EVALUATING TRANSPORT USER BENEFITS: ADJUSTMENT OF THE LOGSUM DIFFERENCE FOR CONSTRAINED TRAVEL DEMAND MODELS

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ABSTRACT

Transport user benefits are of great importance in cost benefit analysis when appraising transport projects. Generally they have the greatest impact on the results of cost-benefit analyses. It is common to adopt the consumer surplus for calculating transport user benefits. The consumer surplus measure is based on the underlying demand model and follows from the integration of the demand curve. If the popular logit model is used for forecasting travel demand, consumer surplus measure takes a closed form (the "logsum") that is easy to calculate. Furthermore, in cost-benefit analyses the change in consumer surplus between an initial and final state is needed, which can be easily derived by the difference of the logsums of the two states. This logsum approach is proven and correct for travel demand models based on the logit model without multiple constraints. However, for travel demand models dealing with two or more sets of constraints the logsum approach fails.

In this paper we describe a mathematical approach for a transport user benefits measure that corresponds to the consumer surplus and is universal for all travel demand models with constraints. We exemplarily derive the measure for a doubly constrained trip distribution. The applicability of the derived approach is shown by a small example.

INTRODUCTION

Transport investments and policies are often associated with high investment costs for society and cost changes for transport users, which might lead to welfare changes. It is therefore essential to anticipate welfare impacts ex ante and give decision makers robust decision support. There are different concepts for evaluating projects and policies; however, cost-benefit analysis (CBA) is normally the method of choice.

Generally, in transport project and policy appraisals, transport user benefits have (in addition to investment costs) the most impact on welfare changes. User benefits result from the changes of user costs such as travel time and travel cost between the "initial state" (the most likely transport situation over the course of the appraisal period if no intervention were to occur) and the "final state" (the initial state with transport intervention). Economic theory – as the theoretical basis of the CBA – provides the concept of the (change in) consumer surplus for the calculation of the transport user benefits. This measure can be derived by the mathematical integral of the demand function (see, for example, Varian, 1992).

Accordingly, there is a direct link between user benefits, costs, and travel demand changes. The applied method of integrating the travel demand function defines the accuracy of the resulting user benefits. The standard approach in practice is the so called "Rule of a Half" (RoH). It has been widely used in project investments and policy analysis since its first introduction in the 1960s (Williams, 1977). The concept assumes a linear path between the user costs in the initial and final state, whereas the real shape of the underlying demand function is neglected. For this reason, the RoH only provides a precise benefit measure for small user cost changes and only if the demand curve is nearly a straight line, otherwise it can be only considered as a rough approximation (De Jong et al. 2007).

Demand curves of travel demand models are generally not straight lines, and the RoH often provides inexact transport user benefits. However, there are other approaches to obtain more precise results. In particular, the antiderivative of the applied travel demand function could be used for calculating a mathematical exact consumer surplus. This approach presupposes that the antiderivative is known. If that is the case, direct integration is possible and would be the best solution.

The widely used multinomial logit model (MNL) provides the antiderivative that is called "logsum term" (or short "logsum"). The MNL is based on the concept of random utility maximization and was microeconomically derived by McFadden (1981). In context of project evaluation, the consumer surplus of the initial and final states can be provided by logsums. Hence, the change in consumer surplus is calculated as the logsum difference.

The logsum approach is a theoretically well-founded and easily calculable method for providing transport user benefits. It is applied by more and more researchers and transportation planners (Zhao et al., 2012). However, it strongly depends on the use of the MNL (or nested logit) as the underlying travel demand function. There are many travel demand models applied in research and practice that are not apparently congruent to the MNL. This is especially the case for travel demand models with constraints. These models are traditionally used for trip distribution and regard origin and destination constraints, which ensure that given total numbers of trips produced in and attracted to each traffic zone equal resulting numbers of trips. If such a model is used for calculating trip distribution, the logsum approach fails and as a rule transport user benefits are approximated by the RoH.

In this paper we want to transfer the logsum approach for these widely used constrained travel demand models and provide a mathematical exact transport user benefit measure for these models. For this purpose an adjustment of the logsum difference is necessary. We will carry out the analysis by the EVA model (Vrtic et al., 2007) as a representative of constrained models, which is characterized by different constraint types. For this reason the paper first introduces travel demand models with constraints, in particular the EVA model and its "logit version". It then goes on with benefit measures and problems of the logsum difference approach with constraint models. After that, the mathematical adjustment of the logsum approach follows. Then, a short and very simple example will serve as an illustration of the approach. Finally, a summery is provided. It has to be noted that the main aim of the paper is to provide a theoretical derivation of an adjusted logsum approach for travel demand models with constraints rather than to show specific applications and comparisons. These questions will be discussed in the future.

TRAVEL DEMAND MODELS WITH CONSTRAINTS

There are lots of different concepts and approaches for modelling travel demand, and the well-known 4-step-algorithm is of particular importance. These steps can be calculated successively or partly combined, whereby a combination of trip distribution and mode choice is most common. Here we focus on this approach and give models with constraints special attention.

Traditionally, constraints are used for trip distribution. The best-known model is the doubly constrained gravity model (see Wilson, 1967), which satisfies the given total numbers of trips originating in and attracted to a travel zone. The total numbers of trips are results of trip generation and input variables of trip distribution.

