Kinetics of fluid demixing in complex plasmas: Domain growth analysis using Minkowski tensors

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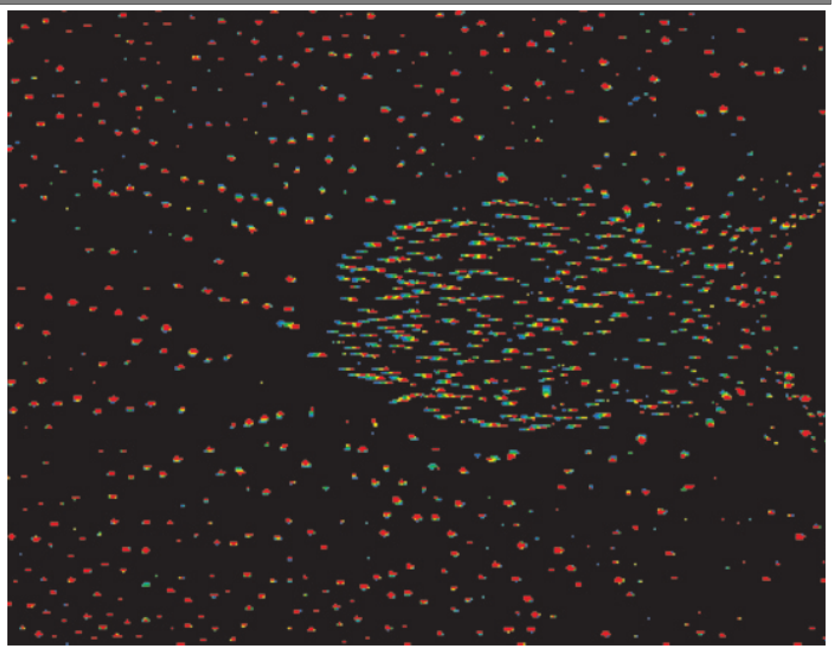
Introduction

Minkowski tensors (MTs) provide new insights into everyday phenomena of many-body physics like demixing of a binary fluid. Ideal model systems to investigate the demixing dynamics are complex plasmas consisting of small microparticles of two different sizes immersed in a (cold) plasma. Here, we present MTs as a proper local and nonlinear measure quantifying the demixing kinetics on the particle level. MTs provide measures for the elongation or metrics for the symmetry of a body allowing a very precise detection of ordered particle phases.

Demixing

When a binary fluid is forced into the immiscible state, it starts to dynamically demix until the thermodynamically stable state of two coexisting fluids is reached. This spinodal decomposition is accompanied by a domain growth that is believed to be selfsimilar in time; i.e., the domain morphology is preserved. This implies a single time-dependent characteristic length that obeys a power-law growth $L(t) \propto t^{\alpha}$.[1]



Phase separation under microgravity conditions in the PK-3 Plus rf discharge chamber on board the ISS.[2]

Methods

Model [3]

We consider a stationary isotropic highly collisional plasma with a effect of plasma production and loss. Plasma production is due to electron impact ionization. Plasma losses are associated with the combined effect of three-body bulk recombination and ambipolar diffusion to the plasma boundaries. The continuity and momentum equations for e.g. ions are:

$$\nabla(n_i v_i) = \nu_I n_e - \nu_L n_i - \beta n_e n_i$$

$$(v_i \nabla) v_i = -(e/m_i)\nabla\Phi - (u_{T_i}^2/n_i)\nabla n_i - \nu_i v_i$$

(here v_i and m_i are the velocity and mass of the ions, ν_I is the ionization frequency, ν_L is the characteristic frequency of ambipolar losses, β is the recombination coefficient, ν_i is the characteristic frequency of ion-neutral collisions, Φ is the electrical potential, and u_{T_i} is the ion thermal velocity.)

