NPI: Real-Time Adaptive Entry Guidance

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Agenda

- **Motivation and introduction**
- **Trajectory computation**
  - Parametric sensitivity analysis of nonlinear programs (offline)
  - Fast solution approximation (online)
  - Repeated online trajectory computation
  - Region of convergence
- **Trajectory tracking**
  - Feedback linearization
  - Drag energy dynamics
- **Guidance system overview**
- **Monte Carlo campaign**
  - Perturbed environment
  - Guidance performance results
- **Processor-in-the-loop test**
- **Software tools**
- **Summary out outlook**

Guided Entry Phase

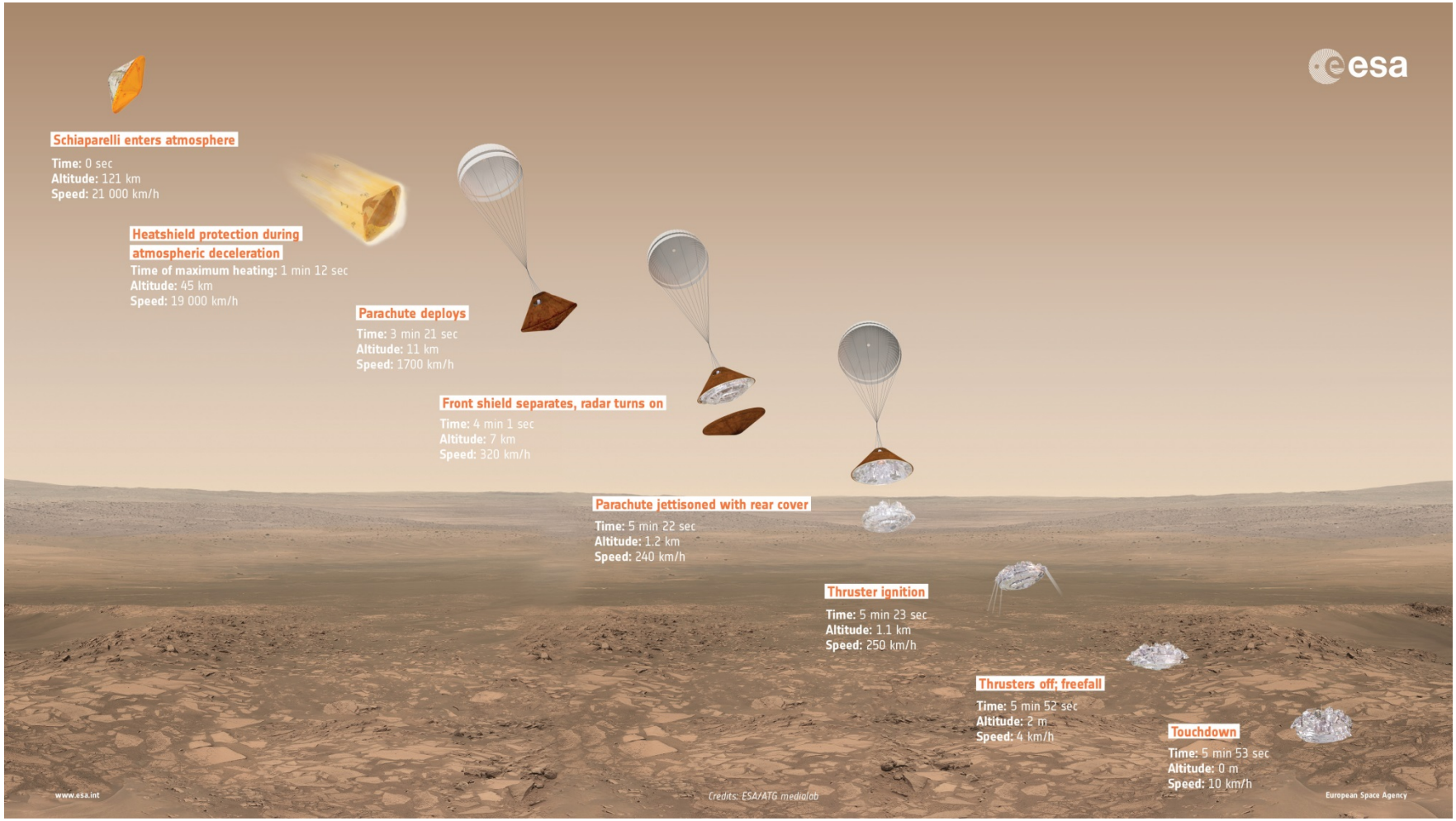
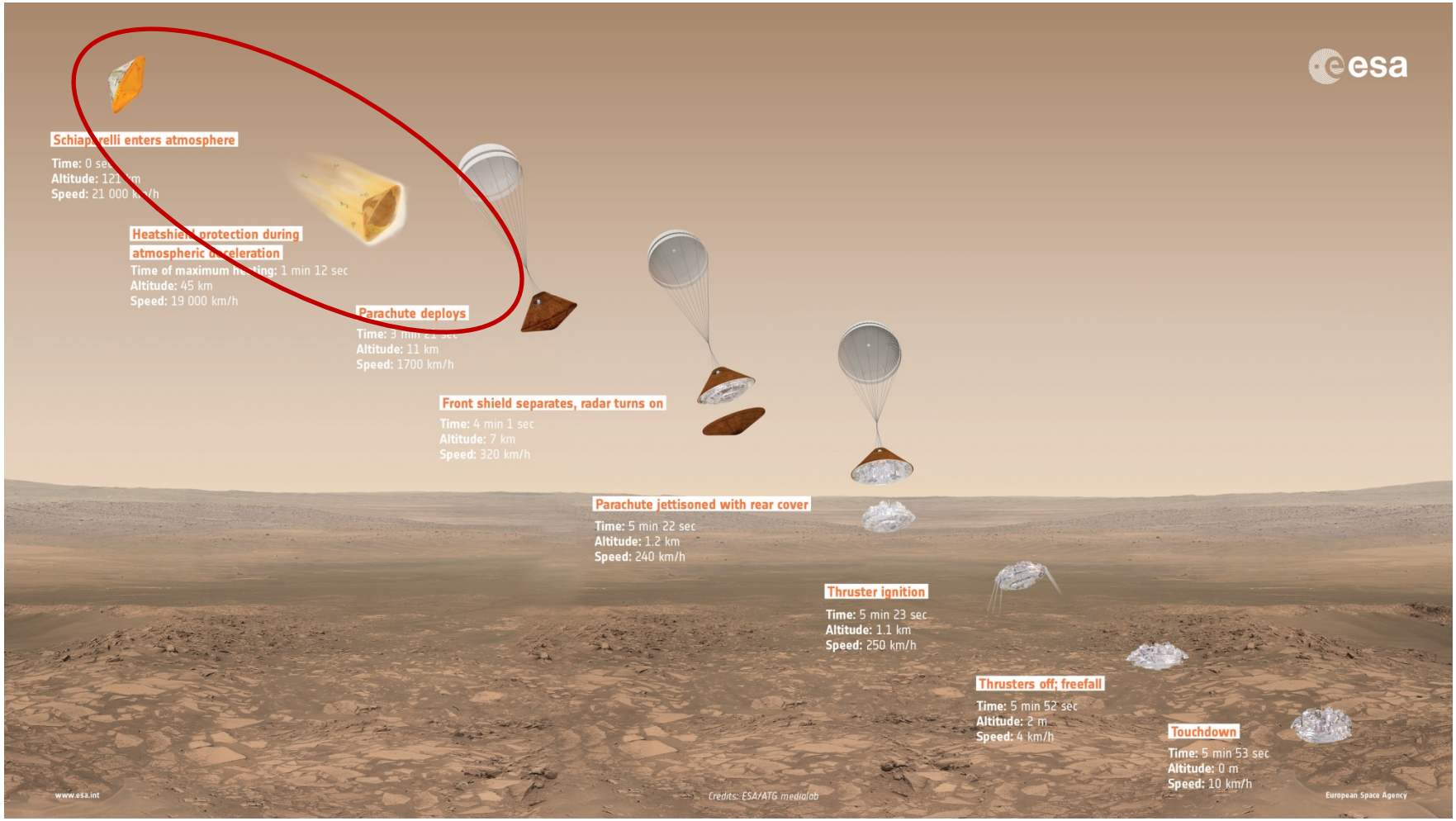