EVA Model

Lohse et al. (1997) extended the doubly constrained gravity model to mode choice and to less restrictive kinds of constraints. The model is called EVA model from the German terms for production (Erzeugung), distribution (Verteilung) and mode choice (Aufteilung). It is more universal and flexible than traditional approaches, which can also be formulated by the EVA approach. The algorithm contains trip generation, joint trip distribution, and mode choice. However, in the following we do not consider trip generation and mode choice, since it has no influence of the topic discussed in this paper. For a more comprehensive discussion of the EVA model see Vrtic et al. (2007).

The EVA approach models travel behavior based on a detailed segmentation of behaviorally homogenous groups of persons, their activities and so forth. For the sake of clarity, we omit differentiation, but all following analyses will be applicable to all groups. The main focus here is on constraints, and the EVA model distinguishes inelastic (or hard) and elastic (or soft) constraints. Inelastic constraints are used if the total number of trips of an alternative (origin or destination) can be derived exclusively from zone characteristics, i.e. transport supply and subsequent competition between zones do not have any effect on trip production. In contrast, elastic constraints are used if transport supply and zone competition have an additional influence.

Since it is not possible to show all facets, in the following the model specification with a set of inelastic constraints for the origins and a set of elastic constraints for the destinations will be used. The doubly constrained model is then:

$$\begin{aligned} & T_{ij} = f\left(g_{ij}\right) a_i b_j d_j^p \\ & \left.\begin{aligned} & \sum_j T_{ij} = O_i \\ & \sum_i T_{ij} = D_j \leq D_j^{max} \end{aligned}\right\} \text{constraints} \end{aligned} \tag{1}$$

with

a_i, b_j balancing factors to satisfy the constraints
f deterrence function with regard to the generalized costs g_{ijk} from zone i to zone j by mode k
O_i total number of trips originating at zone i
D_j total number of trips attracted to zone j
D_j^{max} maximal total number of trips attracted to zone j

T_{ij} trips from zone i to zone j
d_j^P destination specific attractor/potential

Thereby the trips T_{ij} are calculated taking into account generalized costs, destination specific attractors and the two sets of constraints. The constraints are satisfied by the balancing factors, which are solved iteratively. For solving this optimization problem different methods can be used, for example, the Furness algorithm (Furness, 1965). The given O_is are an inelastic set of constraints and are calculated in the trip generation a priori. The same applies to D_j^{max}, which, however, define only upper limits of what zones j can accommodate. The resulting D_js are calculated within the trip distribution under consideration of D_j^{max} and the spatial competition of destination zones. This competition is reflected by different generalized costs and destination specific attractors, which could be, for instance, population in zones or D_j^{max}.

There are different combinations of constraints that are possible and reasonable. For example, imposing inelastic constraints for work or school trips are necessary, because we expect to find workers or students at their places of work and education respectively. Elastic constraints are used for substitutable trips such as for leisure or shopping.

In addition to constraints, the above-mentioned deterrence function has a high importance for modelling results. The EVA model is of high flexibility and nearly every function type can be used. Probably the exponential function is the most important function and will be applied below. In addition to generalized costs, origin and destination specific characteristics can be considered. In equation (1), d_j^P represents such a variable. Considering zone specific values would only have an impact on modelling results in the case of elastic, but not inelastic constraints. The reason is that inelastic constraints are not influenced by zone competition and the resulting total numbers of trips per zone are guaranteed by modifying the balancing factors.

Travel demand models are used for estimating demand changes due to transport interventions. For this reason, forecasting capability is of great relevance. An important question is how constraints should be handled. There are different approaches to answering this question. If information about the (maximal) number of trips in each zone is known for the final state, the same procedure as in the initial state should be executed. If information is not available or valid, balancing factors defined in the initial state could be used. From this it follows that the total number of trips per zone are not restricted by constraints. However, the most common approach is that origin and destination constraints are given and new balancing factors are calculated.

EVA Logit Model

For detailed economic analyses and welfare evaluation, it is reasonable and possible to formulate the EVA model in terms of an MNL. Other authors, such as Anas (1983) and Erlander and Stewart (1990), have already shown how a constrained travel demand model may be defined as an MNL. However, they demonstrated it for doubly constrained gravity models which exclusively consider inelastic constraints. Here we apply the approach for the EVA model with different sets of constraints.

Starting point is the well-known MNL. This model is grounded on the economic concept of random utility maximization (RUM) and the individuals are assumed to choose the most beneficial (optimal) alternative (McFadden, 1981). Unlike the constrained models, the MNL respects per se a maximum of one set of constraints (singly constrained model) and calculates probabilities for the discrete alternatives and not the trips directly. These are definable by multiplying the probabilities with the total number of trips (total demand).

The comparison of alternatives is on the basis of the alternative specific utilities. Therefore utility functions including decision-relevant variables have to be defined. In addition, the utility is decomposed into an observed and an unobserved (random) component. The reason is that not all decision-relevant factors are observable for researchers. The utility that a single decision maker or representative of a homogenous group obtains from alternative ij can be described by (Ben-Akiva and Lerman, 1985):

$$U_{ij} = V_{ij} + \varepsilon_{ij} \quad \forall i, j \tag{2}$$

with

U utility of an alternative

V observable part of utility (indirect utility)

ε unobservable part of utility

In a standard MNL the unobserved utility is assumed to have i.i.d. extreme value distribution with standard variance ($\Pi^2/6$). Then the choice probabilities P_{ij} are given by:

$$P_{ij} = \frac{e^{V_{ij}}}{\sum_{i'} \sum_{j'} e^{V_{i'j'}}} \tag{3}$$

It represents the demand for a representative person. Trips T_{ij} result from a globally fixed MNL:

$$T_{ij} = P_{ij} T \tag{4}$$

with

T total number of trips over all zones (total demand)

