Fast Approximation of Wave Propagation in Complex Geometries

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Abstract

A minimal model to approximate wave propagation in complex structures has been proposed in previous works of the authors. Core principle of the model is a reduction of inhomogeneities, like stiffeners, to interaction parameters and homogeneous areas to phase velocities. Combined with the plate like shape of aircraft parts relevant for SHM, a two-dimensional model is sufficient to simulate these structures. Paths taken by the waves from actuator to sensor are estimated with ray tracing. Finally, signals at selected sensor positions can be calculated with a signal synthesis algorithm. Several characteristic values of the structure significant to guided waves have to be calculated beforehand. Preparation of such a database can be costly to a certain extent, but is only necessary once per part. With a database in place, the proposed technique offers a very fast way to approximate wave propagation in large and complex structures. This paper discusses effects that influence wave propagation, sensor signals and simulation accuracy. To validate the model, measured signals and FEM solutions are compared to results of the minimal model.

1 INTRODUCTION

A long-term target for SHM systems based on guided waves is the coverage of large areas with sparsely distributed sensors. Future systems will need detailed information about the monitored structure and flexible but fast algorithms to reconstruct wave propagation. Multiple stiffeners, thickness changes and different materials cause complex signals in aircraft parts trough reflection and mode conversion of the initial waves. Computation of wave propagation will require a certain amount of approximation to enable calculation within reasonable times. Sensor network optimization and damage localization are fields which can benefit from such fast but simplified algorithms.

Many sophisticated simulation approaches for Lamb wave propagation analysis have been proposed in the last decades [1-6]. The majority of these numerical methods are based on the finite element method (FEM) where the accuracy and the efficiency has been increased using different shape functions. While good progress has been made in this field, realistic aircraft parts are still not easily simulated. The three reasons for this are the large areas relevant for SHM systems, the fine discretization required for the small wavelengths and the presence of many discontinuities. A consequence of this is a large number of degrees of freedom.

Computational costs are additionally increased by the number of time steps necessary in transient analysis. More steps are required for larger geometries with potentially longer times of flight of wave packets.

2 MINIMAL MODEL

Most approaches currently used to simulate Lamb wave aim to accurately reproduce wave propagation. Research on these methods focuses to decrease computational cost while retaining accuracy. A highly specialized algorithm is required to enable simulation of Lamb wave propagation in complex structures. The term *complex* is used here to describe plate-like geometries with multiple stiffeners and build of layered anisotropic material, like carbon fiber reinforced polymers (CFRP). It can be explained by the reasons mentioned above, why conventional simulation approaches are not well suited to solve such models in reasonable times. Accuracy and flexibility have to be sacrificed to a certain degree to gain the aspired boost in speed. A minimal model approach is proposed to handle complex structures with minimal computational effort. The model is therefore reduced to the minimal amount of information necessary to reproduce the behavior of Lamb waves in stiffened CFRP plates. Different properties that are relevant for wave propagation are calculated beforehand and kept in a database. Firstly, this includes phase velocities, which are determined analytically for all base materials present in the structure. Secondly, interaction parameters for inhomogeneities that are part of the structural design are calculated with FEM models. Ray tracing is utilized to search the paths taken by wave groups between actuator and a position of interest. A two-dimensional representation of the geometry is sufficient for this approach. The models consist of areas with homogeneous properties and area borders with interaction parameters. Subsequently, time signals can be calculated analytically based on the geometric information gained by ray tracing and the properties in the database. The individual steps of this process are detailed in the following sections.

Similar to most SHM approaches based on guided waves, the presented minimal model currently utilizes the first symmetric and the first anti-symmetric wave modes S_0 and A_0, respectively. However, the method can easily be adapted for other modes. While the proposed technique is intended primarily for undamaged structures, it is nevertheless possible to include damages the same way as designed inhomogeneities. Potential use cases for such a fast approximation of wave propagation in undamaged structures are calculated reference signals or the optimization of actuator sensor networks. The ray tracing algorithm could also be adapted for damage localization, which is still a challenge for complex structures.

The homogenous areas of the plate like structure are defined by their phase velocities. The required dispersion curves are calculated with an in-house tool based on the stiffness-matrix-method [7]. Here they are not only a function of frequency but also of direction of propagation, since materials can be anisotropic. This can potentially be expanded by further dependencies, like temperature. In the work of Kijanka et al. the temperature influence on wave propagation was successfully modeled by adapting the elastic material properties [8]. Similar results should be achievable by an analogous adaptation of the phase velocities.

