Experiences with Scheduling Problems on Adiabatic Quantum Computers

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Abstract—The realization of quantum computing models is nowadays a grand challenge regarding supercomputer architectures which may potentially exceed the scaling limits of Moore's Law. The Canadian company D-Wave Systems Inc. developed a hardware for quantum annealing which is well suited for solving combinatorial optimization problems. We investigate the formulation of a scheduling problem for quantum annealing and report on our experience with the D-Wave system at NASA Ames.

1 Introduction

Planning and scheduling problems are usually hard combinatorial optimization problems. Therefore new hardware architectures with potential supremacy above classical approaches, like quantum devices, are worth studying.

The realization of quantum computing models is nowadays a grand challenge regarding supercomputer architectures which may potentially exceed the scaling limits of Moore's Law [2]. The Canadian company D-Wave Systems Inc. developed the first commercially available quantum annealer. Quantum annealing is well suited for solving combinatorial optimization problems with quadratic objective function over binary variables without constraints (QUBOs). The execution of QUBO problems on a D-Wave machine requires a mapping of the problem to the interconnection topology of the hardware, the so-called Chimera graph.

2 Satellite Scheduling

Hard planning and scheduling problems as they appear in aerospace research can be mapped and solved on quantum optimizers [4]. In this paper, we will focus on a problem from the field of satellite mission planning based on [3]. The planning of satellite missions can be mapped to a state machine [5]. The goal is to achieve a certain mission objective while obeying several boundary conditions (charge, data storage, etc.).

2.1 Exemplary Formulation for quantum annealers

The D-Wave quantum annealer can be regarded as a heuristic solver of combinatorial optimization problems of the QUBO form

$$Q(\mathbf{x}) = \sum_i h_i x_i + \sum_{ij} J_{ij} x_i x_j \,,$$

where x_i are binary variables. A QUBO is comprised of linear and quadratic terms. It can be represented as an undirected graph in the following way: Each binary variable is represented as a node in the graph. The coefficients of the linear terms are assigned to each node and each non-vanishing coefficient of the quadratic terms is represented as a weighted edge between two nodes. However, the hardware does not allow for arbitrary connections between nodes since the D-Wave chips have a Chimera graph design [4]. This problem can be overcome by representing a logical qubit x_i by several physical qubits on the chip. All of these physical qubits are coupled together ferromagnetically in order to make sure that the physical qubits representing a logical qubit agree on a value. This procedure is called embedding. The result is another QUBO with reduced connectivity to fit onto the Chimera graph hardware. The more the graph representing the problem differs from the Chimera graph, the more physical qubits are needed. In [1] it is shown that in the case of a complete graph of size N the number of physical qubits is $\mathcal{O}(N^2)$. An example of the embedding of a complete graph of size 10 is shown in figure 1.

2.2 QUBO formulation for satellite scheduling

In this section, we will derive a QUBO formulation for a simplified version of the model from [5]. The resulting model exhibits some similarities to the Mars lander mission planning done in [4], [6]. We assume the satellite can occupy three states: charging (c), downlink (d) and experiment (e). We discretize the time and assume time steps $t \in \{0, 1, \ldots, T\}$. The variable x_{st} tells us if the satellite is in the state $s \in \{c, d, e\}$ at time t. With this, the time sequence of these variables represents the schedule we want to optimize. The optimization goal is