For defining the EVA MNL, the observable part of utility is the central element. Again, we show the case with inelastic origin and elastic destination constraints, but other cases can be easily defined as well. Firstly, the utility function is defined by the observable alternative specific attributes (negative) generalized costs and destination utilities. In addition we include origin and destination shadow prices (Neuburger and Wilcox, 1976). The term “shadow” is used, since these “prices” are not definable a priori but derivable from the balancing factors of the EVA model a posteriori (after the iteration process). The given numbers of originating and attracting trips contain additional information about the underlying decision process of transport users. This information is implemented in the utility function by the shadow prices, which serve the emendation of the “a priori utility”. Then, by means of the shadow prices, the utility maximization process provides the probabilities of alternatives producing a result with satisfied constraints. The utility function can then be written:

$$V_{ij} = -g_{ij} + z_j + \theta_i + \tau_j \quad \forall i, j \tag{5}$$

with

θ_i, τ_j shadow prices of origin i and destination j

z_j observable destination utility

By implementing this utility function in the MNL we get the EVA MNL for estimating trips T_{ij}:

$$\begin{gathered} T_{ij} = P_{ij} T = \frac{e^{\left(-g_{ij} + z_j + \theta_i + \tau_j\right)}}{\sum_{i'} \sum_{j'} e^{\left(-g_{i'j'} + z_{j'} + \theta_{i'} + \tau_{j'}\right)}} T \\ \sum_j T_{ij} = O_i \\ \sum_i T_{ij} = D_j \le D_j^{max} \end{gathered} \tag{6}$$

The EVA MNL is in fact a doubly constrained MNL and defined by this equation, although shadow prices are unknown. However, it can be shown that shadow prices are easily derivable from the EVA model by mathematical redrafts as follows (Winkler, 2015):

$$\begin{aligned} a_i &= e^{\theta_i} \rightarrow \ln\left(a_i\right) = \theta_i \\ b_j &= e^{\tau_j} \rightarrow \ln\left(b_j\right) = \tau_j \end{aligned} \tag{7}$$

Hence, the derivation of the shadow prices is straightforward by logarithmic transformations of the balancing factors. The same applies to destination utilities that are defined by logarithmic transformation of the destination attractors.

LOGSUM DIFFERENCE AND ITS FAILURE IN CASE OF CONSTRAINED MODELS

Measuring transport user benefits is often necessary for evaluation transport investments and policies by CBA. For this purpose the concept of change in consumer surplus is often used. When applying an MNL for estimating change in travel demand, change in consumer surplus can be easily derived by the logsum difference. It is a simple mathematical formula and consistent with microeconomics. The logsum difference as a measure for change in consumer surplus has been investigated and used in transport for some time (see, for a comprehensive literature synthesis, Ma et al., 2015).

For deriving the change in consumer surplus on the base of MNL, expected maximum utility of a decision maker or representative has to be defined and it is:

$$\begin{aligned} E\left(U\right) &= E\left(\mathrm{Max}\left(U_{ij}, \forall i, j\right)\right) \\ &= \ln\left(\sum_i \sum_j e^{V_{ij}}\right) + C \end{aligned} \tag{8}$$

where C is an unknown constant that represents that the absolute level of utility is not measurable. This mathematical expression corresponds with the antiderivative of the MNL (see, for example, Small and Rosen, 1981). The argument in parentheses is the denominator of the MNL and the term $\ln\sum_i \sum_j \exp\left(V_{ij}\right)$ is called "logsum", which is simply the log of the denominator. However, for evaluating transport investments and policies we are more interested in change in utility that is defined by the difference between initial (0) and final (1) state conditions. For this purpose a difference in logsum terms is applied:

$$\Delta E\left(U\right) = \ln\left(\sum_i \sum_j e^{V_{ij}^1}\right) - \ln\left(\sum_i \sum_j e^{V_{ij}^0}\right) \tag{9}$$

The unknown constant C enters expected maximum utility both before and after change and drops out. On the basis of this change in expected maximum utility, the required change in consumer surplus can be derived, which is the monetized equivalent. Hence we get the change in consumer surplus by normalizing the logsum difference by the marginal utility of income λ (of a representative traveler of a homogenous group):

$$\Delta E\left(CS\right) = \frac{1}{\lambda}\left[\ln\left(\sum_i \sum_j e^{V_{ij}^1}\right) - \ln\left(\sum_i \sum_j e^{V_{ij}^0}\right)\right] \tag{10}$$

This normalization presumes that λ is constant and income effect is not accounted for (see, for example, Williams, 1977 and Small and Rosen, 1981). This assumption is reasonable for many problems. However, if income effects occur, different and more complex approaches for deriving monetary user benefits have to be used. Microeconomics applies the “compensating variation” (CV) as the Hicksian (or compensated) version of consumer surplus (see, for example, Varian, 1992). There has been extensive research for deriving CV for discrete choice models, see, for example, McFadden, 1995, Karlström and Morey, 2001 and Dagsvik and Karlström, 2005, but this is not the topic here.

