Terminal Reliability of Road Networks with multiple Destination Options

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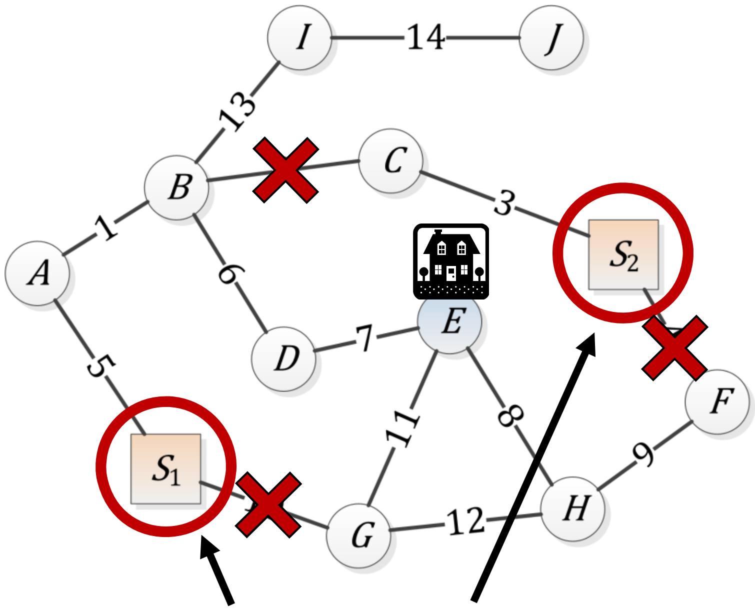
Individual consequences of road network vulnerability

Virtual road network



„Supply nodes“

(e.g., fire department, hospital, …)

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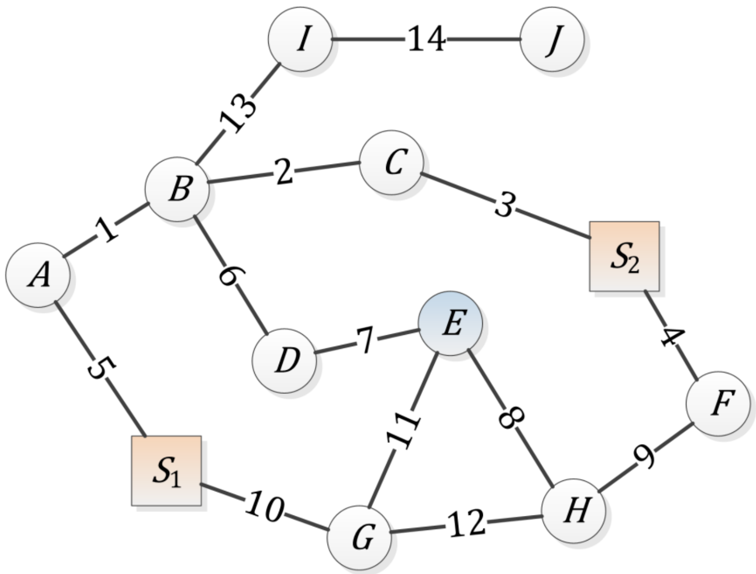
Problem statement

(1) How likely is it that a given node of the network loses its connection to *all* supply nodes of a given type?

(2) What are the most critical links in the network in terms of negatively affecting the isolation risk of many nodes when failing?



(Mathematical) assumptions:

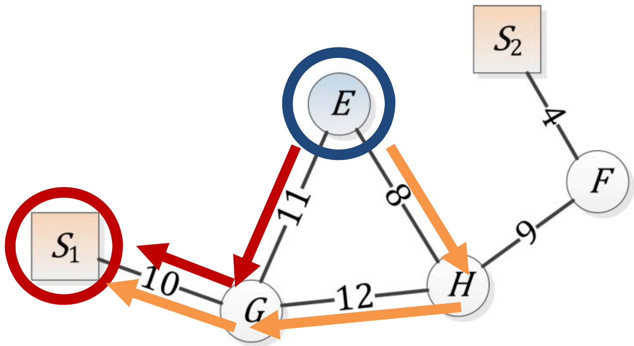
- Bidirectional links
- Stochastically independent link failures
- Link failure probabilities $p_i \in [0,1]$
- Link availability is represented by indicator functions, i.e.

$$I_i = \begin{cases} 1 & \text{if link } i \text{ is available} \\ 0 & \text{if link } i \text{ is not available} \end{cases}$$

Terminal reliability in road networks (1/2)

