Uncertainty Analysis in Railway Asset Management using the Point Estimate Method

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Motivation

- Degradation
- Failures
- Reliability

Uncertainty

- Detection
- Diagnosis
- Prognosis

Modelling

- Statistical models
- Physical models
- Hybrid models

Preventive and condition-based maintenance
Models and Uncertainty Analysis

(Variable) **Input** $X$

- Data / Measurements
- Parameters

( Mathematical) **Model** $f$

**Output** $Y$

- Current degradation state
- Detected failure mode
- Future health condition
- Estimated life time
- …

Uncertainty propagation

$\mathbb{P}X \checkmark \rightarrow \mathbb{P}Y = ?$

Sources of uncertainty in modelling

- Data / Measurements ✓
- Model parameters ✓
- Model structure ✗

**Mathematical notation**

$f: \mathbb{R}^n \rightarrow \mathbb{R} \quad Y = f(X)$

where

$X = (X_1, \ldots, X_n)^T$

$(X_i$ stochastically independent)
How to derive the output distribution?

- Standard approach: **Monte Carlo simulation (MC)**

  \[ \mathbb{P}^X \Rightarrow \text{Uncertainty propagation} \Rightarrow \mathbb{P}^Y = ? \]

  - Random sample based on \( \mathbb{P}^X \)
  - Random sample based on unknown \( \mathbb{P}^Y \)
  - Numerical estimate of \( \mathbb{P}^Y \) based on \( y^{(1)}, \ldots, y^{(N)} \)
  - True \( \mathbb{P}^Y \) when \( N \to \infty \)

In general, **large samples** necessary!

- **Approximate results** only!
Alternative approach: Point Estimate Method (PEM)

- Often it is **sufficient to know** some **basic statistics** of the output distribution
  
  Mean: $\mathbb{E}(Y)$ \hspace{1cm} Variance: $\text{Var}(Y)$ \hspace{1cm} $k$-th moment: $\mathbb{E}(Y^k)$  

- **Approximation scheme** (= core element of PEM):
  
  $$\int_{\Omega} g(x) \, \text{pdf}_X(x) \, dx \approx w_0 \, g(GF[0]) + w_1 \sum g(GF[\pm \vartheta]) + \cdots + w_m \sum g(GF[\pm \vartheta, \ldots, \pm \vartheta])_{\text{m times}}$$

with suitable **weights** $w_j$ and **deterministic sample points**

- $GF[0]$
- $GF[\pm \vartheta]$
- $GF[\pm \vartheta, \pm \vartheta]$
- \ldots

$g = f \quad \mathbb{E}(Y)$

$g = f^2 \quad \mathbb{E}(Y^2)$

$\text{Var}(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2$
Standard scheme ($m = 2$)

\[
\int_{\Omega} g(x) \, \text{pdf}_X(x) \, dx \approx w_0 \, g(\text{GF}[0]) + w_1 \sum g(\text{GF}[\pm \vartheta]) + w_2 \sum g(\text{GF}[\pm \vartheta, \pm \vartheta])
\]

where

\[
w_0 = 1 + \frac{n^2 - 7n}{18}, \quad w_1 = \frac{4-n}{18}, \quad w_2 = \frac{1}{36}, \quad \vartheta = \sqrt{3}
\]

- Weights determined such that scheme is **exact for polynomials** $g : \mathbb{R}^n \rightarrow \mathbb{R}$ (degree $\leq 5$) in case of standard Gaussian inputs $X_i$ for $i = 1, \ldots, n$
- **Transformation** of the sample points in case of other input distributions

Number of sample points

\[
N = 2n^2 + 1
\]
Example of use: Track degradation

- Simple **Petri net model** for track degradation (without maintenance):

- **Stochastic transitions**: $T_r \sim \mathcal{W}(\beta_r, \eta_r)$
  (i.e., Weibull distributed duration [in days] before jumping to the next “health state”)

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_r$ (shape)</td>
<td>1.5</td>
<td>1.5</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>$\eta_r$ (scale)</td>
<td>600</td>
<td>500</td>
<td>370</td>
<td>280</td>
</tr>
</tbody>
</table>
Example of use: Track degradation