In the present analysis we suppose a constant marginal utility of income, which, however, has to be defined. Usually, a price or cost variable c enters the indirect utility V and the negative of its coefficient β_c is λ per definition (Train, 2003). The derived change in consumer surplus applies for a representative traveler. It can be interpreted as the average consumer surplus in the group under consideration. The change in total consumer surplus (sum over all travelers of the group) can then be calculated by multiplying the monetized logsum difference by total demand and it is:

$$\Delta E(TCS) = -\frac{1}{\beta_c}\left[\ln\left(\sum_i\sum_j e^{V_{ij}^1}\right) - \ln\left(\sum_i\sum_j e^{V_{ij}^0}\right)\right]T \tag{11}$$

After this brief introduction of the logsum difference, we shall discuss the problem with applying it in the EVA MNL. The bases for changes in consumer surplus are project-induced changes of variables in the indirect utility function. If generalized costs decrease after intervention, indirect utility and consumer surplus increase. However, the EVA MNL comprises shadow prices as additional “bonus” components and shadow prices change as a consequence of changing costs. This means that if costs decrease, (positive) shadow prices also decrease. The question arises which consequence does it have?

It can be shown (see Winkler, 2015) that it is:

$$T = \sum_i\sum_j e^{\left(-g_{ij}+z_j+\theta_i+\tau_j\right)} \tag{12}$$

In the standard case $T^0 = T^1$ it follows:

$$\begin{aligned}\Delta E(U) &= \left[\ln\left(\sum_i\sum_j e^{\left(-g_{ij}^1+z_j^1+\theta_i^1+\tau_j^1\right)}\right) - \ln\left(\sum_i\sum_j e^{\left(-g_{ij}^0+z_j^0+\theta_i^0+\tau_j^0\right)}\right)\right]\\ &= \ln\left(T^1\right) - \ln\left(T^0\right)\\ &= 0\end{aligned} \tag{13}$$

From this it follows that, regardless of the changes of generalized costs, the logsum difference provides no (or incorrect) benefits. However, due to cost decreases, transport users in fact receive real benefit. Therefore, the logsum difference obviously fails for constrained models.

There is only a little literature concerning this question. Williams (1976) and later Martinez and Araya (2000) showed approaches for deriving transport user benefit measure on the basis of doubly constrained gravity models. These derivations are based on the maximum entropy model and it is not easily transferable to other models. Furthermore, a rigorous relationship between the transport user benefit measure within the entropy framework and the logsum benefit measure within random utility theory has not been established so far (Geurs et al., 2010). Here we will close the gap and show a universal approach, which adjusts the logsum difference to multiple-constrained models.

ADJUSTMENT OF THE LOGSUM DIFFERENCE

We have shown that the logsum difference fails for the EVA MNL. For distinguishing the real change in expected maximum utility $\Delta E(U)$ and the incorrect measure, we define the logsum difference applied to the EVA MNL provides $\Delta E\left(U^{*}\right)$ and it is $\Delta E\left(U^{*}\right) \neq \Delta E\left(U\right)$. In the following we want to clarify how the logsum difference should be adjusted to the EVA MNL. First of all, we derive the approach for the representative traveler. We then move over to the change in (total) consumer surplus.

The starting point of the analysis is the differential:

$$\Delta E\left(U^{*}\right) = E\left(U^{*1}\right) - E\left(U^{*0}\right) = \int_{C} dE\left(U^{*}\right) \tag{14}$$

where $dE\left(U^{*}\right)$ represents the demand function (Varian, 1992). Therefore it is:

$$dE\left(U^{*}\right) = \sum_{i}\sum_{j} P_{ij}\left(\mathbf{V}\right) dV_{ij} \tag{15}$$

Here **V** is a vector of all alternative-specific deterministic utilities, since the probabilities depend on all alternatives. Hence it can be written:

$$\Delta E\left(U^{*}\right) = \int_{C}\sum_{i}\sum_{j} P_{ij}\left(\mathbf{V}\right) dV_{ij} \tag{16}$$

This integral is a line integral and its solution depends on the path C between the final and initial state. Due to the fact that the antiderivative of the MNL, and hence of the EVA MNL, is known, a path independent solution of equation (16) is possible. However, as shown above, in the case of constraints the logsum difference does not provide the wanted change in expected maximum utility. For this reason a more detailed analysis including a segmentation of the EVA MNL is necessary. First of all the total differential dV_{ijk} can be expressed by partial differentials (Chiang and Wainwright, 2005):

$$dV_{ij} = -\frac{\partial V_{ij}}{\partial g_{ij}} dg_{ij} + \frac{\partial V_{ij}}{\partial z_{j}} dz_{j} + \frac{\partial V_{ij}}{\partial \theta_{i}} d\theta_{i} + \frac{\partial V_{ij}}{\partial \tau_{j}} d\tau_{j} \tag{17}$$

All components are composed additive and without additional weightings. Hence, the figures of the partial derivatives equal one and it follows:

$$dV_{ij} = -dg_{ij} + dz_{j} + d\theta_{i} + d\tau_{j} \tag{18}$$

For the sake of simplicity we define $P_{ij}\left(\mathbf{V}\right) = P_{ij}$ and it is:

$$dE\left(U^{*}\right) = \sum_{i}\sum_{j} P_{ij}\left(-dg_{ij} + dz_{j} + d\theta_{i} + d\tau_{j}\right) \tag{19}$$

Next we can insert it in equation (16) and obtain the integral with respect to the bounds of integration:

$$\Delta E\left(U^{*}\right) = \int_{\mathbf{V}^{0}}^{\mathbf{V}^{1}}\sum_{i}\sum_{j} P_{ij}\left(-dg_{ij} + dz_{j} + d\theta_{i} + d\tau_{j}\right) \tag{20}$$