Solving the Poisson equation (using linearization) leads to a double Yukawa interaction potential:

$$\Phi(r) = \frac{1}{r}\left(Z_{SR}^{*} e^{-r/\lambda_{SR}} + Z_{LR}^{*} e^{-r/\lambda_{LR}}\right)$$

Simulation [4]

MD simulations with the Langevin thermostat were employed: 729000 particles in a cubic box with mean inter-particle distance $d = 0.3mm$ and periodic boundary con-ditons. Particle sizes were $2a_1 = 3.4\mu m$ and $2a_2 = 9.2\mu m$. Different ratios $\Lambda = \lambda_{LR}/\lambda_{SR}$ were analyzed.

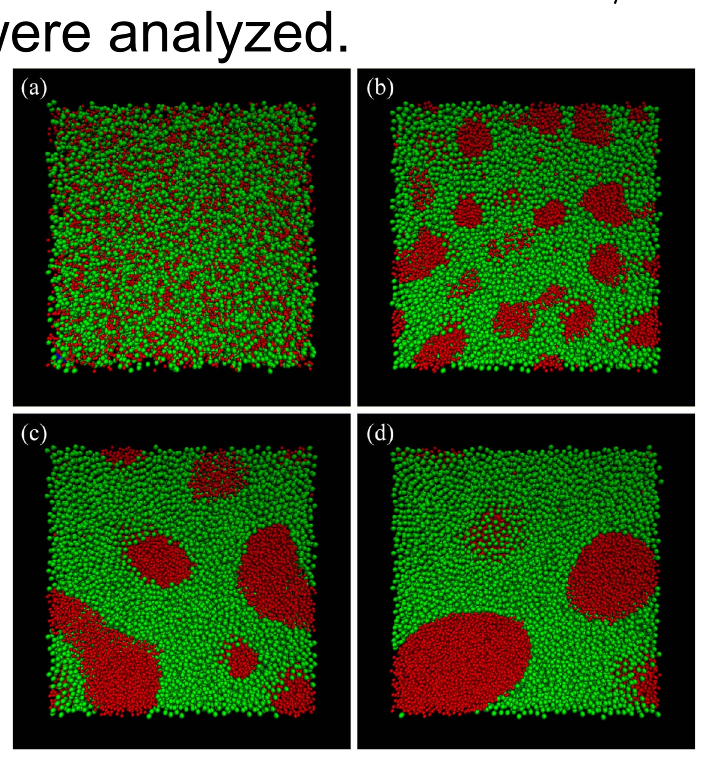


Illustration of the demixing simulation [4]

Minkowski Valuations[5]

Functionals → **Tensors**

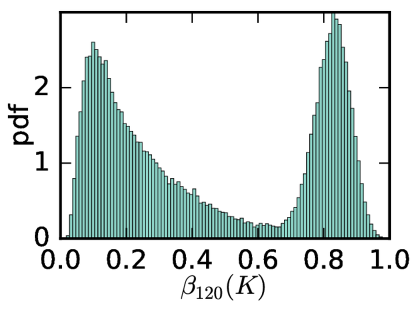
$$W_0(K) = \int_K \mathrm{d}^D r \quad\longrightarrow\quad W_0^{a,0}(K) := \int_K \mathrm{d}^D r \; \mathbf{r}^{\odot a}$$

$$W_\nu(K) = \int_{\partial K} G_\nu(r)\,\mathrm{d}^{D-1} r \qquad W_\nu^{a,b}(K) := 1/D \int_{\partial K} \mathrm{d}^{D-1} r \; G_\nu(r)\; \mathbf{r}^{\odot a} \odot \mathbf{n}^{\odot b}$$

Rank 2 isotropy index:

$$\beta_\nu^{a,b}(K) := \frac{\lambda_{min}\left(W_\nu^{a,b}(K)\right)}{\lambda_{max}\left(W_\nu^{a,b}(K)\right)}$$

provides a measure for the elongation of a body K



Histogram method to calculate the demixed domain size. Ordered domains correspond to β close to 1.