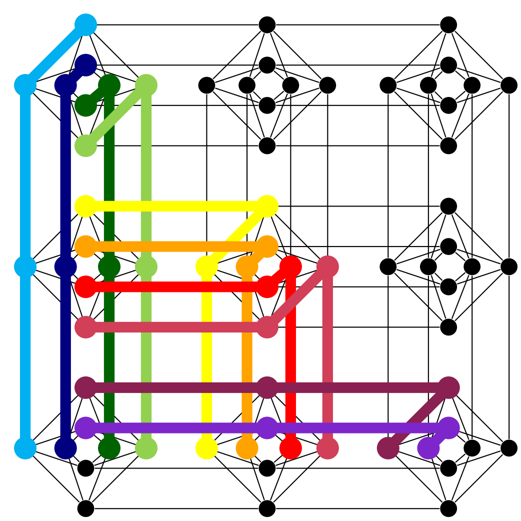

Fig. 1. Embedding of a complete graph of size 10 into the Chimera graph

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to record as much data as possible during the mission. There are two satellite variables which may change over time: The charge of the battery C and the data stored on the memory D. The rate with which these variables are changing depending on state s are denoted by c_s and d_s respectively. For example the experiment state will increase the data $d_d > 0$ and decrease the charge $d_c < 0$. Both the battery and the memory define an upper and lower limit for the charge and the data, respectively. Not every state can be occupied at each instance in time. For example the charging through solar panels is only possible in the sunlight, or the downlink is only possible in the vicinity of a ground station. Therefore for each state s there is a subset of times $\tau_s \subseteq \{0, 1, \ldots, T\}$ at which the satellite can occupy this state. To enforce this constraint, we remove all the variables $x_{st} \in \{x_{st} | t \in \tau_s\}$. For the sake of simplicity, we assume that each state has minimum duration of 1.

The QUBO $Q = \sum_i Q_i$ is comprised of the following contributions:

1) At each time step the satellite can only occupy a single state. This is enforced by

$$Q_1 = p_1 \sum_t \left(\sum_s x_{st} - 1 \right)^2 .$$

2) At all times, the charge must be in between the upper c_{max} and lower c_{min} limit of the battery.

$$c_{\text{min}} < c_0 + \sum_s c_s \sum_{\tau < t} x_{s\tau} < c_{\text{max}} .$$

Here, c_0 is the charge at the mission start. In order to enforce this inequality, we need to introduce slack variables

$$y_t := c_0 + \sum_s c_s \sum_{\tau < t} x_{s\tau} - c_{\text{min}} \in \{0, c_{\text{max}} - c_{\text{min}}\} .$$

We need to represent these slack variables in terms of binary variables:

$$y_t = \sum_\alpha 2^\alpha y_{t\alpha}$$

The contribution to the QUBO reads

$$Q_2 = p_2 \sum_t \left(c_0 + \sum_s c_s \sum_{\tau < t} x_{s\tau} - c_{\text{min}} - \sum_\alpha 2^\alpha y_{t\alpha} \right)^2 .$$

3) Analogously, we can obtain the contribution from the memory constraint as

$$Q_3 = p_3 \sum_t \left(d_0 + \sum_s d_s \sum_{\tau < t} x_{s\tau} - d_{\text{min}} - \sum_\alpha 2^\alpha z_{t\alpha} \right)^2 .$$

4) The contribution which enforces the maximal downlink reads

$$Q_4 = -p_4 \sum_s d_s \sum_{t=1}^{T} x_{st} .$$

The penalty weights p_i need to be chosen in such a way that the hard constraints are fulfilled, i.e.,

$$\sum_{i=1}^{3} Q_i = 0 ,$$

and the value of Q_4 is as small as possible.

3 Experiments on D-Wave's 2X System

Due the statistical nature of the machine, multiple runs are necessary to solve a QUBO with a certain success probability. Usually one solves the same the problem thousands of times before investigating the statistics. The success probability p is then given by

$$p = \frac{N_{\text{successful}}}{N_{\text{total}}} ,$$

where $N_{\text{successful}}$ and N_{total} are the number of successful and total runs, respectively. As it is done for example in [4], one typically uses the expected run time to obtain a 99% success probability

$$T = \frac{\ln(1 - 0.99)}{\ln(1 - p)} T_{\text{Anneal}}$$

as a measure of performance. Here T_{Anneal} is the run time of a single run of the adiabatic quantum computer. Usually the value is set to values around $T_{\text{Anneal}} = 20\mu\text{sec}$.

4 Conclusion

We explored the applicability of quantum annealing to a selected space planning problem. For the satellite scheduling problem, we found an adequate QUBO formulation.

Our experiments on D-Wave's 2X System indicate that this problem can be solved. As common for heuristic solvers, multiple runs and a statistical analysis of the results were necessary to guarantee a high solution quality. Due to the small problem sizes, runs on the D-Wave machine were extremely fast and global solutions to the combinatorial optimization problems were found with very high probability.

Absolute performance and scalability of quantum annealing hardware for general problems is hard to assess from experiments. One reason for this is the fact that only problems of moderate size could be executed on the available D-Wave hardware.

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