Fingerprints of different interaction mechanisms on the collective modes in complex (dusty) plasmas


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Complex Plasma: Interdisciplinary Research Field

- Solid particles in the plasma background
- Particles are charged (mainly by collecting electrons and ions)
- Classical system of strongly interacting particles
- Interdisciplinary research area

Astrophysical topics:

Laboratory:

Industry:

Fusion:

From Kretschmer, Selwyn, Sharpe, et al.
Waves in complex plasmas (few examples)

- Low damping due to particle-neutral gas collisions
- Simple determination of dispersion relation
- Individual particle motion can often be resolved
- Various topics: acoustic waves, shock waves, Mach cones, instabilities, etc.

Ratynskaia et al. (2004)  
Yaroshenko et al. (2004)  
Fortov et al. (2000)
Motivation:

• Central to the topic of wave phenomena in dusty (complex) plasmas is the dust acoustic velocity (DAV)

• Is DAV a universal quantity in complex (dusty) plasmas?

• What determines the magnitude of DAV when the interparticle interactions are of Yukawa type?

• What happens when interactions differ from the Yukawa type?
Theory of dust acoustic waves (DAW)

• Continuity and momentum equations for the particle component

\[
\frac{\partial n_d}{\partial t} + \nabla (n_d v_d) = 0 ,
\]

\[
\frac{\partial v_d}{\partial t} + (v_d \cdot \nabla) v_d = -\frac{eZ}{m_d} \nabla \phi - \frac{\nabla (n_d T_d)}{m_d n_d} - \sum_\beta v_d \beta (v_d - v_\beta) .
\]

• Boltzmann distribution for ions and electrons

\[
n_{e(i)} \approx n_0 \exp(\pm e \phi / T_{i(e)}).
\]

• Poisson equation

\[
\nabla^2 \phi = -4\pi e (n_i - n_e + Z n_d)
\]

• Standard linearization procedure [perturbations are proportional \(\sim \exp(-i\omega t + ikr))\]

\[
\omega^2 \frac{k^2}{k^2} = \gamma_d v_d^2 T_d + \frac{\omega_{pd}^2 \lambda_D^2}{1 + \lambda_D^2 k^2}
\]

Rao, Shukla, Yu (1990)
Dust acoustic velocity

• In the long-wavelength limit DAW exhibits acoustic dispersion with the DA velocity

\[ C_{DA} = \omega_{pd} \lambda_D \equiv \sqrt{\frac{|Z|}{T_d}} \sqrt{\frac{T_i}{P\tau}} \sqrt{\frac{1}{1 + (1 + P)\tau}} v_{Td} \]

• Here \( P = \left| Z \right| n_d / n_e \) is the Havnes parameter, \( \tau = T_e / T_i \) is the electron–to–ion temperature ratio (normally \(~100\)), \( Z \) is the particle charge number (normally \(~1000\))

• DA velocity can be remarkably higher than the thermal velocity (plasma related effect)
Strong-coupling effects: Fluid approach with proper thermodynamics

- The standard expression for the sound velocity of single-component fluids reads

\[ c_s = v_T \sqrt{\gamma \mu} \]

where \( \gamma = C_p / C_v \) is the adiabatic index, \( \mu = (1/T)(\partial P / \partial n)_T \) is the isothermal compressibility modulus, and \( v_T \) is the particle thermal velocity.

- Proper equation of state is required.

- The simple practical equations of state for 3D and 2D Yukawa fluids and crystals have been recently worked out in papers by Khrapak and Thomas, PRE (2015); Khrapak, Kryuchkov, Yurchenko and Thomas, JCP (2015)

- DA sound velocity can be considerably modified due to strong coupling.
Dependence on coupling and screening

- Weak-coupling => conventional DAV scale

\[ c_0 = \omega_p \lambda_D. \]

- Weak dependence of \( c_s/c_0 \) on \( \Gamma \) deep in the fluid regime

- \( c_s/c_0 \) is sensitive to the screening parameter

- \( c_s/c_0 \) drops by almost one order on the way from near-OCP to \( \kappa = 5 \).