Rank 4 symmety metric:

consider $W_1^{04}(K) = 1/3 \cdot \int_{\partial K} \mathrm{d}^2 r\, \mathbf{n}(\mathbf{r}) \otimes \mathbf{n}(\mathbf{r}) \otimes \mathbf{n}(\mathbf{r}) \otimes \mathbf{n}(\mathbf{r})$ rewrite as

$$M = \begin{bmatrix} S_{xxxx} & S_{xxyy} & S_{xxzz} & \sqrt{2}S_{xxyz} & \sqrt{2}S_{xxxz} & \sqrt{2}S_{xxxy} \\ S_{xxyy} & S_{yyyy} & S_{yyzz} & \sqrt{2}S_{yyyz} & \sqrt{2}S_{yyxz} & \sqrt{2}S_{yyxy} \\ S_{xxzz} & S_{yyzz} & S_{zzzz} & \sqrt{2}S_{zzyz} & \sqrt{2}S_{zzxz} & \sqrt{2}S_{zzxy} \\ \sqrt{2}S_{xxyz} & \sqrt{2}S_{yyyz} & \sqrt{2}S_{zzyz} & 2S_{yzyz} & 2S_{yzxz} & 2S_{yzxy} \\ \sqrt{2}S_{xxxz} & \sqrt{2}S_{yyxz} & \sqrt{2}S_{zzxz} & 2S_{yzxz} & 2S_{xzxz} & 2S_{xzxy} \\ \sqrt{2}S_{xxxy} & \sqrt{2}S_{yyxy} & \sqrt{2}S_{zzxy} & 2S_{yzxy} & 2S_{xzxy} & 2S_{xyxy} \end{bmatrix}$$

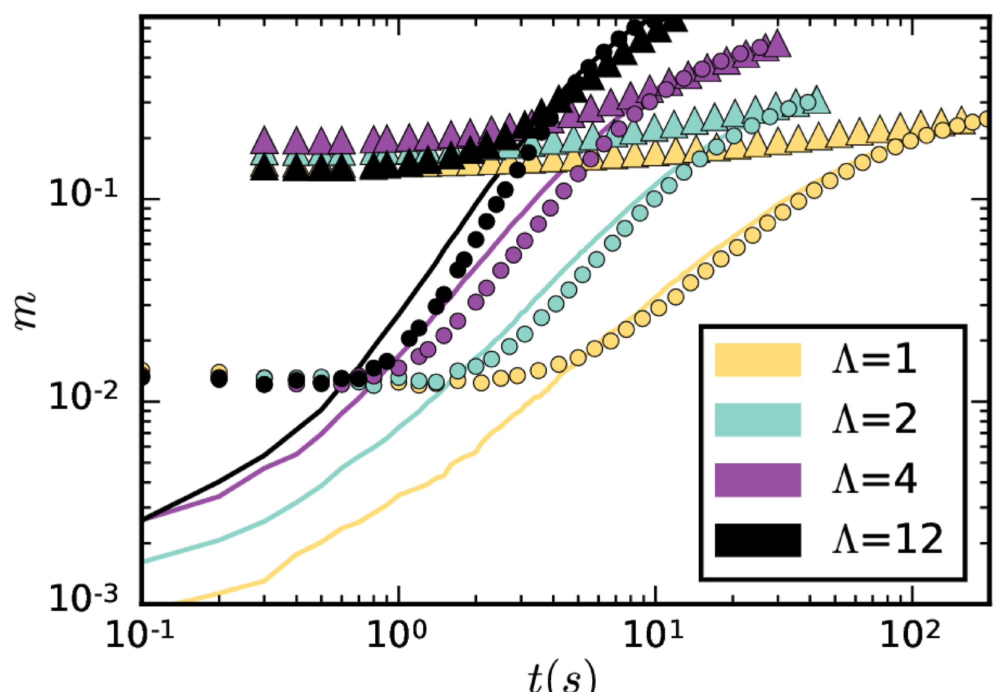
where $S = W_1^{04}(K)/W_1(K)$

The eigenvalues ζ_i of M are a symmetry fingerprint of a body K and induce a metric on the space of bodies:

$$\Delta(K_1, K_2) := \left(\sum_{i=1}^{6} (\zeta_i(K_1) - \zeta_i(K_2))^2\right)^{1/2}$$