Image credit: ESA

Guided Entry Phase



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Guidance Task

Find an admissible control sequence that steers the vehicle on a feasible trajectory form the current state to the desired terminal state.

Problem: Nonlinear dynamic system with state and control constraints

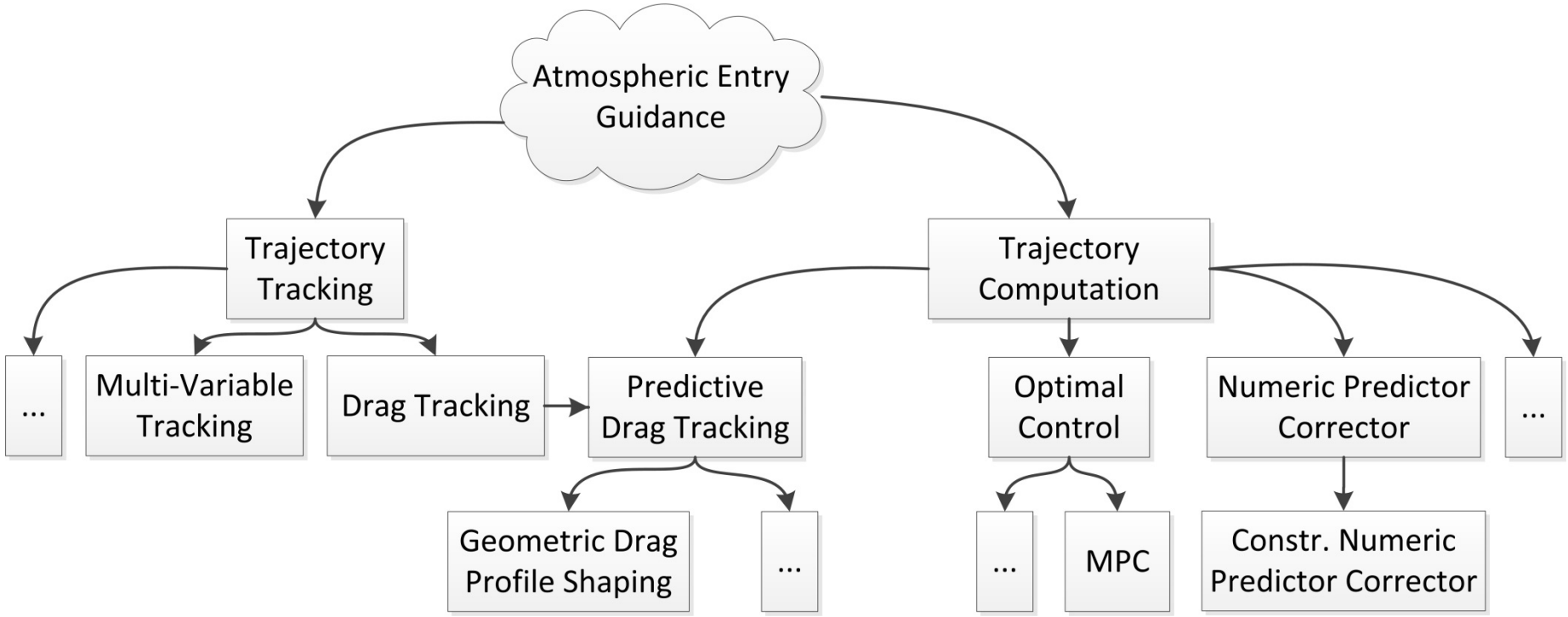
Req.: Planning of the control sequence and state trajectory over the entire future path

Req.: Mathematical model

Challenges:

1. Constrained nonconvex optimization problem
2. High computational power demand
3. Model dependency

Approaches to Entry Guidance



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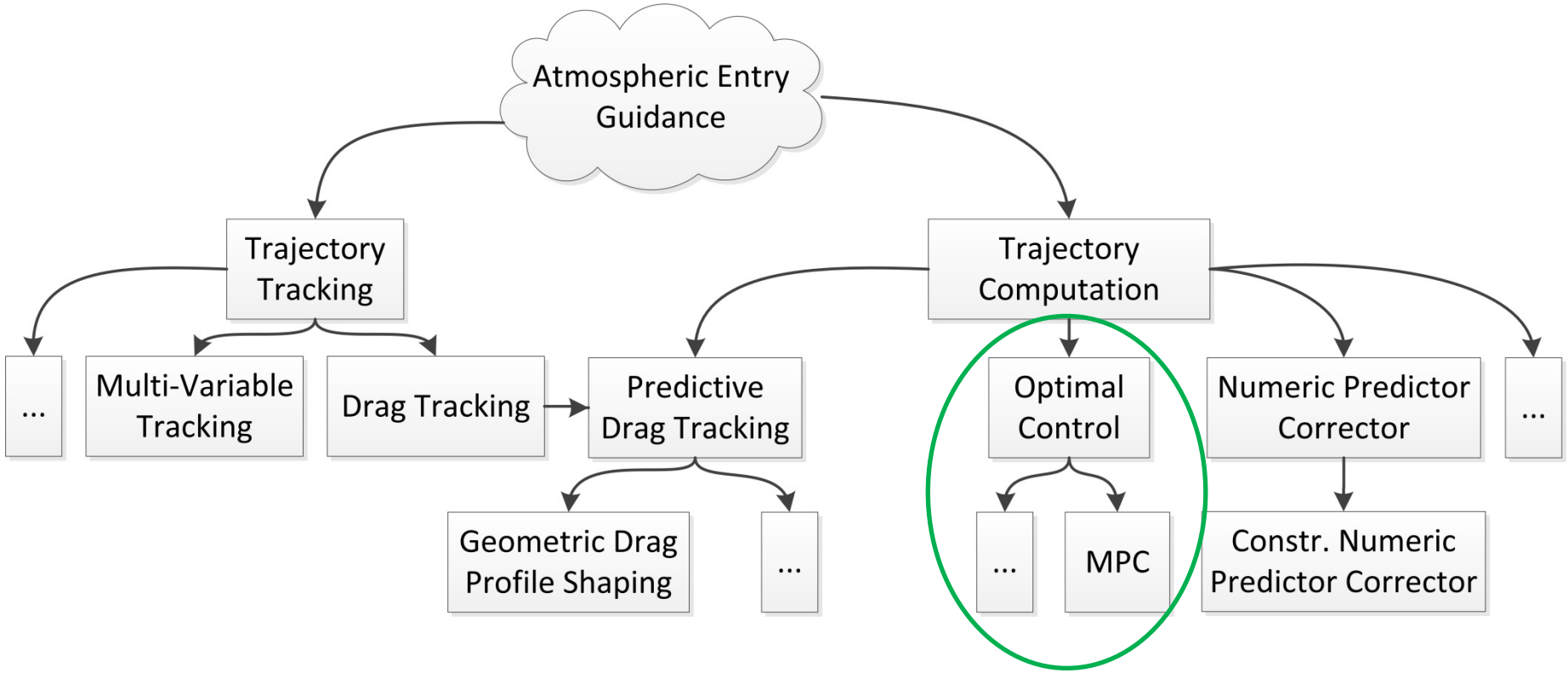
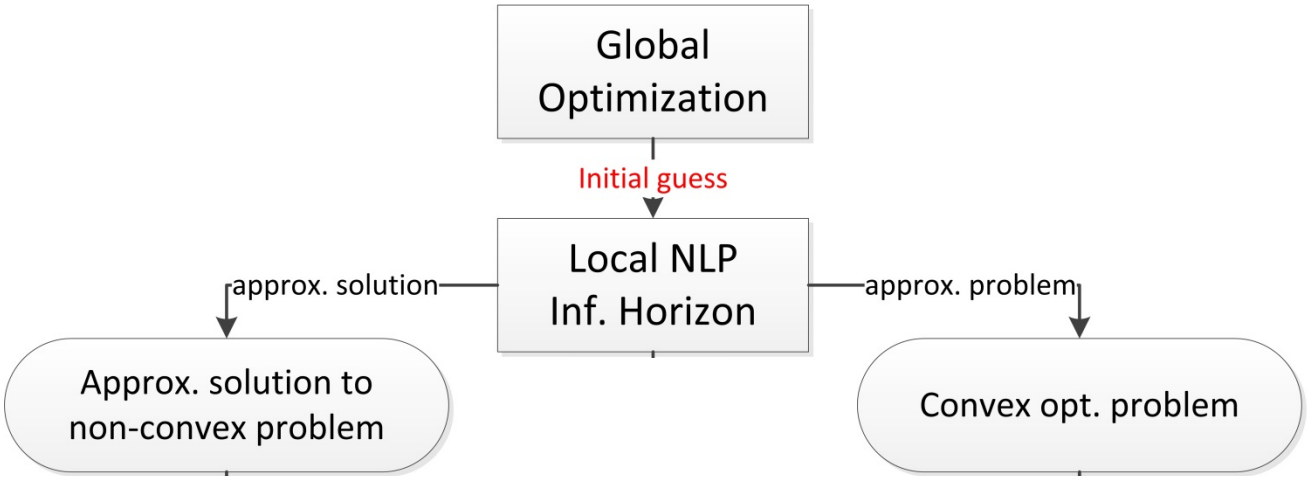