Estimated track reliability based on MC simulation

Sample points
Example of use: Track degradation

- Duration [in days] before reaching state $P_{s+1}$ after renewal or new construction:

$$\tilde{T}_s := \sum_{r=1}^{s} T_r$$

<table>
<thead>
<tr>
<th>Analytical solution</th>
<th>$\mathbb{E}(\cdot)$</th>
<th>$\sigma(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{T}_1$</td>
<td>541.6</td>
<td>367.8</td>
</tr>
<tr>
<td>$\tilde{T}_2$</td>
<td>993.0</td>
<td>478.7</td>
</tr>
<tr>
<td>$\tilde{T}_3$</td>
<td>1324.8</td>
<td>523.7</td>
</tr>
<tr>
<td>$\tilde{T}_4$</td>
<td>1574.6</td>
<td>545.1</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>$\Delta_{\text{rel}}$ (MC)</th>
<th>$\mathbb{E}(\cdot)$</th>
<th>$\sigma(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{T}_1$</td>
<td>$-0.68%$</td>
<td>$+0.50%$</td>
</tr>
<tr>
<td>$\tilde{T}_2$</td>
<td>$-1.00%$</td>
<td>$-0.03%$</td>
</tr>
<tr>
<td>$\tilde{T}_3$</td>
<td>$-0.56%$</td>
<td>$+0.50%$</td>
</tr>
<tr>
<td>$\tilde{T}_4$</td>
<td>$-0.49%$</td>
<td>$+0.67%$</td>
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</tbody>
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<table>
<thead>
<tr>
<th>$\Delta_{\text{rel}}$ (PEM)</th>
<th>$\mathbb{E}(\cdot)$</th>
<th>$\sigma(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{T}_1$</td>
<td>$+0.00%$</td>
<td>$-0.03%$</td>
</tr>
<tr>
<td>$\tilde{T}_2$</td>
<td>$+0.00%$</td>
<td>$-0.03%$</td>
</tr>
<tr>
<td>$\tilde{T}_3$</td>
<td>$+0.01%$</td>
<td>$-0.05%$</td>
</tr>
<tr>
<td>$\tilde{T}_4$</td>
<td>$+0.01%$</td>
<td>$-0.07%$</td>
</tr>
</tbody>
</table>

**10,000** Sample points

**25 (!) Sample points**
Example of use: System reliability

- **Composite system** with three (stochastically) **independent components**:

  ![Composite System Diagram]

- **Exponential life distributions**: $X_i \sim \text{Exp}(\beta_i) \rightarrow \text{Reliability}: R_{X_i}(t) = \exp\left(-\frac{t}{\beta_i}\right)$

- **System reliability** (= probability that the system “survives” until time $t$):

  $$R_{\text{sys}}(t) = R_{X_1}(t)R_{X_2}(t) + R_{X_3}(t) - R_{X_1}(t)R_{X_2}(t)R_{X_3}(t)$$
Example of use: System reliability

\[ R_{sys}(t) = \exp \left( -\frac{t}{\beta_1} - \frac{t}{\beta_2} \right) + \exp \left( -\frac{t}{\beta_3} \right) - \exp \left( -\frac{t}{\beta_1} - \frac{t}{\beta_2} - \frac{t}{\beta_3} \right) \]

- Parameters \( \beta_i \) fixed \( \rightarrow \) Deterministic function!
- But, \( \beta_i \) uncertain \( \rightarrow \) Random variable for each \( t \)!

- Apply PEM (in comparison to MC) for estimating \( \mathbb{E}(R_{sys}(t)) \) and \( \text{Var}(R_{sys}(t)) \) depending on \( t \)

Exemplary assumptions!
Example of use: System reliability

Expected system reliability based on PEM and MC simulation

10,000 Sample points

MonteCarlo

PEM

19 (!) Sample points

$E(R_{sys}(t))$ vs $t$

0 1 2 3 4 5 6 7 8 9 10
Example of use: System reliability

**Variance of the system reliability** based on PEM and MC simulation

10,000 Sample points

19 (!) Sample points
Example of use: Failure diagnosis using decision trees

- Decision tree:

- Results:

- Further information: see supplementary material!
Summary: Assets and drawbacks of PEM

- Flexible approach (Easy to apply)
- Reduction of computing time because of small samples (compared to MC)
- Exact results (for “polynomial models” with given maximum degree)
- Applicable to various types of distributions (by transforming sample points)
- Reproducible results (deterministic approach)
- Approximate calculation of the full output distribution possible by combining PEM with further approaches (e.g., polynomial chaos expansion)

- Basic statistics of the output distribution only (when using the original PEM)
- Stochastically independent inputs required
- No general guarantee concerning the accuracy of the results (i.e., no convergence as $N \to \infty$)
Thanks for your attention!

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