2.1 Interaction of Lamb Waves with Discontinuities

Designed features of the structure, like stiffeners, are inhomogeneities in terms of the minimal model. It is assumed that these inhomogeneities are sharply confined the interaction of waves with the structure is accordingly concentrated. An example are stiffeners, like the omega stringer in Figure 1. The B-scan below the stringer illustrates the propagation of the S_0

mode at the bottom surface. Attenuation and geometric spread are omitted. The S_0 mode interacts at start and end of each stringer foot, which is clearly visible through conversion to the A_0 mode. Based on this assumption, geometric features in the ray tracing model are represented by lines. The areas enclosed by these lines have phase velocities depending on the material. Thus, the thickness dimension can be omitted and 3D structures can be reduced to 2D models, like the model of a plate in Figure 3. Obstacles, like stiffeners, are modeled with multiple lines and enclosed areas. For example, a wave propagating orthogonal through the depicted stringer crosses four parallel lines, which define the outer and the inner edge of every stringer foot. The geometry of such a line is independent of the angle of a crossing ray, as it is a one-dimensional obstacle. It is therefore assumed that the interaction parameters are also independent of the angle. The effect of the incident angle and thus refraction on wave propagation is included in the model by the ray paths inside the stringer determined by ray tracing.

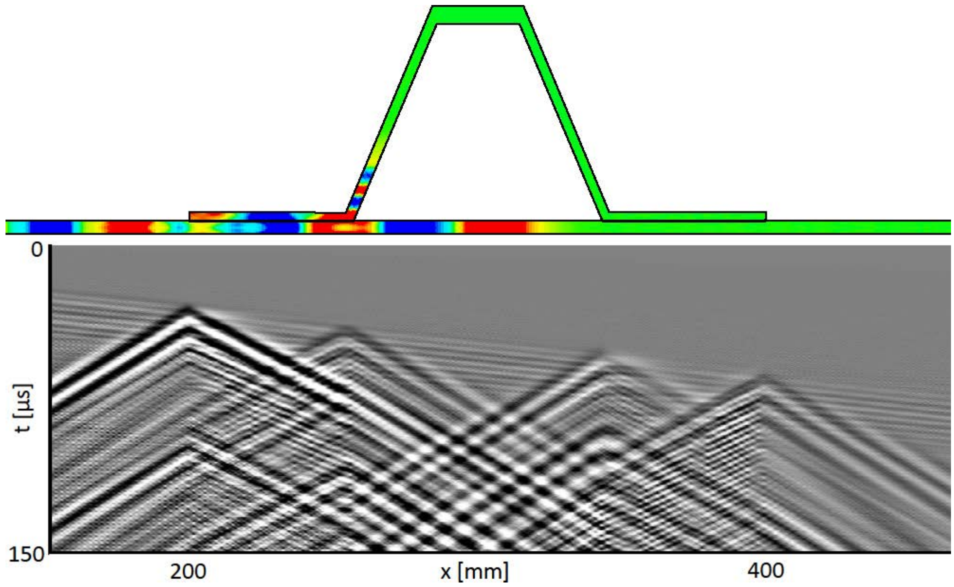

Figure 1: Lamb wave propagation in an omega stringer: cross section (top) and B-scan (bottom).

Analytic solutions to obtain interaction parameters are only known for special cases, like pure changes of material properties without change in geometry. FEM models allow estimations of the interaction parameters of arbitrary changes in geometry and material and are therefore preferred. This numerical approach requires a certain effort for modeling, computation and post-processing. However, parameters can be reused for every occurrence of same inhomogeneity. A cross section of the plate with discontinuity is simulated as 2D plane strain model to obtain parameters. The computational costs are further reduced by limiting the model size with non-reflecting boundaries, which are based on the work of Liu and Jerry [9]. A_0 and S_0 modes can be generated separately by either symmetric or anti-symmetric forces.

The interaction of a single Lamb waves group with an inhomogeneity often leads to multiple new wave packets. Transmitted and reflected parts are known from other wave types. Additionally, conversion of Lamb waves occurs at asymmetric discontinuities [6]. Similarly, the converted mode can be classified as transmitted or reflected, depending on its propagation direction. The interaction parameters are defined as amplitude ratios of the leaving to the incident wave packet. This approach is similar to approach are frequency responses, commonly used to describe linear time-invariant systems in signal processing application [10]. The employed ratios are a reasonable compromise to minimize database size while maintaining accuracy, but they require the excitation signal to be identical in the simulation and the real application.