Optimization based Guidance



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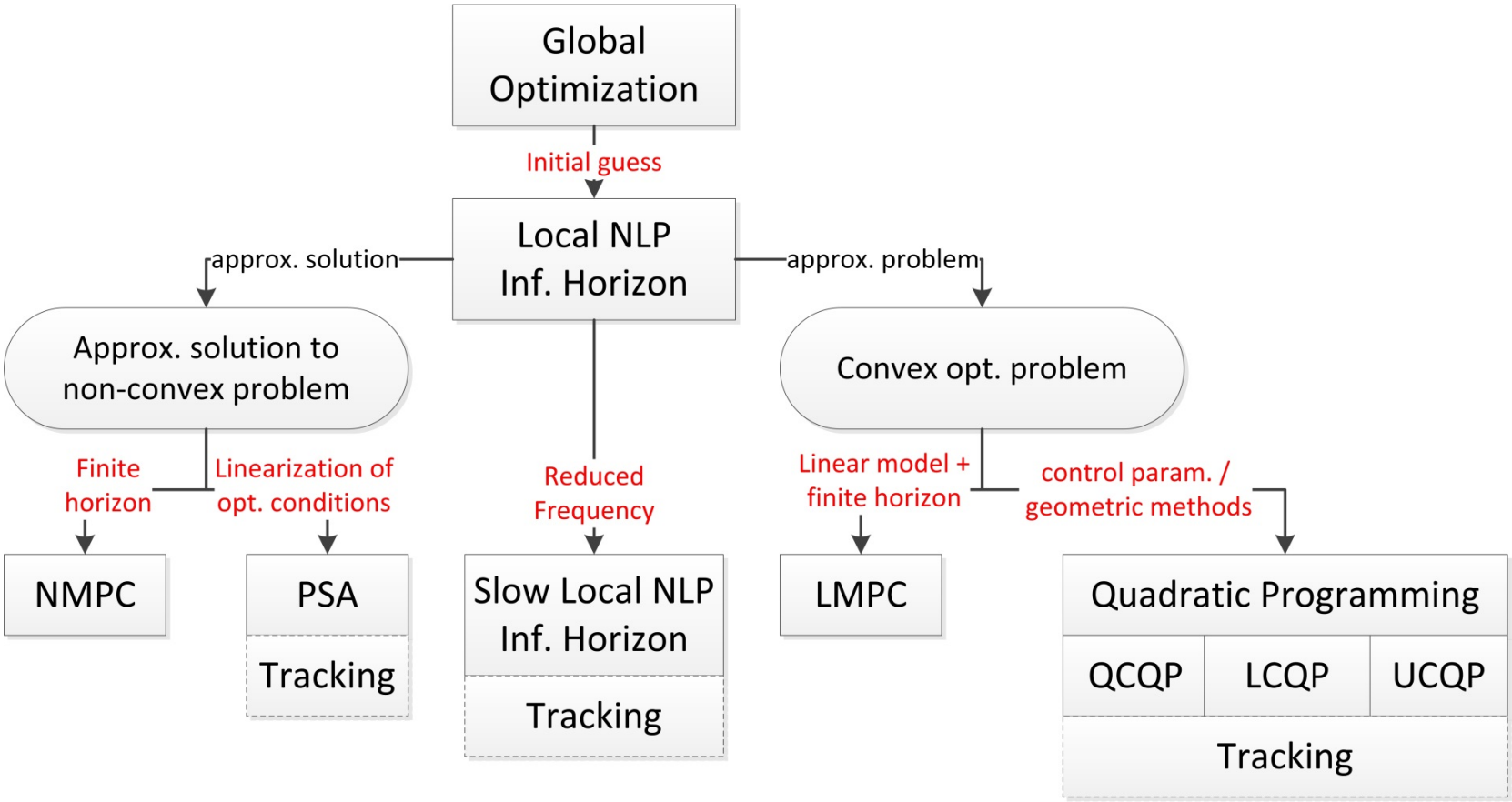

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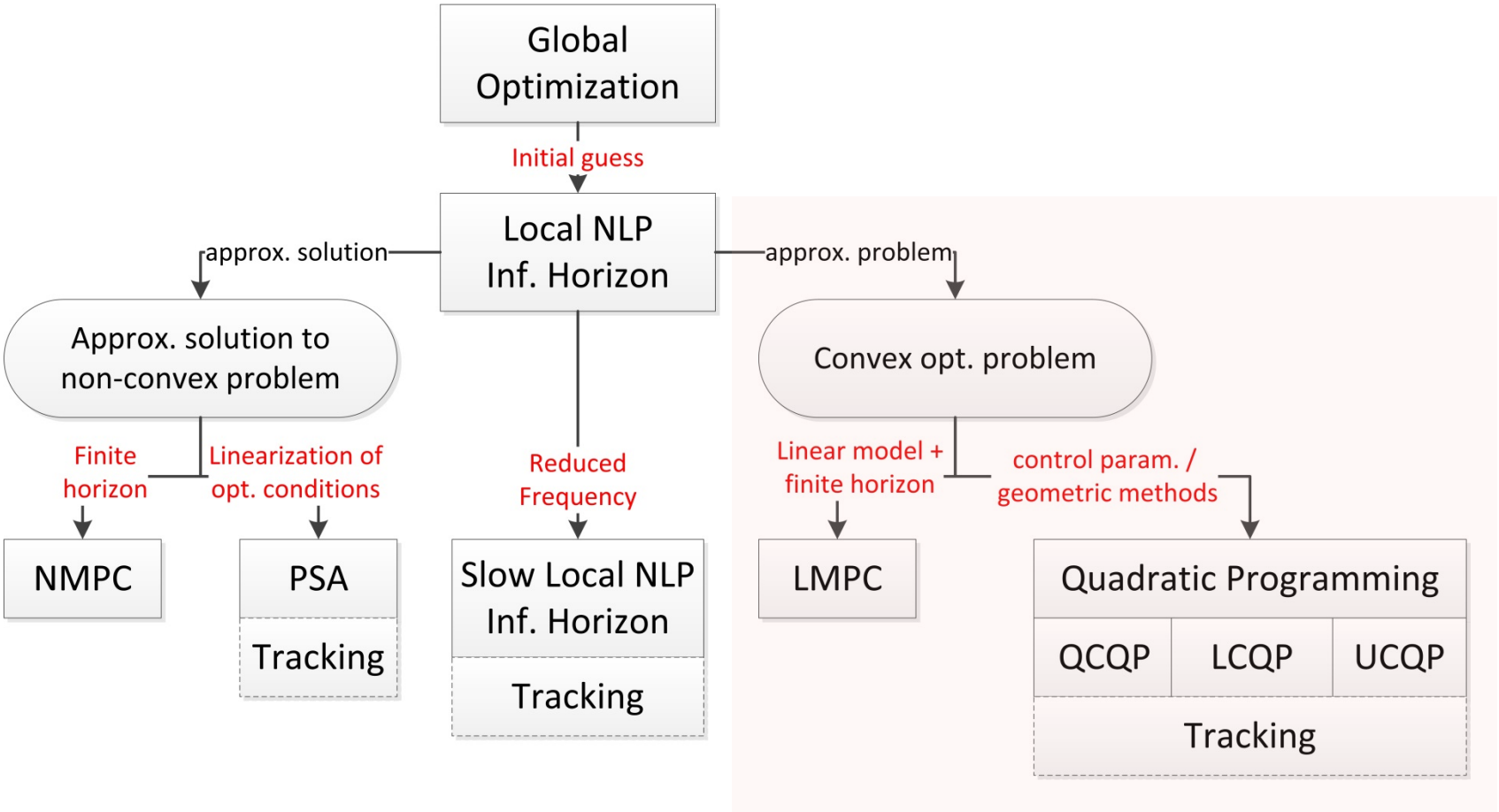