Reliability of the connection for a given O-D-pair, i.e. single destination

Example: $E \rightarrow S_1$



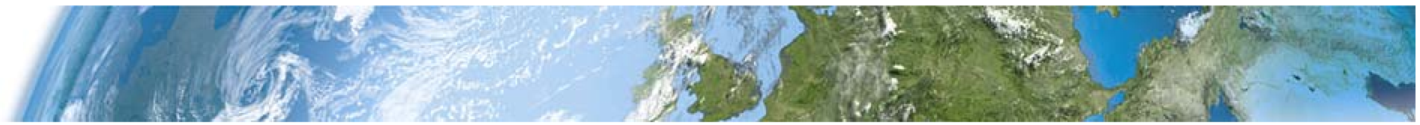
$\left(\text{Reduced network}\right)$

- (Minimal) Paths ($r = 2$):
  $P(1) = \{10{,}11\}$
  $P(2) = \{8{,}10{,}12\}$

- Availability of a connection between E and S_1 → "*Structure function*":

$$\psi(I_4, I_8, I_9, I_{10}, I_{11}, I_{12}) \coloneqq 1 - \prod_{k=1}^{r}\left(1 - \prod_{i \in P(k)} I_i\right) = I_{10}I_{11} + I_8I_{10}I_{12} - I_8I_{10}I_{11}I_{12}$$

Boolean algebra, i.e.
$(I_i)^k = I_i \quad \forall k \in \mathbb{N}$



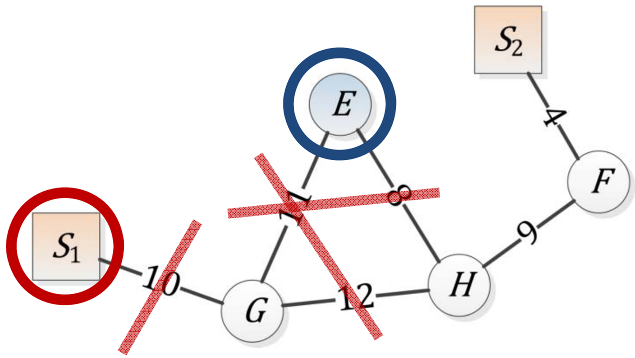
Terminal reliability in road networks (2/2)

- "*System reliability*" (i.e., probability of at least one available connection):

$$R_{\text{sys}} = \mathbb{E}\big(\psi(I_4, I_8, I_9, I_{10}, I_{11}, I_{12})\big) = R_{10}R_{11} + R_8R_{10}R_{12} - R_8R_{10}R_{11}R_{12}$$

where $R_i \coloneqq 1 - p_i$ ("*Link reliability*")

- Equivalent representation of ψ using minimal cut sets ($r' = 3$):
  $Z(1) = \{10\}$
  $Z(2) = \{8,11\}$
  $Z(3) = \{11,12\}$



$$\Rightarrow \qquad \psi(I_4, I_8, I_9, I_{10}, I_{11}, I_{12}) \coloneqq \prod_{k=1}^{r'} \left(1 - \prod_{i \in Z(k)} (1 - I_i)\right)$$

Simplifications and algorithms

- The full paper contains further mathematical information as follows:
  - Two Lemmas simplifying the Boolean calculations in context of the structure function ψ

Lemma 1: *Let $P(k)$ for $k = 1, \dots, r$ be the (minimal) paths between two selected nodes of a given network with (stochastically independent) link indicator functions I_i and link reliabilities R_i for $i = 1, \dots, m$. Moreover, let $i_0 \in \{1, \dots m\}$ be such that $i_0 \in P(k)$ for all k. Then,*

$$R_{\text{sys}} = \mathbb{E}\big(\psi(I_1, \dots, I_m)\big) = R_{i_0} \cdot \mathbb{E}\big(1 - \prod_{k=1}^{r}\big(1 - \prod_{i \in P(k)\setminus\{i_0\}} I_i\big)\big). \quad (8)$$

Lemma 2: *Let $P(k)$ for $k = 1, \dots, r$ be the (minimal) paths between two selected nodes of a given network with (stochastically independent) link indicator functions I_i and link reliabilities R_i for $i = 1, \dots, m$. Moreover, let C_j for $j = 1, \dots, l$ be disjoint subsets of $\{1, \dots, m\}$ such that for each $k \in \{1, \dots, r\}$ there is a $j \in \{1, \dots, l\}$ with $P(k) \subseteq C_j$. Then,*

$$R_{\text{sys}} = \mathbb{E}\big(\psi(I_1, \dots, I_m)\big) = 1 - \prod_{j-1}^{l}\big(1 - R_{\text{sys}}^{(j)}\big) \quad (9)$$