2.2 Ray tracing

All wave groups traveling through a sensor position have to be calculated to generate a time signal. A high frequency approximation is used to identify all relevant wave packets [11]. This allows the use of geometrical optics to describe wave propagation with plane waves in structures consisting of sharply defined boundaries between homogeneous areas. Particle motion in plane waves is independent of the coordinate normal to the propagation direction [12]. The term *high frequency* relates to the fact that wavelengths needs to be small compared geometrical feature. For features smaller than the half wavelength, wave specific effects like diffraction and interference get relevant. Rays parallel to the direction of wave propagation are used to approximate the taken paths. To obtain a signal at a single point, all relevant paths from an actuator to this point have to be found. In accordance with Fermat's principle, a relevant path for a wave packet is the path with the shortest travel time. The identification of these paths is influenced by reflection and refraction. Snell's law can be applied for isotropic material to determine the angle of refraction. With two modes present in plates at the same frequency in most applications, the refraction angles of every wave packet can be calculated with:

$$\frac{\sin\varphi_{11}}{c_{p11}} = \frac{\sin\varphi_{12}}{c_{p12}} = \frac{\sin\varphi_{21}}{c_{p21}} = \frac{\sin\varphi_{22}}{c_{p22}} \tag{1}$$

The phase velocity c_p and thus is the angle φ is different for every combination of material and mode, which are denoted by the indices (Figure 2). For example, the index 11 identifies the incident wave and its reflection, while the index 21 is the transmitted part of the same mode. Refraction angles larger than 90° result in total reflection and the corresponding ray will not be formed. While Snell’s law is easily applied in isotropic materials, its utilization for anisotropic composite materials is complicated by the angular dependent phase velocities. Refraction of mechanical waves in anisotropic media has been discussed by different authors [13-15]. However, an algorithm for the refraction angle suited for ray tracing has yet to be identified. With the phase velocity depending on the angle that has to be calculated, a possible approach is an iterative algorithm. It is started with the angle of the incident wave and is stopped as soon as the angular change between two steps falls below a previously selected limit.

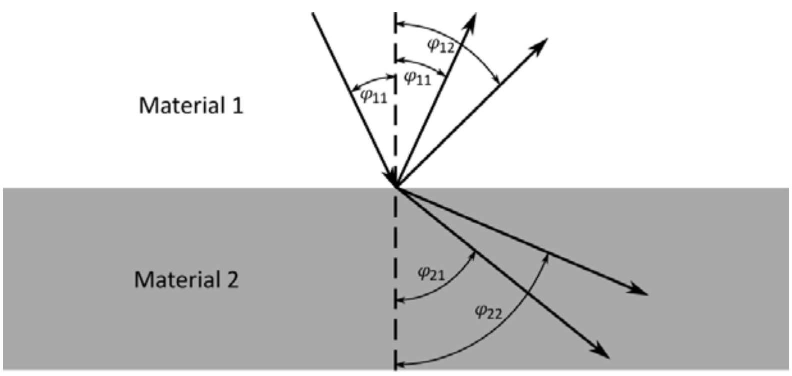

Figure 2: Refraction at a discontinuity.

The model for ray tracing is two dimensional with areas of different material properties and their borders with interaction parameters (Figure 3). To find all relevant paths between actuator and sensor pairs, a large number of rays are emitted. This approach is also known as *ray launching*. For a set number of interactions, the rays transmit, reflect and convert at all discontinuities they encounter, increasing the number of rays from step to step.

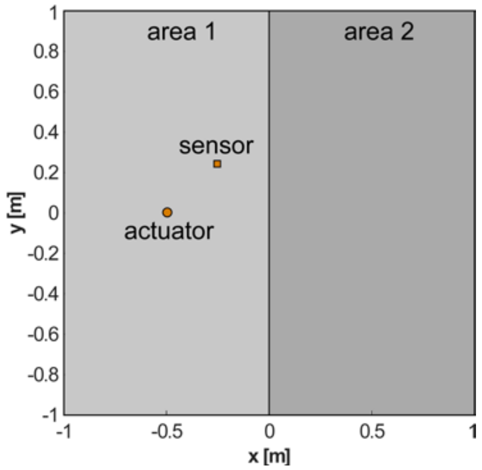

Figure 3: 2D model of a plate with two base materials.