Optimization based Guidance

Property	NMPC Inf. Horizon	NMPC
Computational power required	--	-
OBS complexity	--	--
Respect path constraint	++	++
Region of attraction	++	+
Optimality	++	+
Method universality	++	++
Online optimization	yes	yes

Optimization based Guidance

Property	NMPC Inf. Horizon	NMPC	Slow NLP / Tracking	PSA / Tracking
Computational power required	--	-	o/+	+/+
OBS complexity	--	--	--	+
Respect path constraint	++	++	+/-	o/-
Region of attraction	++	+	+/?	?/?
Optimality	++	+	+/?	+/?
Method universality	++	++	+/o	o/o
Online optimization	yes	yes	yes	no

Optimization based Guidance

Property	NMPC Inf. Horizon	NMPC	Slow NLP / Tracking	PSA / Tracking
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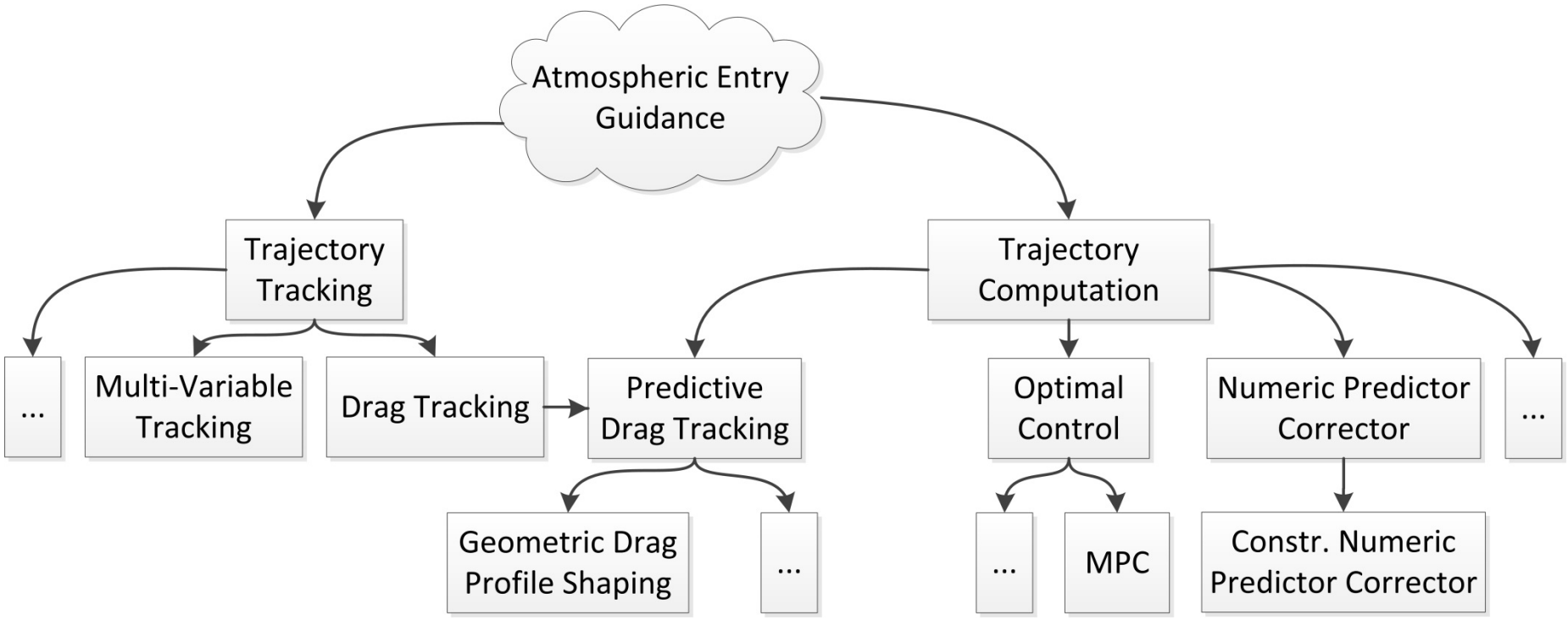
Approaches to Entry Guidance