Furthermore, a separation of the integral is necessary and possible (Merziger et al., 1999):

$$\Delta E\left(U^{*}\right) = -\int_{\mathbf{g}^{0}}^{\mathbf{g}^{1}}\sum_{i}\sum_{j} P_{ij}\, dg_{ij} + \int_{\mathbf{z}^{0}}^{\mathbf{z}^{1}}\sum_{i}\sum_{j} P_{ij}\, dz_{j} + \int_{\boldsymbol{\theta}^{0}}^{\boldsymbol{\theta}^{1}}\sum_{i}\sum_{j} P_{ij}\, d\theta_{i} + \int_{\boldsymbol{\tau}^{0}}^{\boldsymbol{\tau}^{1}}\sum_{i}\sum_{j} P_{ij}\, d\tau_{j} \tag{21}$$

$\Delta E\left(U^{*}\right)$ results from the sum of all integrals. For each integral the antiderivative is given by the logsum formulae. However, all integrals can only be solved one by one with respect to the considered changing variable (g, z etc.) while all other components have to be constant. It has to be taken into account that benefit contributions of different variables depend on the sequence of integrations (path dependency). Hence, singly benefit contributions cannot be defined unambiguously, but the sum over all is independent.

The requested change in expected maximum utility is only a function of real and observable changing attributes between the initial and final state (Williams, 1976). In the present analysis these are generalized costs g and destination utilities z and it is:

$$\Delta E(U) = -\int_{\mathbf{g}^0}^{\mathbf{g}^1} \sum_i \sum_j P_{ij}\, dg_{ij} + \int_{\mathbf{z}^0}^{\mathbf{z}^1} \sum_i \sum_j P_{ij}\, dz_j \tag{22}$$

Now we can reformulate equation (21) and express $\Delta E(U)$ indirectly:

$$\Delta E(U) = \Delta E(U^*) - \int_{\boldsymbol{\theta}^0}^{\boldsymbol{\theta}^1} \sum_i \sum_j P_{ij}\, d\theta_i - \int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \sum_i \sum_j P_{ij}\, d\tau_j \tag{23}$$

In this equation the logsum difference of the EVA MNL is adjusted by the shadow price integrals. However, again, their figures depend on sequence of calculation. Consequently the unambiguousness of the requested $\Delta E(U)$ depends on the solution of the integrals of shadow prices. These integrals are only one-dimensional because their differentials only have one dimension (I or j). Since a change in θ_i has no influence on probability of destination (same applies reciprocally for τ_i), it can be written:

$$\Delta E(U) = \Delta E(U^*) - \int_{\boldsymbol{\theta}^0}^{\boldsymbol{\theta}^1} \sum_i P_i\, d\theta_i - \int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \sum_j P_j\, d\tau_j \tag{24}$$

where P_i, and P_j are the probabilities of choosing origin I and destination j. Hence, these probabilities represent representative traveler’s demand for origins and destinations. However, the demand functions for these probabilities are not explicitly defined, but result implicitly from the summation of alternative-specific probabilities P_{ij}. Consequently, antiderivatives do not exist and integration cannot readily be achieved, but it is necessary for deriving $\Delta E(U)$.

For the solution of the integrals, it is of great importance whether the initial and final probabilities P_i and P_j are equal or not. Generally, they are equal in the case of inelastic and unequal in the case of elastic constraints. Next we shall derive the solution for the constant case through the example of $P_i^0 = P_i^1$. Afterwards we turn to the elastic constraints ($P_j^0 \neq P_j^1$). The following derivations are transferable one to one to other dimensions.

Case $\mathbf{P_i^0 = P_i^1}$

The probabilities are obviously constant and accordingly independent from shadow prices. Hence there are no formal interdependences of probabilities of the origins and the shadow prices. As a consequence, the probabilities can be taken outside the integrals. Thus, the integration is path-independent because of the independent integrability of all integration variables θ_i. So we obtain:

$$\int_{\boldsymbol{\theta}^0}^{\boldsymbol{\theta}^1} \sum_i P_i\, d\theta_i = \sum_i P_i \cdot \int_{\theta^0}^{\theta^1} d\theta_i \tag{25}$$

Then the integration of the differentials $d\theta_i$ provides the integration variables θ_i. Next we insert the bounds of integration and the solution for the inelastic constraint case is:

$$\int_{\boldsymbol{\theta}^0}^{\boldsymbol{\theta}^1} \sum_i P_i \, d\theta_i = \sum_i P_i \cdot \left(\theta_i^1 - \theta_i^0\right) \qquad (26)$$

The probability for choosing origin i is defined by the ratio of total number of trips originating at zone i and total demand. These numbers are known from the trip generation and it can be written:

$$\int_{\boldsymbol{\theta}^0}^{\boldsymbol{\theta}^1} \sum_i P_i \, d\theta_i = \sum_i \frac{O_i}{T} \cdot \left(\theta_i^1 - \theta_i^0\right) \qquad (27)$$

This is an unambiguous solution and all relevant variables are given by the solved EVA MNL.

Case $\mathbf{P_j^0 \neq P_j^1}$

If destination probabilities change between initial and final state, we need another approach, since we cannot take the probabilities outside the integral readily. So, in principal, it is necessary to integrate the (unknown) functions of the resulting probabilities P_js. But we only have information about the P_js for the two equilibrium (initial and final) states. Hence, we have to make an assumption about the shape of the function in between. According to the RoH we assume a linear form.

