Collective modes in strongly coupled complex plasmas and related systems

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Motivation:

- Theory of collective motion in liquids is an active area of research since the second half of XX century (starting from neutron scattering measurements).

- In particular, the theory for monoatomic liquids (liquid metals and rare gases) has been worked out.

- To what extent are the developed theories applicable to complex (dusty) plasmas, representing classical systems of strongly interacting particles?

- Alternatively, using complex (dusty) plasmas is it possible to check the accuracy and applicability limits of these early theories?
Quasi-crystalline (quasi-localized charge) approximation (QCA/QLCA)

• Generic expressions for the longitudinal and transverse dispersion relations:

\[
\omega_L^2 = \frac{n}{m} \int \frac{\partial^2 V(r)}{\partial z^2} g(r) [1 - \cos(kz)] \, dr,
\]

\[
\omega_T^2 = \frac{n}{m} \int \frac{\partial^2 V(r)}{\partial y^2} g(r) [1 - \cos(kz)] \, dr.
\]

are equivalent to the model of collective motion in liquids by Zwanzig (1967), quasi-crystalline approximation (QCA) by Hubbard&Beeby (1969), Takeno&Goda (1971). Similar expressions occur from the analysis of frequency moments of \(S(k,\omega)\).

• In the context of plasma physics QCA is also known as QLCA after Kalman and Golden who applied the approximation to one-component-plasma and related charged systems.
Application to complex (dusty) plasmas

- **Yukawa interaction potential**

- First applied by Rosenberg and Kalman (1997) in the regime of long-wavelengths and weak screening

- Kalman et al. (2000) computed $g(r)$ using the HNC scheme get results in good agreement with MD modeling by Ohta and Hamaguchi (2000)
How accurately RDF should be known?

• The simplest model which takes into account excluded volume effects

• The integration can be performed analytically

• The parameter $R$ is evaluated requiring consistency for energy or pressure

\[
\omega_L^2 = \omega_p^2 e^{-R\kappa} \left[ (1 + R\kappa) \left( \frac{1}{3} - \frac{2 \cos Rq}{R^2 q^2} + \frac{2 \sin Rq}{R^3 q^3} \right) \right. \\
- \frac{\kappa^2}{\kappa^2 + q^2} \left( \cos Rq + \frac{\kappa}{q} \sin Rq \right) \left. \right]
\]

\[
\omega_T^2 = \omega_p^2 e^{-R\kappa} (1 + R\kappa) \left( \frac{1}{3} + \frac{\cos Rq}{R^2 q^2} - \frac{\sin Rq}{R^3 q^3} \right).
\]

Khrapak et al. (2016)
Long-wavelength regime

- In the long-wavelength regime such a simple approximation is very useful

- Explicit expression for R: Roughly $R(\kappa) \simeq 1 + \kappa/10$

MD data by Ohta and Hamaguchi (2000)
One-component-plasma (OCP) limit

• Again, simple explicit expressions:

\[ \omega_L^2 = \omega_p^2 \left( \frac{1}{3} - \frac{2 \cos Rq}{R^2q^2} + \frac{2 \sin Rq}{R^3q^3} \right) \]

\[ \omega_T^2 = \omega_p^2 \left( \frac{1}{3} + \frac{\cos Rq}{R^2q^2} - \frac{\sin Rq}{R^3q^3} \right) \]

• Good accuracy at strong coupling

Left: Hansen et al. (1974); Right: Schmidt et al. (1997)
Deviations from pure Yukawa interaction in complex (dusty) plasmas

- Electron and ion collection ➔ Power-law long-range asymptotes
- Non-linear ion-particle interaction ➔ Variability of the effective screening length
- Plasma production and loss ➔ Double-Yukawa interaction potential
- Ion flows ➔ Wake-mediated interaction

• Can QLCA be used to discriminate between different interactions in complex plasmas?
Representative examples of interaction

- Double-Yukawa potentials
  \[
  V(r) = \frac{Q^2}{r} \left[ \epsilon_1 \exp(-r/\lambda_1) + \epsilon_2 \exp(-r/\lambda_2) \right]
  \]

- Yukawa + inverse square \( r \)
  \[
  V(r) = \frac{Q^2}{r} \left[ (1 - \epsilon) e^{-r/\lambda_D} + (\epsilon \lambda_D / r) \left( 1 - e^{-r/\lambda_D} \right) \right]
  \]

<table>
<thead>
<tr>
<th>Case</th>
<th>Functional form</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Eq. (2)</td>
<td>( \epsilon_1 = \epsilon_2 = 0.5, \lambda_1 = 0.7\lambda_D, \lambda_2 = 6.3\lambda_D )</td>
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<tr>
<td>2</td>
<td>Eq. (2)</td>
<td>( \epsilon_1 = 0.8, \epsilon_2 = 0.2, \lambda_1 = \lambda_D, \lambda_2 = 10\lambda_D )</td>
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<tr>
<td>3</td>
<td>Eq. (2)</td>
<td>( \epsilon_1 = \epsilon_2 = 0.5, \lambda_1 = \lambda_D, \lambda_2 = \infty )</td>
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<tr>
<td>4</td>
<td>Eq. (3)</td>
<td>( \epsilon = 0.05 )</td>
</tr>
<tr>
<td>5</td>
<td>Eq. (3)</td>
<td>( \epsilon = 0.1 )</td>
</tr>
</tbody>
</table>

Khrapak et al. (2017)
Fingerprints of interactions: Double Yukawa class

Long-range asymptote of the interaction potential affects the dispersion at long wavelengths, which can be measured experimentally.

Khrapak et al. (2017)
Fingerprints of interactions: Yukawa + $1/r^2$

Long-range asymptote of the interaction potential affects the dispersion at long wavelengths, which can be measured experimentally

Khrapak et al. (2017)
Is QCA/QLCA equally good for hard and soft interactions?

- Systematic study of IPL fluids near the fluid-solid transition
- Agrawal & Kofke (1995) data on coexistence fluid densities of the IPL model
- MD simulations for a number of IPL exponents \((10 \leq n \leq 100)\)
- Analysis: Structure, dynamics, longitudinal mode dispersion
Structure (RDF) and dynamics (VAF)

- With increasing the exponent $n$ structure and dynamics tend to HS-like

- Raveche-Mountain-Streett criterion of freezing is not very accurate when potential softness varies in a wide range

- More accurate criterion can be based on the height of the minimum of $g(r)$

Chart 13

Couedel&Klumov (2016)
Dispersion of the longitudinal mode

- QCA/QLCA is reasonably accurate only for sufficiently soft potentials with \( n < 20 \)

Khrapak, Klumov, Couedel (2017)
Sound velocities

- Elastic QCA longitudinal sound velocity overestimates that measured in MD experiment and diverges at large $n$

\[ C_L^2 = \frac{(3n+1)v_p^2}{5}, \]

- Instantaneous sound velocity is close to that measured in MD, but also diverges at large $n$

\[ C_\infty = \frac{K_\infty}{mp} = C_L^2 - \frac{4}{3} C_T^2 \]

- The HS sound velocity remains finite

Khrapak, Klumov, Couedel (2017)
Conclusion

- We have discussed different aspects of describing theoretically collective modes in simple fluids with applications to complex (dusty) plasmas

- QLCA/QCA approach
  - Simplification based on excluded volume arguments produces useful analytical expressions
  - Possibility to discriminate between different interactions using the long-wavelength dispersion relation
  - Applicability is limited by sufficiently soft interactions
Thank you for your attention!

Acknowledgments

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