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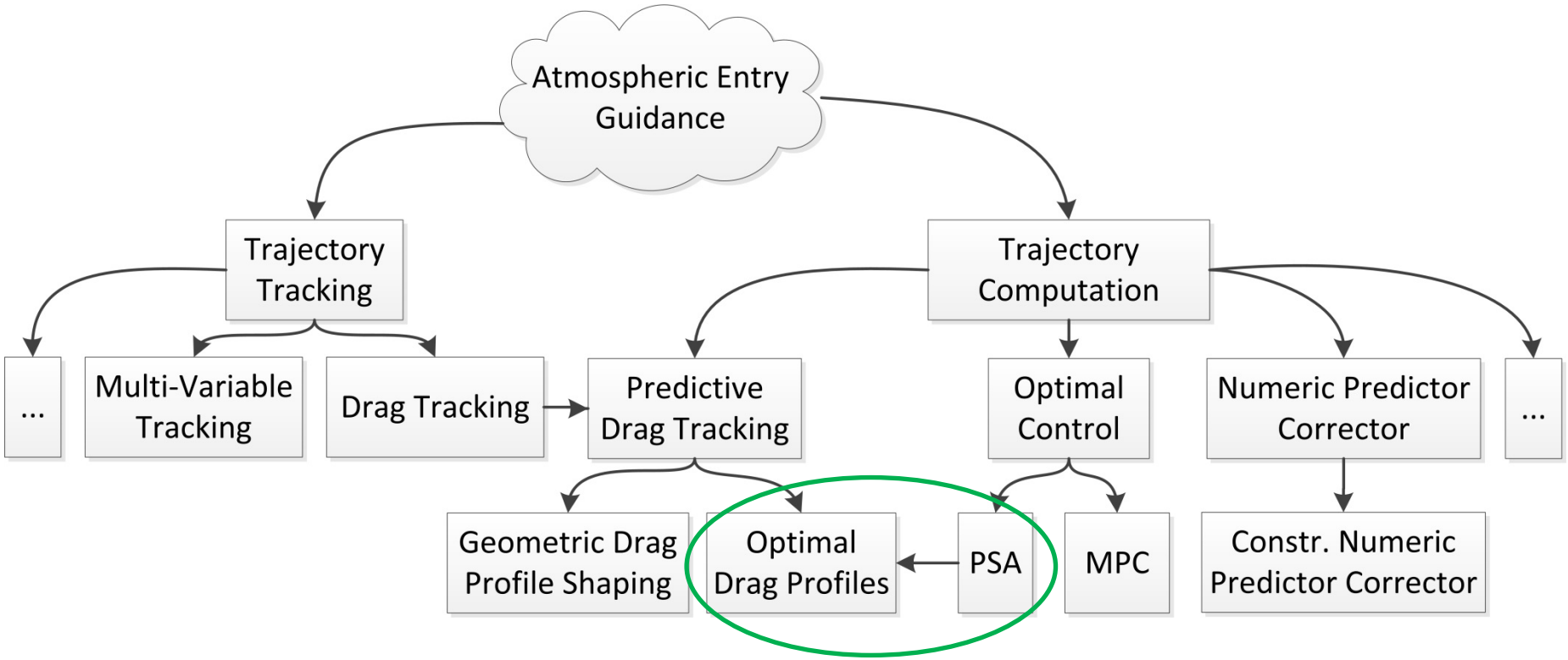

- Optimal solution approximation based on parametric sensitivity analysis to obtain a fast update of the optimal control sequence
- Combination with drag tracking

Trajectory computation

- ❑ Parametric sensitivity analysis of nonlinear programs (offline)
- ❑ Fast solution approximation (online)
- ❑ Repeated online trajectory computation
- ❑ Region of attraction

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Offline Phase: Problem Formulation and Transcription

OCP(p)

Formulation as parametric Optimal Control Problem:

$$\min \quad J(x,u,p) = g\big(x(t_0), x(t_f), p\big) + \int_{t_o}^{t_f} l(x(t),u(t),p)\ dt$$

