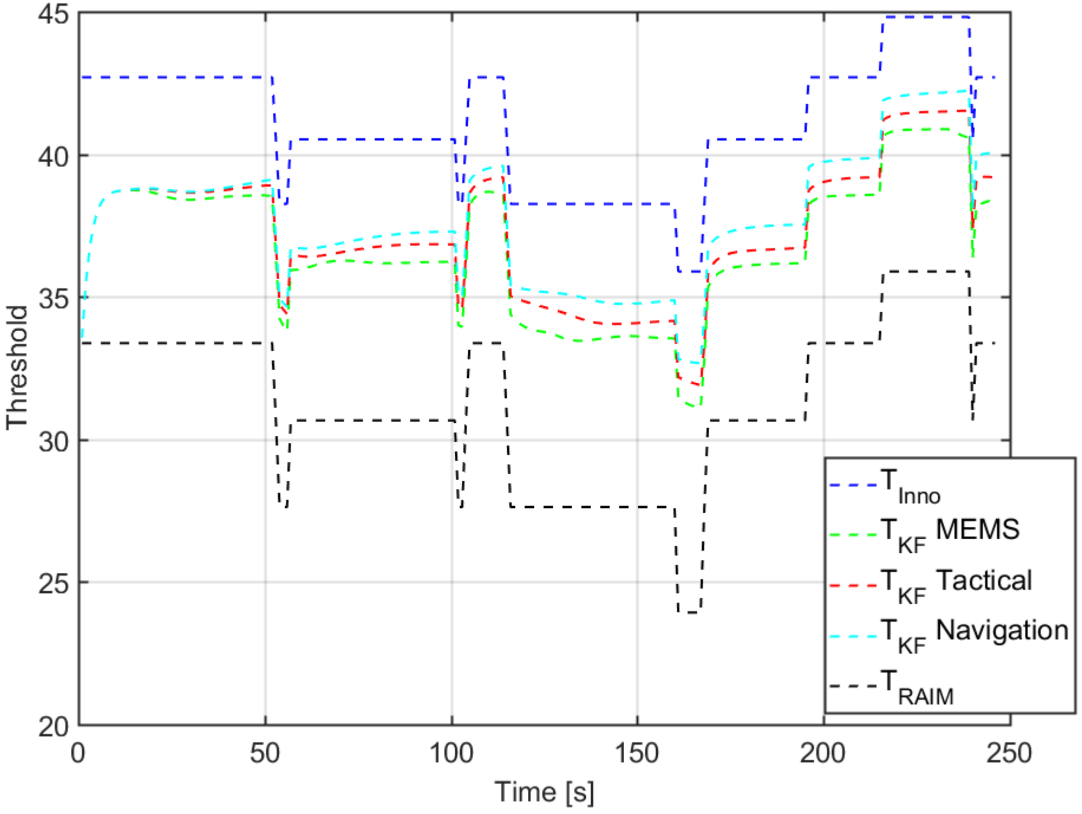
Evaluations: Slopes & VPL

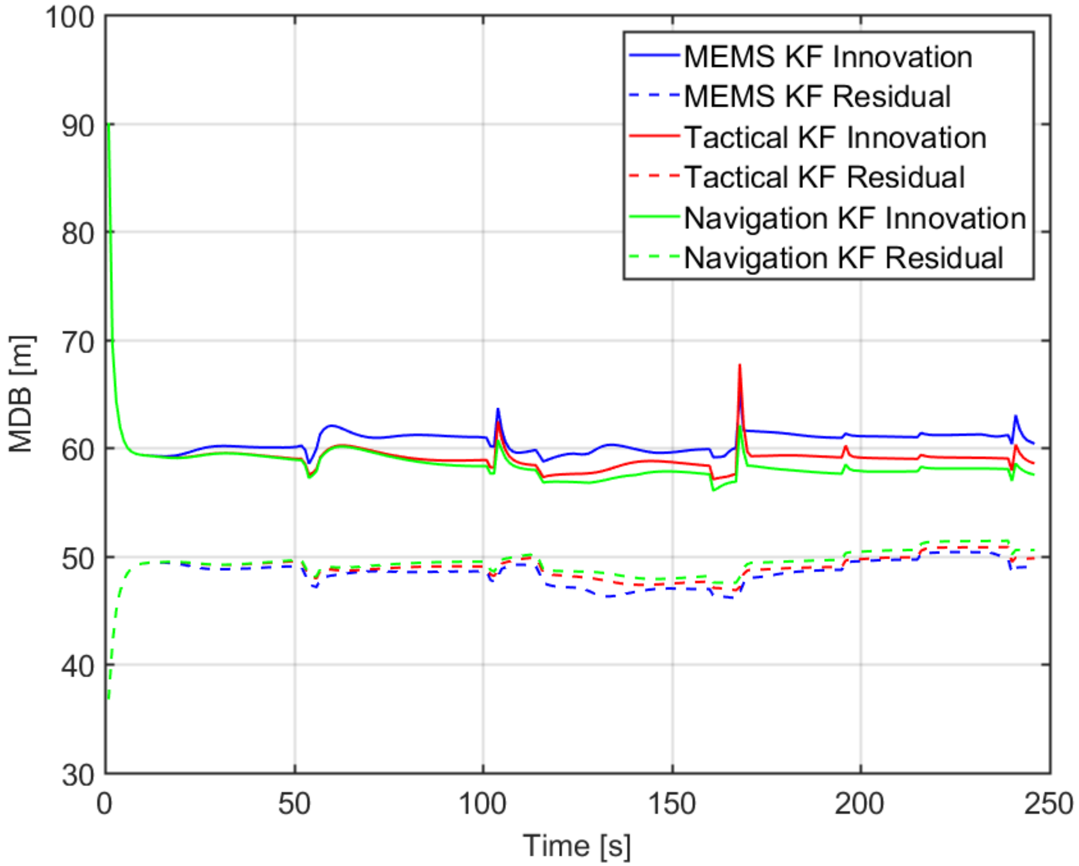




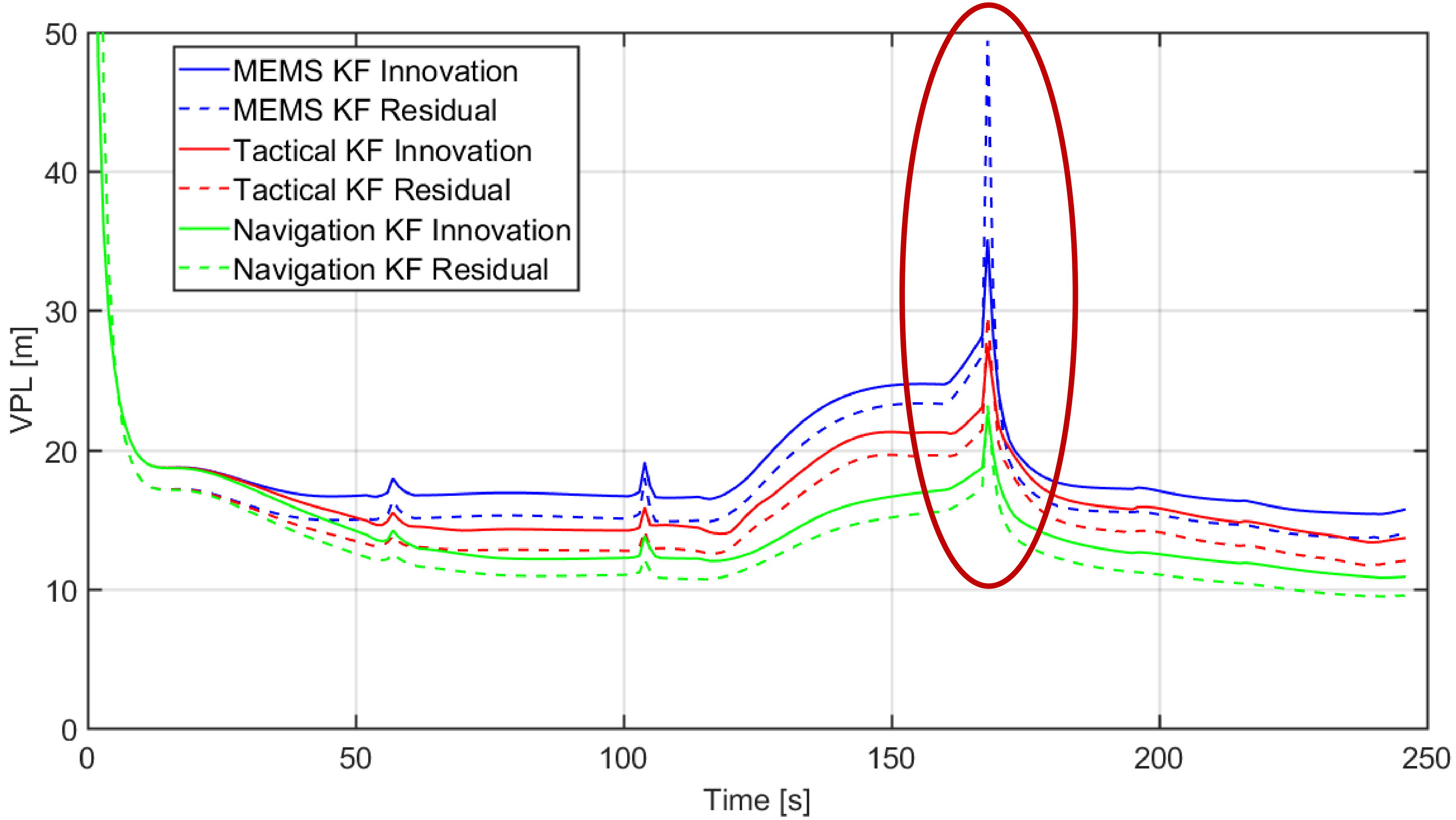
Evaluations: Inertial Quality Influence

	Accelerometer			Gyroscope		
	Noise [ug/sqrt(Hz)]	Bias Noise [ug]	tau [s]	Noise [deg/h/sqrt(Hz)]	Bias Noise [deg/h]	tau [s]
Navigation	15	20	3000	0.01	0.005	10000
Tactical	50	160	3000	2	0.5	10800
MEMS	120	150	25	10	15	600





Evaluations: Inertial Quality Influence (VPL)



Conclusions & Outlook

- Distribution of residual-based test is modulated by the prediction uncertainty $n_k - 4 < dof_{\mathrm{R}} < n_k$
- Threshold and other statistics must be computed numerically at each time
- Residuals and innovation show complementary behaviors in Fault Detection
- In the single fault case, residual-based shows better performance than innovation one in general

- Will look at sequential faults and multiple faults

Thank you

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