where

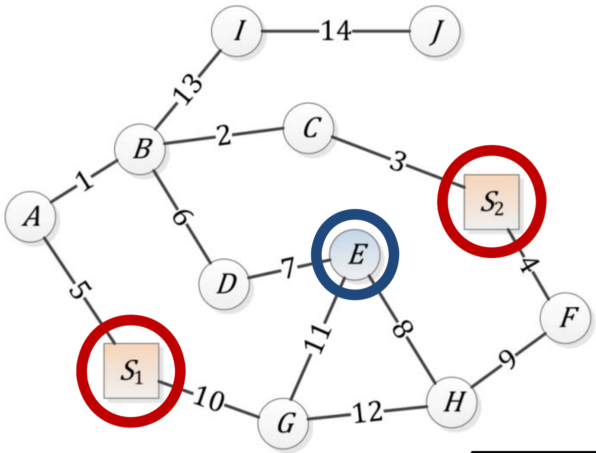
$$R_{\text{sys}}^{(j)} := \mathbb{E}\Big(1 - \prod_{k:P(k)\subseteq C_j}\big(1 - \prod_{i \in P(k)} I_i\big)\Big). \quad (10)$$

  - A recursive algorithm for deriving all minimal cut sets based on given sets of all (minimal) paths

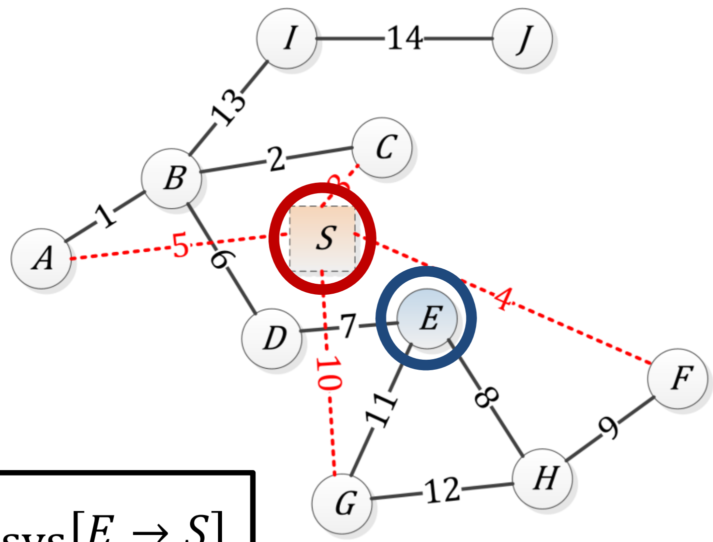
- An exact recursive algorithm for computing the system reliability based on the (minimal) paths without the need of Boolean algebra is available and will be described in a future publication

Terminal reliability with multiple destination options

- System reliability: $R_{\text{sys}}[E \rightarrow S_1 \vee S_2] =$ Probability that there is a connection between E and S_1 or S_2 (or both) available
- Note that: $R_{\text{sys}}[E \rightarrow S_1 \vee S_2] \neq 1 - (1 - R_{\text{sys}}[E \rightarrow S_1])(1 - R_{\text{sys}}[E \rightarrow S_2])$