With the assumption of a linear path of integration it can be written:

$$\int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \sum_j P_j \, d\tau_j = \sum_j \left(\frac{P_j^0 + P_j^1}{2}\right) \cdot \int_{\tau^0}^{\tau^1} d\tau_j \qquad (28)$$

Now the integral can be solved analogically to the inelastic constraint case and we obtain:

$$\int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \sum_j P_j \, d\tau_j = \sum_j \left(\frac{P_j^0 + P_j^1}{2}\right) \cdot \left(\tau_j^1 - \tau_j^0\right) \qquad (29)$$

Replacing probabilities by total numbers of trips gives:

$$\int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \sum_j P_j \, d\tau_j = \frac{1}{2} \cdot \sum_j \left(\frac{D_j^0 + D_j^1}{T}\right) \cdot \left(\tau_j^1 - \tau_j^0\right) \qquad (30)$$

Again, all necessary variables are given by the EVA MNL.

Now we are able to formulate the change in expected maximum utility of a representative traveler. For the example under consideration with one set of inelastic constraints for origins and one set of elastic constraints for destinations, we obtain:

$$\Delta E(U) = \left[\ln\left(\sum_i \sum_j e^{\left(-g_{ij}^1 + z_j^1 + \theta_i^1 + \tau_j^1\right)}\right) - \ln\left(\sum_i \sum_j e^{\left(-g_{ij}^0 + z_j^0 + \theta_i^0 + \tau_j^0\right)}\right)\right] + \sum_i \frac{O_i}{T} \cdot \left(\theta_i^0 - \theta_i^1\right) + \frac{1}{2} \cdot \sum_j \left(\frac{D_j^0 + D_j^1}{T}\right) \cdot \left(\tau_j^0 - \tau_j^1\right) \qquad (31)$$

Obviously the logsum difference is corrected by the influence of the shadow prices. Furthermore, it is crucial to note that if shadow prices are unchanged, no correction is necessary. The last two terms cancel out and the logsum difference provides the change in consumer surplus. That is the case when generally no constraints occur or when constraints are only complied in the initial case. Then, in the final state initial shadow prices are considered as additional origin, destination or mode utilities.

Finally, with $\ln\left(\sum_i\sum_j \exp\left(-g_{ij}^1 + z_j^1 + \theta_i^1 + \tau_j^1\right)\right) - \ln\left(\sum_i\sum_j \exp\left(-g_{ij}^0 + z_j^0 + \theta_i^0 + \tau_j^0\right)\right) = \text{logDiff}$ equation (31) can be written as:

$$\Delta E(U) = \text{logDiff} + \sum_i \frac{O_i}{T} \cdot \left(\theta_i^0 - \theta_i^1\right) + \frac{1}{2} \cdot \sum_j \left(\frac{D_j^0 + D_j^1}{T}\right) \cdot \left(\tau_j^0 - \tau_j^1\right) \tag{32}$$

Total change in consumer surplus

So far, we have derived the change in expected maximum utility of a representative traveler. Now we move on to the solution for the needed (total) change in consumer surplus. For this purpose $\Delta E(U)$ has to be monetized by $-\beta_c$ (negative marginal utility of the cost variable) and multiplied by the total number of trips T. In case of $T^0 = T^1$, i.e. total demand is fixed and no project-induced new generated trips occur, we have:

$$\Delta E(TCS) = -\frac{1}{\beta_c}\left[\sum_i O_i\left(\theta_i^0 - \theta_i^1\right) + \frac{1}{2}\sum_j \left(D_j^0 + D_j^1\right) \cdot \left(\tau_j^0 - \tau_j^1\right)\right] \tag{33}$$

The logsum difference cancels out, since, as already partly shown in equation (13), it is:

$$\ln\left(\sum_i\sum_j e^{\left(-g_{ij}^1 + z_j^1 + \theta_i^1 + \tau_j^1\right)}\right) - \ln\left(\sum_i\sum_j e^{\left(-g_{ij}^0 + z_j^0 + \theta_i^0 + \tau_j^0\right)}\right) = \ln\left(T^1\right) - \ln\left(T^0\right) = \ln\left(\frac{T^1}{T^0}\right) \tag{34}$$

and log(1) = 0. Hence, the change in total consumer surplus results from the change in shadow prices. However, these changes are caused by changes of observable costs and (destination) utilities, therefore the project-induced transport user benefits are indirectly measured by this approach. If, for example, a cost reduction for trips originating in zone i occurs, this zone attains better accessibility. As a result of this, and under the assumption of respecting constraints, the shadow price of this zone decreases. Conversely, if shadow prices increase, negative benefits result. Hence, benefits changes are based on changes in accessibility of zones.