Three rays are emitted in the left picture of Figure 5. A single mode is used here for the sake of clarity and thus mode conversion does not occur. At most obstacles both transmission and reflection occur and reduce opacity of the rays to demonstrate the amplitude reduction at these points. The amplitude is unchanged at the outer border of the plate as only reflection is present without transmission. In the second example of Figure 5 more rays are emitted into the top half of the plate and four paths are found. Potentially more paths can be found with more interaction steps and rays released in every direction. Increasing the amount of calculated steps above a certain point will add little to the accuracy, because resulting paths are both longer and cross more inhomogeneities. The traveled distance and all interactions reduce the amplitude of the wave packet associated with this path, resulting in a later arrival and lower amplitude compared to shorter paths. Once the relevant paths are found, geometric information is gained for the subsequent step of signal synthesis. This information includes directions, distances, plate materials and the boundary lines at which waves interact.

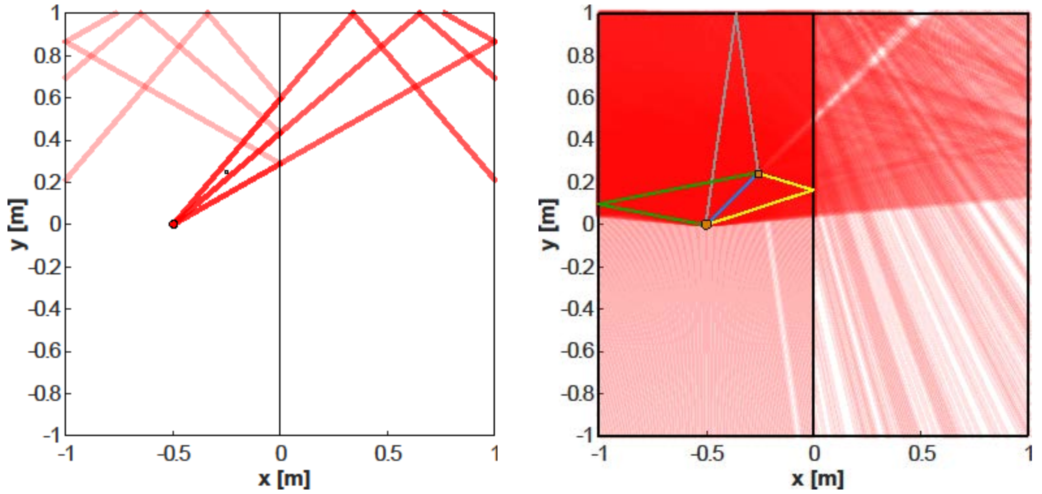

Figure 4: ray tracing with different amounts of rays.

2.3 Signal Synthesis

Ray tracing is used to cover the geometric part of wave propagation. This way, the signal synthesis only needs to be employed for a single location. In contrast, numerical methods based on the FEM compute solutions for all nodes in the model and are therefore computationally more expensive. A superposition of plane harmonic waves is used to calculate the time signal of a wave packet for a single path [16, 17]. The used equation for plane waves traveling inside undisturbed areas is:

$$g(x,t) = \sum_{f=f_1}^{f_n} \left(A(f) \cdot \exp\left(i\, 2\, \pi f \left(\frac{x}{c_p(f)} - t \right) \right) \right). \qquad (2)$$

The amplitude g at a location x and the time t is the sum over all n frequencies steps f_n present at the excitation signal A. The phase velocity c_p is a function of wave mode, plate material and the direction of wave propagation. For piecewise homogenous structures, boundary conditions and coefficients for the interaction have to be included in Equation (2). The boundary condition b is based on the assumption of displacement equality at the discontinuity between two areas and can be written as:

$$b(f) = \sum_{s=1}^{\mathrm{m}} \left(\frac{x_s}{c_{ps}(f)} - \frac{x_s}{c_{ps+1}(f)} \right). \qquad (3)$$

At each interaction step s the position x_s of the interaction and the phase velocities c_{ps} of both areas are known from raytracing. All m interactions have to be added up to cover the whole path. Amplitude changes at discontinuities are included by multiplying the interaction parameters M_s of every interaction step. The modified form of Equation (2) is then:

$$g(x,t) = \sum_{f=f_1}^{f_n} \left(A(f) \cdot \prod_{s=1}^{m} (M_s(f)) \cdot \exp\left(i\, 2\, \pi f \left(\frac{x}{c_p(f)} - t \right) \right) \right). \qquad (4)$$

Different attenuation effects influence wave propagation [18]. Among these, an important one is the amplitude reduction through geometric spreading, which is implemented by the factor $1/\sqrt{x}$. This effect has great relevance to obtain distinct wave packets for paths of different length.