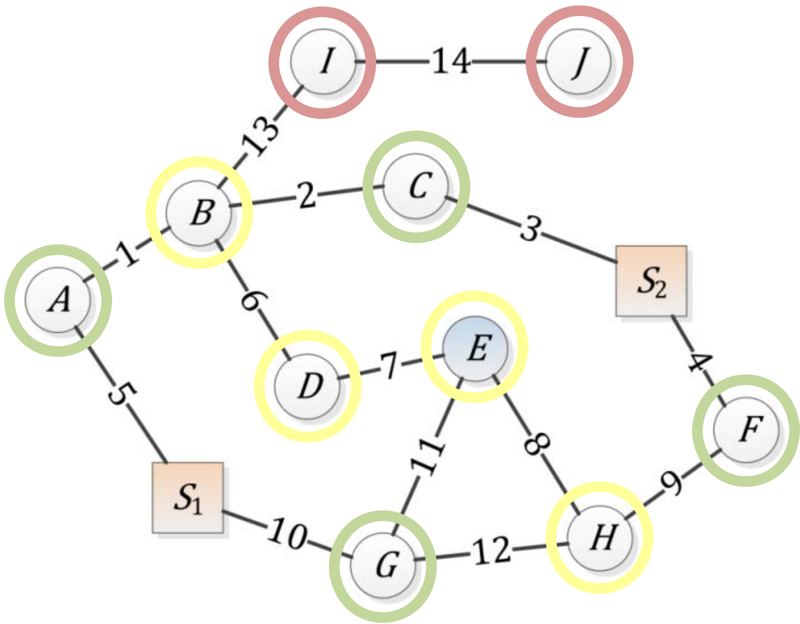
Graph modification



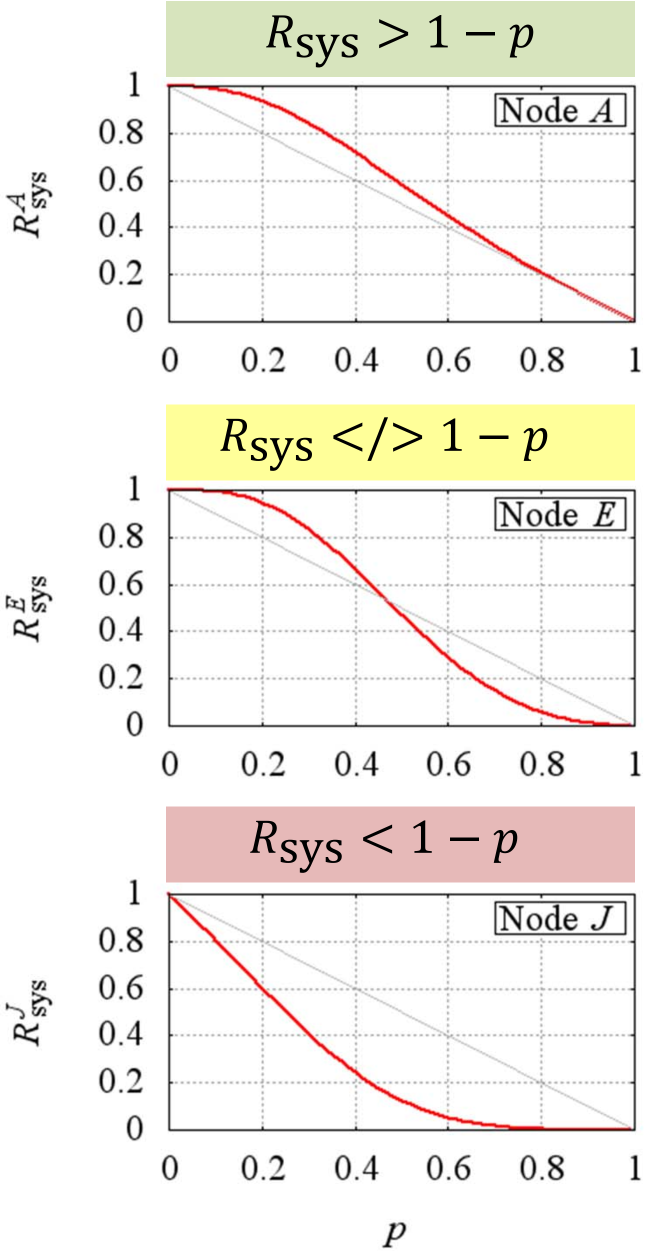
$\Rightarrow$ $R_{\text{sys}}[E \rightarrow S_1 \vee S_2] = R_{\text{sys}}[E \rightarrow S]$

System reliability per node (Results)

- Simplification: $p_i \equiv p \;\; \forall i$



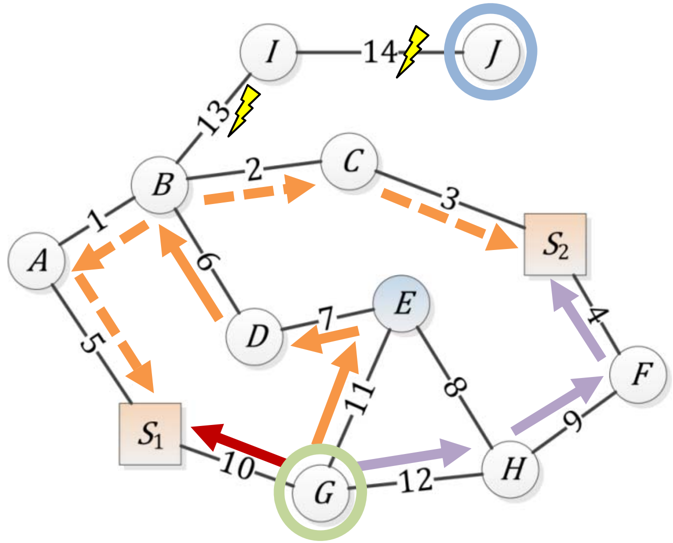
- 3 Groups of nodes:
  $\{A, C, F, G\}$ | $\{B, D, E, H\}$ | $\{I, J\}$