It might be possible that a transport investment or policy leads to a change in the total number of trips and it is $T^0 \neq T^1$. In this case an issue arises comparable to the shown changing probabilities of destinations. For calculating transport user benefits, we have to take into account the shape of the overall demand curve between T^0 and T^1, but we do not have information about that shape and only know total demand for the two equilibrium (initial and final) states. Hence, again, we have to make an assumption about it and use a linear form. Then, $\Delta E(U)$ (equation (32)) has to be multiplied by $(T^0 + T^1) / 2$ and it is:

$$\Delta E(TCS) = -\frac{1}{\beta_c}\left[\text{logDiff}\left(\frac{T^0 + T^1}{2}\right) + \sum_i \left(\frac{O_i^0 + O_i^1}{2}\right)\left(\theta_i^0 - \theta_i^1\right) + \frac{1}{2}\sum_j \left(\frac{D_j^0 + D_j^1}{2}\right)\left(\tau_j^0 - \tau_j^1\right)\right] \tag{35}$$

This equation provides the universal solution for calculating transport user benefits based on the MNL – with or without constraints. If no constraints and shadow prices occur, the logsum difference multiplied by the (mean) total demand provides the change in consumer surplus. The same applies to the constrained case, if shadow prices are the same in the initial and final state. In these cases the last two terms on the right side cancel out. In contrast, if constraints are respected in both calculations, the derived adjustment of the logsum difference has to be taken into account. Then, under consideration of the mathematical simplification in equation (34), it can be written

$$\Delta E(TCS) = -\frac{1}{\beta_c}\left[\ln\left(\frac{T^1}{T^0}\right)\left(\frac{T^0+T^1}{2}\right)+\sum_i\left(\frac{O_i^0+O_i^1}{2}\right)\left(\theta_i^0-\theta_i^1\right)+\frac{1}{2}\sum_j\left(\frac{D_j^0+D_j^1}{2}\right)\left(\tau_j^0-\tau_j^1\right)\right] \tag{36}$$

If shadow prices of single dimensions (e.g., destinations) are equal in the initial and final state, the corresponding correction term disappears, while the others still apply.

The meanings of the benefit terms in equation (35) are the same as explained for the case with fixed total demand. However, the first term enters as an additional benefit term that represents the positive benefits of new generated trips. Negative impacts of these additional trips (like competition for destinations) are taken into account by origin and destination specific shadow prices. That is, if new trips occur due to transport investment, shadow prices decrease less than without these additional trips.

EXAMPLE

The basic principle of the developed approach can be illustrated by a simple synthetic example, which is defined by

- five travel zones
- one homogenous group of persons
- inelastic origin and destination constraints

More realistic examples and case studies will be discussed in the future. However, a simple example such as this might be helpful for initial understanding.

First of all we want to show the concept of calculation of travel demand by the EVA MNL. For this the costs of the initial and final state are summarized in TABLE 1. For the final state we suppose a reduction of costs for the relations from zone 1 to 4 and vice versa (marked in yellow).

TABLE 1: Costs

generalized costs [$] - initial state

gij	1	2	3	4	5
1	4,08	3,25	6,25	5,50	3,25
2	4,08	4,08	4,78	4,58	5,00
3	6,25	4,37	4,08	4,00	4,00
4	5,50	4,58	4,42	4,08	4,08
5	4,50	6,33	4,00	3,25	3,67

generalized costs [$] - final state

gij	1	2	3	4	5
1	4,08	3,25	6,25	3,25	3,25
2	4,08	4,08	4,78	4,58	5,00
3	6,25	4,37	4,08	4,00	4,00
4	3,25	4,58	4,42	4,08	4,08
5	4,50	6,33	4,00	3,25	3,67

The result of the trip calculation (by equation (6)) for the initial state is shown in TABLE 2. In this example we consider inelastic constraints for origins and destinations and the resulting O_is and D_js equal the given O_is and D_js (constraints are satisfied). The total number of trips T (500 trips) is defined by the sum of the given O_is and Z_js, respectively. The shadow prices are derived from the iteratively calculated factors a_i and b_j (see, equation (7)).

Trip distribution is analogously calculated for the final state and the results are also shown in TABLE 2. It is obvious that trips T_{14} and T_{41} have increased since the corresponding costs have decreased. The origin and destination constraints are again satisfied, but by new shadow prices.

TABLE 2: Resulting trips

initial state

Tij	1	2	3	4	5		Oi-resulting	Oi-given	θi
1	3.0	32.4	1.9	1.4	11.3		50	50	2.611
2	9.8	45.5	27.1	11.2	6.4		100	100	3.785
3	0.4	13.5	21.5	7.9	6.8		50	50	2.850
4	2.3	26.7	37.9	17.8	15.4		100	100	3.751
5	9.4	7.0	86.6	61.8	35.2		200	200	4.162
						500	**500**	**500**	
Dj-resulting	25	125	175	100	75	**500**			
Dj-given	25	125	175	100	75	**500**			
τj	2.581	4.116	4.299	3.211	3.065				

final state

Tij	1	2	3	4	5		Oi-resulting	Oi-given	θi
1	1.4	27.9	1.6	9.6	9.4		50	50	2.386
2	5.9	49.4	27.7	10.3	6.7		100	100	3.790
3	0.3	14.3	21.4	7.1	7.0		50	50	2.833
4	11.6	25.6	34.1	14.5	14.2		100	100	3.632
5	5.8	7.8	90.3	58.4	37.7		200	200	4.190
						500	**500**	**500**	
Dj-resulting	25	125	175	100	75	**500**			
Dj-given	25	125	175	100	75	**500**			
τj	2.068	4.194	4.313	3.128	3.107				

The results of the travel demand for the initial and final states are the base of the calculation of the change in consumer surplus. For applying the adjusted logsum difference (see, equation (36)), all O_is and D_js as well as all shadow prices are necessary. All other components in equation (36) cancel out. The result of the calculated change in consumer surplus is summarized in TABLE 3. It should be noted that decreasing shadow prices – and therefore a positive difference – induce positive benefits.