An exemplary signal consisting of the individual wave packets and the corresponding paths is shown in Figure 6. Information about the paths is listed in the legend. Besides the path number, it contains the mode progression along the path.

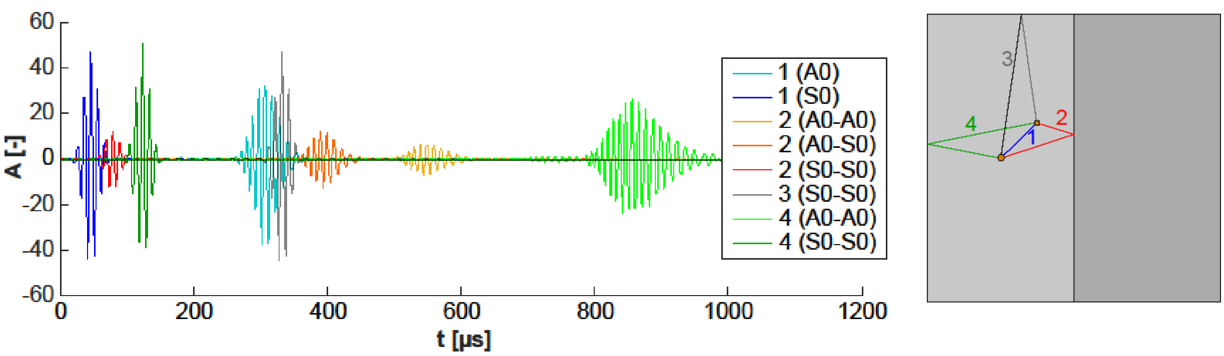

Figure 5: signal with individual wave packets of every path

3 EXPERIMENTAL RESULTS

The validation of the minimal model is done with experiments and FEM simulations on different specimen with increasing complexity. Results on aluminum plates are presented in the following section, while experiments on CFRP plates are discussed in forthcoming publications.

Displacement on the plate surface is measured with a scanning laser Doppler vibrometer that has been developed to compute all three displacement components from three individual recordings [19]. The non-contact measurement of the velocities is well suited to acquire data at arbitrary positions on the surface of arbitrary geometries. The investigated plates have a thickness of 4 mm and displacements are recorded along a straight line from the actuator to create B-scans. Lamb wave are excited with a piezoelectric disk and a sinus burst with a center frequency of 100 kHz.

FEM results are calculated with implicit transient dynamic simulations, which allow the direct modelling of piezoelectric actuators. The plate geometries are identical to the experiments, but symmetry boundary conditions are used to reduce model size when applicable.

The amplitude of the exited wave packets is not only defined by the excitation signal but also by the frequency and mode selective properties of the actuator, which are also influenced by eigenmodes of the coupled ceramic and plate [20, 21]. These electromechanical properties of the transducer are not part of minimal model. To account for this, amplitudes of the primary modes excited by the actuator are transferred from the experimental results to the signal synthesis algorithm.

3.1 Aluminum Plate

An aluminum plate without discontinuity is used in a first comparison. Signals are determined with the different methods for a distance of 500 mm from the actuator to the plate edge. The calculation with the proposed algorithm takes about ten seconds and requires about 500 MB of memory. The FEM simulation of a quarter of the plate takes nine hours and 18 GB of memory. The displacement component parallel to the x-axis is extracted with help of 3D-measurements. These in-plane displacements are in the same order of magnitude for A_0 and S_0 Lamb wave modes.

Time signals at x=250 mm are compared in Figure 7. Envelopes of the signals are used to highlight the wave packets. Amplitudes, position of the wave groups and even the phase is in good agreement. Only the FEM results show discrepancies, which is presumably caused by a mismatch between the real and the modeled actuator. An exact representation of a certain actuator configuration is an elaborate task, as many hard to determine factors influence the electromechanical behavior. This includes the adhesive layer, an optional encapsulation and the electrical connection.