Isolation risk per node (Results)

- Isolation risk: $Q_{\text{sys}} \coloneqq 1 - R_{\text{sys}}$

Node	Isolation risk ($p = 1\%$)
A	0.0104%
B	0.0008%
C	0.0104%
D	0.0107%
E	0.0006%
F	0.0103%
G	0.0005%
H	0.0006%
I	1.0008%
J	1.9908%

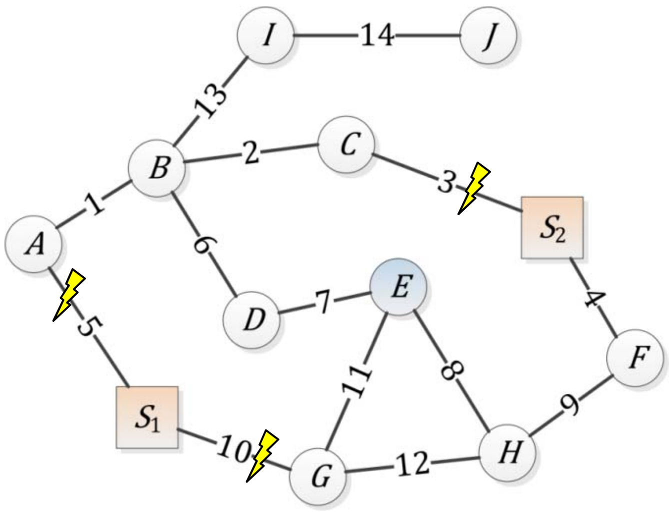


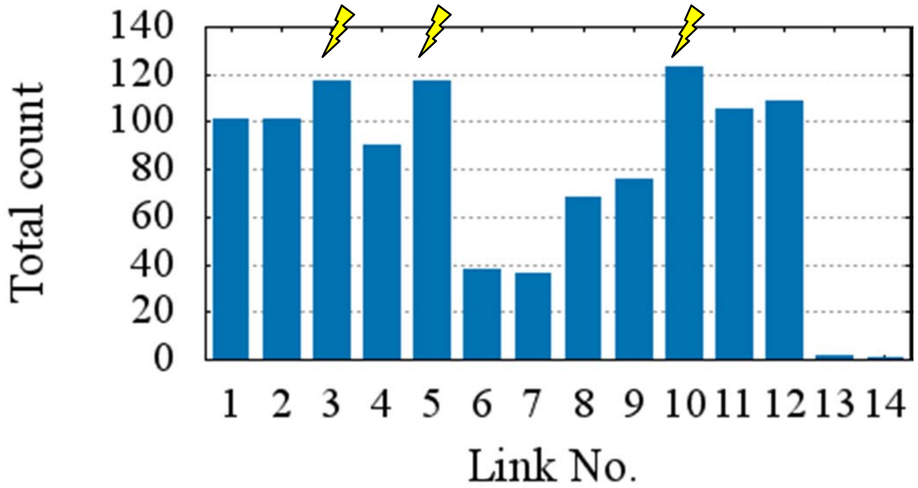
- Individual characteristics of system reliability and isolation risk per node strongly depend on the topological layout of the network
- $Q_{\text{sys}} \ll p$ for "*small*" p whenever there are two or more completely disjoint paths towards the supply nodes

Critical links (Results)

- **<u>Heuristic definition</u>:**

  Links that are <u>element of many cut sets</u> are more critical than others
  (because they negatively affect the isolation risk of many nodes when failing)





- **<u>Possible extension</u>:** <u>Weighting</u> based on the size of the individual cut sets

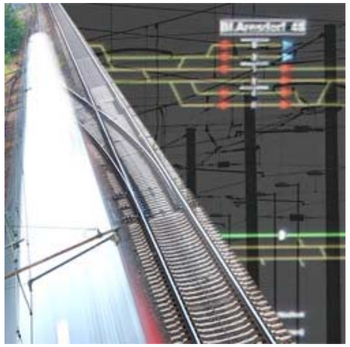
Conclusion and further development

Main observations:

- Standard concept of terminal reliability is applicable also in case of multiple destination options (via graph modification)
- System reliability and isolation risk per node strongly depend on the topological structure of the network
- Scalability is the most crucial aspect with regard to practical applications in context of large-scale networks

Future tasks:

- Extended analysis where different types of supply nodes needs to be accessible at the same time
- Consideration of correlated link failures
- Integration of failing supply nodes (instead of link failures only)
- Further discussion concerning definition and assessment of link criticality


Thanks for your attention!

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