Results

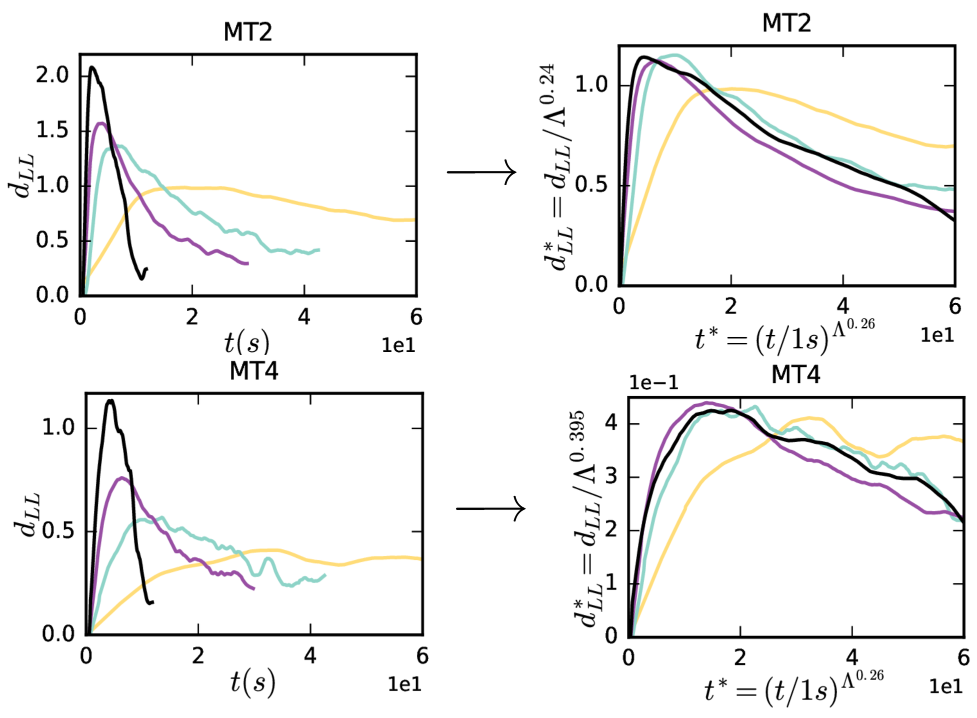
Dynamic range and sensitivity



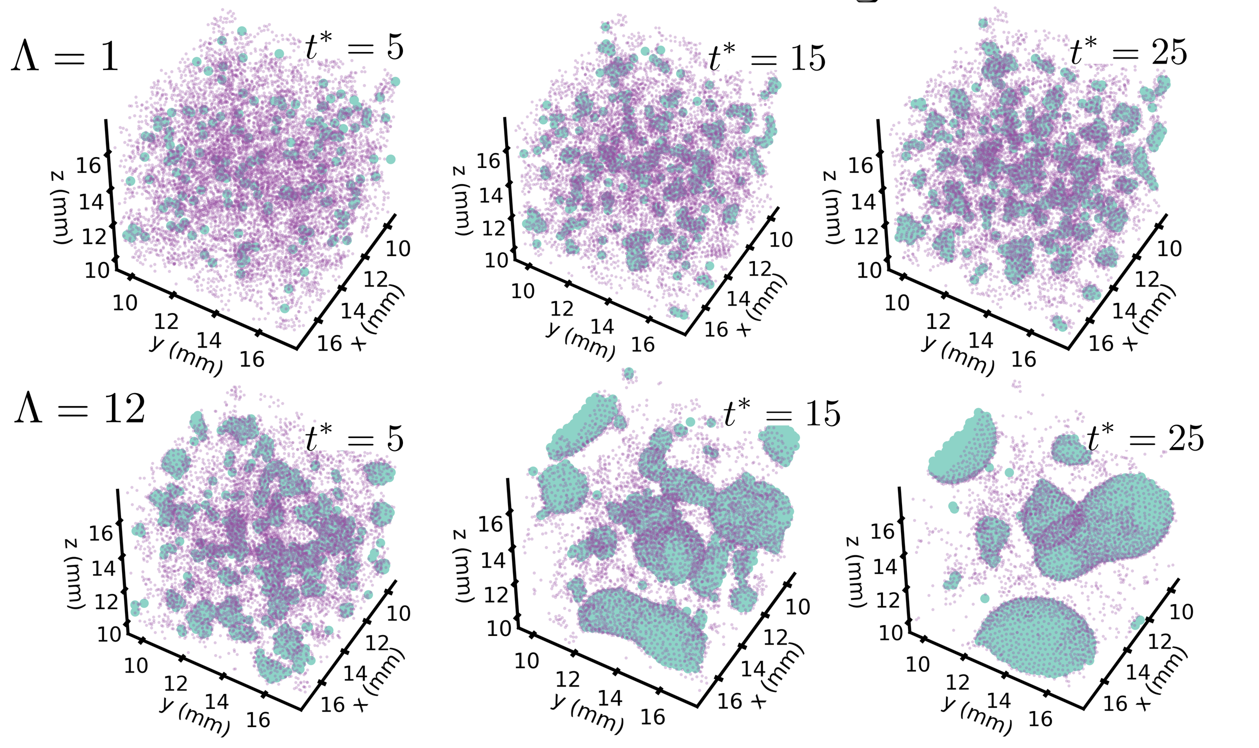
Growth of the species 1 domains. The evolution of domain size measures is plotted. Triangles are obtained by the a power spectrum (PS) method, lines by the volume MT0 method, dots by the MT2 isotropy index method. Different colors indicate different screening length ratios Λ. The dynamic range and sensitivity of MT measures is higher than for the PS measure.

Hints of universal behavior

Proper scaling of the slope of Minkowski measures points towards universal behavior:



Qualitative difference in demixing behavior



Species 1 particles detected by the MT2 analysis as contributions to demixed domains are color coded in turquoise, the remaining species 1 particles are color coded in violet and are smaller in size. To assure visibility particles of species 2 are not shown. They are homogeneously dispersed. For $\Lambda = 1$ only agglomeration, for $\Lambda = 12, 4, 2$ followed by cascades of merging domains.

Conclusion

Minkowski Tensor analysis are likely to become a useful tool for further investigation of this (and other) demixing processes. It is capable to reveal (nonlinear) local topological properties, probing deeper than (linear) global power spectrum analysis, however still providing easily interpretable results founded on a solid mathematical framework.

We demonstrate that the MTs are superior to common linear measures in all respects, namely in the sensitivity at the onset of demixing, in the dynamic range, etc. Only by using MTs we detect qualitative differences in the system evolution depending on the interparticle interactions. For an interaction potential with one length scale phase separation only occurs due to agglomeration of neighboring particles, whereas for a potential with two distinct length scales the agglomeration phase is followed by merging cascades of already demixed domains forming ever larger domains. Proper scaling of the slope of the temporal evolution of MTs hints towards universal behavior of the demixing parameters for two-scale potentials.

References:
[1] Bray, Advances in Physics, 2002
[2] Ivlev et al., EPL, 2009
[3] Khrapak et al., Physics of Plasmas, 2010
[4] Wysocki et al., PRL, 2010
[5] Schröder-Turk et al., New Journal of Physics, 2013

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