For a comparison along the whole length between actuator and plate edge, amplitude progression is plotted in Figure 8. As the highest amplitude of every signal envelope is depicted here, the curves are dominated by the A_0 mode. The decreasing trend indicates the spread of energy proportional to $1/\sqrt{x}$. The visible oscillations are a result of the interference between crossing wave groups.

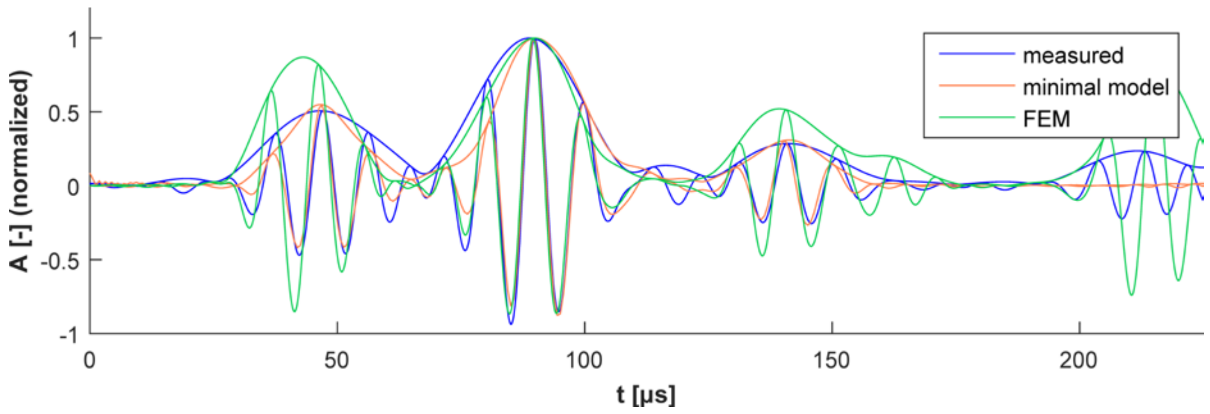

Figure 6: Time signals at a distance of 250 mm from the actuator.

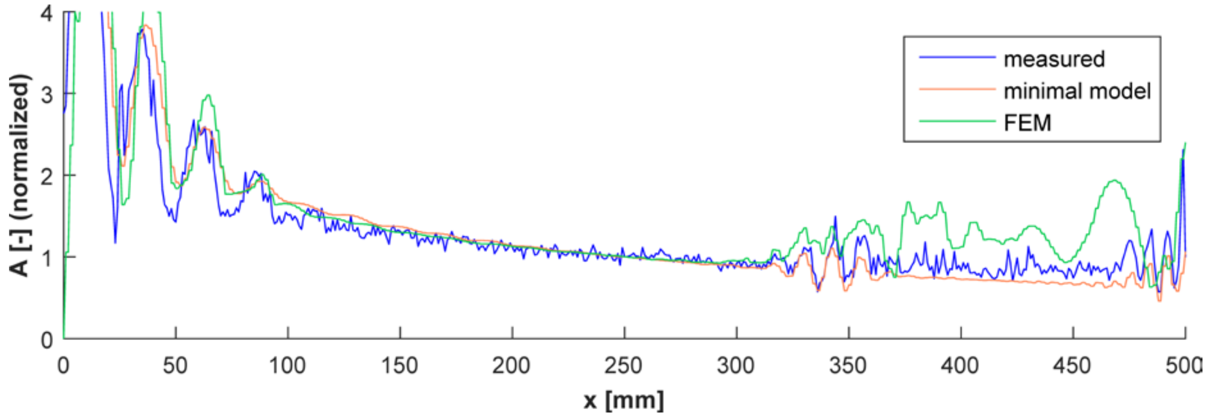

Figure 7: Amplitude progression between actuator and plate edge.

3.2 Aluminum plate with cutout

A second aluminum plate is machined to reduce its thickness in a certain area, as depicted in Figure 9. Trough manufacturing inaccuracy, the milled out area is uneven and thinner. An averaged thickness of 1.64 mm is measured and hence used for simulation purpose. The B-scans are perpendicular to the border of the cutout and 600 mm long.