A reduction of shadow prices is primary achieved for traffic zones that are directly affected by the cost changes (here: zones 1 and 4). The reason is that the accessibility of these zones has been improved and transport users need not be so “forced” to choose these zones (for satisfying constraints) by shadow prices. However, all constraints have to be satisfied in the final state, too. Therefore, all other zones are affected indirectly by the cost changes. Although their accessibilities have not changed, these zones loose attractiveness compared to zones 1 and 4 and shadow prices increase for satisfying constraints.

The change in consumer surplus results from shadow price differences and the total numbers of trips, and it is calculated for each zone. However, it is important to note that only the sum over all zones is interpretable as the transport users benefit measure needed for evaluating transport investments by CBA, because the single values are only “proxies” for defining benefits of “real affected” transport users. The resulting change of consumer surplus for the example is $23.87.

TABLE 3: Change in consumer surplus

total number of trips originating at zone i		difference of origin shadow prices			ΔCS_i [$]
O_1	50	$\theta^O_1-\theta^M_1$	0,224		11,22
O_2	100	$\theta^O_2-\theta^M_2$	-0,005		-0,49
O_3	50	$\theta^O_3-\theta^M_3$	0,018		0,88
O_4	100	$\theta^O_4-\theta^M_4$	0,120		11,96
O_5	200	$\theta^O_5-\theta^M_5$	-0,027		-5,45
				Σ(i)	18,11

total number of trips attracted to zone j		difference of destination shadow prices			ΔCS_j [$]
D_1	25	$\tau^O_1-\tau^M_1$	0,513		12,82
D_2	125	$\tau^O_2-\tau^M_2$	-0,078		-9,69
D_3	175	$\tau^O_3-\tau^M_3$	-0,014		-2,52
D_4	100	$\tau^O_4-\tau^M_4$	0,083		8,34
D_5	75	$\tau^O_5-\tau^M_5$	-0,042		-3,19
				Σ(j)	5,76
				Σ(i,j)	**23,87**

Now the result of the adjusted logsum difference approach should be compared with the widely used traditional RoH, whose applicability for travel demand models with constraints has recently been proved (Winkler, 2015). The RoH is an approximation of the user benefits and it depends on the change in costs and trips for each origin-destination-relation:

$$\Delta E\left(TCS\right)=\sum_{i}\sum_{j}\frac{1}{2}\left(g^0_{ij}-g^1_{ij}\right)\left(T^0_{ij}+T^1_{ij}\right) \tag{37}$$

As we can see in equation (37) and TABLE 4, transport user benefits only occur for relations with changing costs.

The comparison of the results shows that the approximate solution (RoH) overestimates the exact measure (adjusted logsum difference) with 17%. However, the difference between the RoH and an exact measure (logsum or adjusted logsum differences) can be positive or negative and it strongly depends on the amount of the cost changes and the shape of the underlying demand function (Ma et al., 2015). Therefore, it is recommended to use the exact measure for deriving transport user benefits.

TABLE 4: Change in consumer surplus – calculated by the RoH

Δgij	**1**	**2**	**3**	**4**	**5**
1	0,00	0,00	0,00	2,25	0,00
2	0,00	0,00	0,00	0,00	0,00
3	0,00	0,00	0,00	0,00	0,00
4	2,25	0,00	0,00	0,00	0,00
5	0,00	0,00	0,00	0,00	0,00

ΣTij/2	**1**	**2**	**3**	**4**	**5**
1	2,2	30,1	1,8	5,5	10,4
2	7,9	47,5	27,4	10,8	6,5
3	0,4	13,9	21,4	7,5	6,9
4	6,9	26,1	36,0	16,2	14,8
5	7,6	7,4	88,5	60,1	36,4

ΔCS	**1**	**2**	**3**	**4**	**5**	
1	0,00	0,00	0,00	12,38	0,00	
2	0,00	0,00	0,00	0,00	0,00	
3	0,00	0,00	0,00	0,00	0,00	
4	15,62	0,00	0,00	0,00	0,00	
5	0,00	0,00	0,00	0,00	0,00	
					$	**28,00**

CONCLUSIONS

In this paper we have seen – by the example of the EVA model – that multiple-constrained travel demand models are expressible in terms of a MNL. This is the basis for an economic analysis of these models and the starting point for the derivation of a transport user benefit measure that is usually needed for evaluating transport investments by CBA. For this purpose the change in consumer surplus is often used. This measure is provided by the MNL and called "logsum difference", which is a very practical and mathematically exact approach. However, the logsum difference fails for travel demand models with constraints and only approximations can be used.

For this reason we have derived an approach that adjusts the logsum difference for travel demand models with two sets of constraints. This approach provides the change in consumer surplus for models with different constraints (elastic and/or inelastic). Furthermore, it contains the logsum difference for the MNL without constraints as an exception. For the sake of clarity, the analyses presented have been focused on the case with inelastic constraints for the origin and elastic constraints for destination choice. Other cases can be analogically derived, but equation (35) provides the general solution for the change in consumer surplus that covers all possible combinations of constraints.

The approach derived is suitable for measuring benefits caused by changing costs, but also for changing attractions. Attraction changes can be expressed by different origin or destination utilities or by a changed given total number of trips originating at or attracted to zones. Hence, it is possible to evaluate short-run and long-run impacts of transport investments and the derived approach can be used for a wide range of evaluations.

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