Khrapak and Thomas (2015)
**Strongly coupled effects: Quasi-localized charge approximation (QLCA)**

- Generic expressions for the longitudinal dispersion relations:

\[
\omega_L^2 = \frac{n}{m} \int \frac{\partial^2 V(r)}{\partial z^2} g(r)[1 - \cos(kz)] dr
\]

was obtained in models of collective motion in liquids by Zwanzig (1967), quasi-crystalline approximation by Hubbard&Beeby (1969), Takeno&Goda (1971). Similar expressions occur from the analysis of frequency moments of \( S(k,\omega) \).

- In the context of plasma physics it is known as QLCA after Kalman and Golden applied the approximation to one-component-plasma and related systems.

- For non-correlated particles \([g(r)=1]\) with Yukawa interactions the integration can be performed analytically and the result is the **conventional DAW dispersion relation**.
Dispersion relations at strong coupling

• Yukawa interaction potential

• First applied by Rosenberg and Kalman (1997) in the regime of long-wavelengths and weak screening

• Kalman et al. (2000) computed $g(r)$ using the HNC scheme get results in good agreement with MD modeling by Ohta and Hamaguchi (2000)
Relations between the fluid (thermodynamic) and QLCA approaches

• The sound velocities evaluated using the fluid thermodynamic approach and QLCA are very close, QLCA yields systematically slightly higher values.

• The difference is because QLCA is a theory for high-frequency perturbations.

TABLE I. Reduced sound velocity \( c_s/\omega_\nu a \) of Yukawa fluids as calculated from the QLC approximation and present fluid model for several phase state points. QLCA data are from Ref. [29]. For details see the text.

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>( \Gamma / \Gamma_m )</th>
<th>QLCA</th>
<th>Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.12</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
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<td>0.70</td>
<td>0.96</td>
<td>0.94</td>
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<tr>
<td>2.0</td>
<td>0.70</td>
<td>0.41</td>
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<tr>
<td>3.0</td>
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<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>3.0</td>
<td>0.70</td>
<td>0.21</td>
<td>0.19</td>
</tr>
</tbody>
</table>

QLCA results by Kalman et al. (2000) and Donko et al. (2008)
Actual interactions in complex plasmas

• Can deviate from single Yukawa shape:
  • Electron and ion collection ➔ Power-law long-range asymptotes
  • Non-linear ion-particle interaction ➔ Variability of the effective screening length
  • Plasma production and loss ➔ Double-Yukawa interaction potential
  • Ion flows ➔ Wake-mediated interaction

• Can QLCA be used to discriminate between different interactions in complex plasmas?
Representative examples of interaction

- Double-Yukawa potentials

\[ V(r) = \frac{Q^2}{r} \left[ \epsilon_1 \exp\left(-r/\lambda_1\right) + \epsilon_2 \exp\left(-r/\lambda_2\right) \right] \]

- Yukawa + inverse square \( r \)

\[ V(r) = \frac{Q^2}{r} \left[ (1-\epsilon)e^{-r/\lambda_D} + (\epsilon \lambda_D/r) \left(1-e^{-r/\lambda_D}\right) \right] \]

| Table I. Summary of the model interaction potentials considered in this study (Cases 1 - 5). |
|---|---|---|---|
| Case | Functional form | Parameters |
| 1 | Eq. (2) | \( \epsilon_1 = \epsilon_2 = 0.5, \lambda_1 = 0.7\lambda_D, \lambda_2 = 6.3\lambda_D \) |
| 2 | Eq. (2) | \( \epsilon_1 = 0.8, \epsilon_2 = 0.2, \lambda_1 = \lambda_D, \lambda_2 = 10\lambda_D \) |
| 3 | Eq. (2) | \( \epsilon_1 = \epsilon_2 = 0.5, \lambda_1 = \lambda_D, \lambda_2 = \infty \) |
| 4 | Eq. (3) | \( \epsilon = 0.05 \) |
| 5 | Eq. (3) | \( \epsilon = 0.1 \) |
Double Yukawa class
Yukawa + inverse square class

Chart 15
Conclusion

• Dust acoustic velocity is NOT a universal quantity in complex (dusty) plasmas

• If the Yukawa potential can be regarded as a reasonable approximation for actual interactions, then DAV depends strongly on the screening parameter

• Deviations from Yukawa interactions have pronounced effect on the dispersion relations, especially in the long wavelength regime

• New possibility to experimentally discriminate between different interactions predicted theoretically
Thank you for your attention!