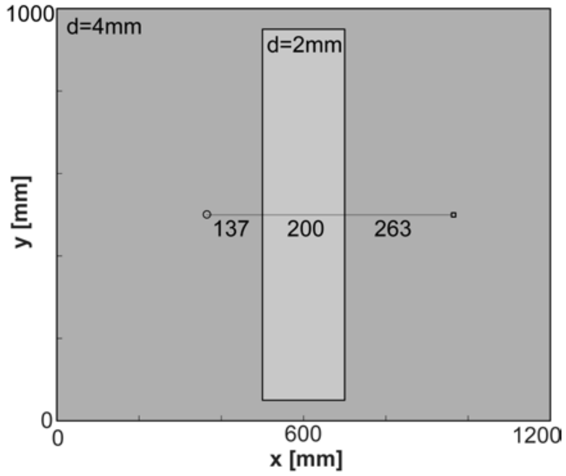

Figure 8: Schematic of the aluminum plate with cutout

With inhomogeneities present in the structure, interaction parameters have to be calculated with the FEM to be used in the signal synthesis. After this step, computational costs are similar to the previous plate without cutout. In contrast, the FEM model size increases, as the plate used in the experiments is larger and half of it has to be modeled. As a consequence of this, the simulation takes 37 hours and requires 55 GB of memory.

Amplitude progression is again used to compare the results (Figure 10). The plateau at the location of the cutout is caused by the distribution of a similar amount of energy over a smaller cross section. While amplitudes are very similar in the first section, minimal model amplitudes are lower after the first inhomogeneity, indicating discrepancies in the interaction parameters. Further investigations are necessary to identify the reasons for this.

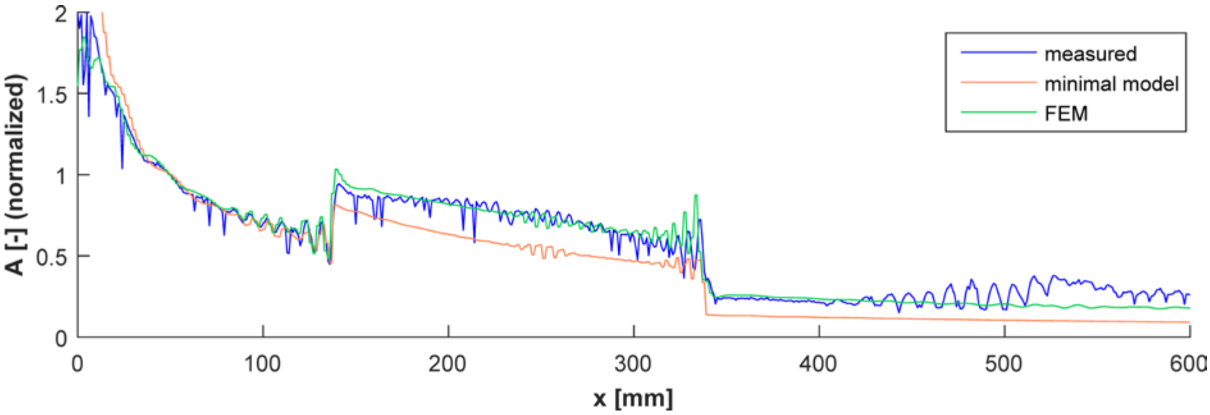

Figure 9: Amplitude progression in an aluminum plate with cutout.

4 CONCLUSIONS

The proposed minimal model reduces complex structures to minimize the computational costs of Lamb wave propagation simulation. A 2D-model is used to represent plate like structures that are common in aircraft. The areas in this model are assumed to be homogeneous. Thus, only phase velocities are required to cover wave propagation, but effects of inhomogeneity, like continuous mode conversion [22], cannot be modeled. Amplitude changes on arrival of a wave at the area borders are defines by interaction parameters. The paths Lamb waves take between an actuator and a sensor position are influenced by these inhomogeneities and identified by ray tracing. Based on these paths, a signal synthesis algorithm is able to calculate time signals.

In comparison with FEM the proposed technique requires a fraction of the time to calculate signals for a few locations. This can be useful for certain applications, like optimization algorithms or fast adaptions of reference signals based on environmental conditions. However, more work has to be carried out prior to the use of the minimal model, as databases for interaction parameters and dispersion curves need to be determined.

The principle feasibility to simulate Lamb wave propagation with the proposed method is shown by comparison with FEM and measured results on aluminum plates. While good agreement is found for most characteristics, the cause of some discrepancies and the influence of anisotropy have still to be examined.

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