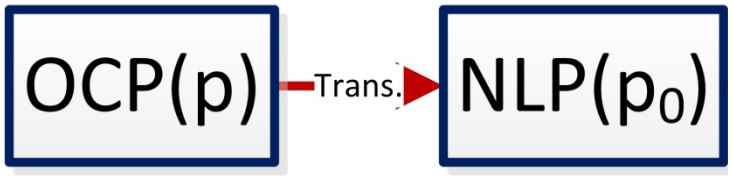
$$\text{w.r.t.} \quad \dot{x}(t) = f(x(t),u(t),p)$$

$$\psi_0(x(t_0),p) = 0$$

$$\psi_f\big(x(t_f),p\big) = 0$$

$$C(x(t),u(t),p) \leq 0 \qquad t\epsilon[t_0,t_f]$$

Offline Phase: Problem Formulation and Transcription



- Direct optimization
  - Discretization (Runge-Kutta-4, Linear control interpolation)
  - Transcription into a parametric nonlinear program (using single or multiple shooting)

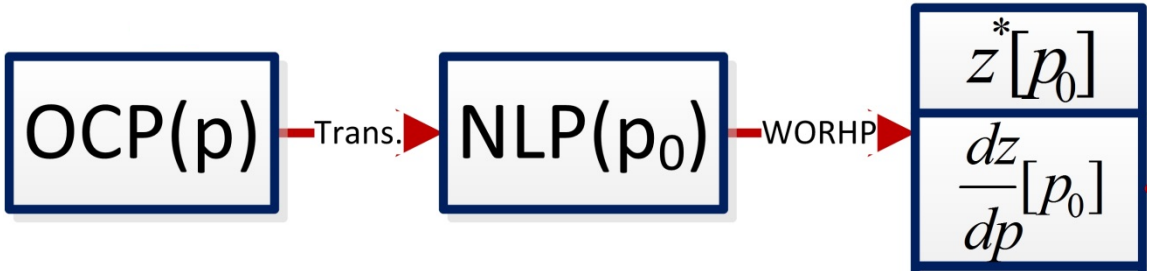
- Choice of a nominal parameter set $p_0 = 0$

$$\begin{aligned} \min_z \quad & F(z, p_0) \\ \text{w.r.t.} \quad & g_l \leq G(z, p_0) \leq g_u \end{aligned}$$

Offline Phase: Sensitivity Analysis I

- Obtain nominal optimal solution $z_0 = z^*(p_0)$

Offline Phase: Sensitivity Analysis I



- Obtain nominal optimal solution $z_0 = z^*(p_0)$
- Obtain parametric sensitivity $\frac{dz}{dp}[p_0]$

Linearization of the Necessary Optimality Conditions

Active constraints: $G^a(z,p)$

Lagrange multipliers: η^a

Lagrange function: $L(z,\eta^a,p) = F(z,p) + (\eta^a)^T G^a(z,p)$

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Lagrange function: $L(z,\eta^a,p) = F(z,p) + (\eta^a)^T G^a(z,p)$

Necessary optimality conditions (KKT):

$$K(z(p),\eta^a(p),p) = \begin{bmatrix} \nabla_z L(z,\eta^a,p) \\ G^a(z,p) \end{bmatrix} = \begin{bmatrix} \nabla_z F(z,p) + (\eta^a)^T \nabla_z G^a(z,p) \\ G^a(z,p) \end{bmatrix} = 0$$

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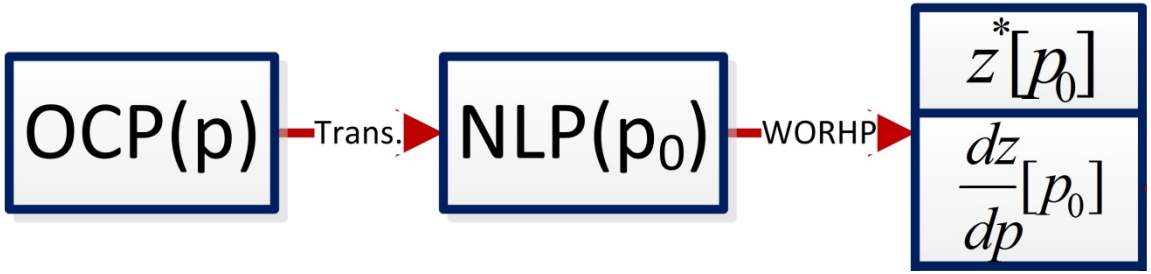
Differentiation of $K(z(p),\eta^a(p),p) \equiv 0$ with respect to p at point p_0:

$$\begin{bmatrix} \nabla_z{}^2 L(z_0,\eta_0^a,p_0) & \nabla_z G^a(z_0,p_0)^T \\ \nabla_z G^a(z_0,p_0) & 0 \end{bmatrix} \begin{bmatrix} \frac{dz}{dp}[p_0] \\ \frac{d\eta^a}{dp}[p_0] \end{bmatrix} + \begin{bmatrix} \nabla_{zp}{}^2 L(z_0,\eta_0^a,p_0) \\ \nabla_p G^a(z_0,p_0) \end{bmatrix} = 0$$

Literature: [Fiacco] [Büskens]

Offline Phase: Sensitivity Analysis I



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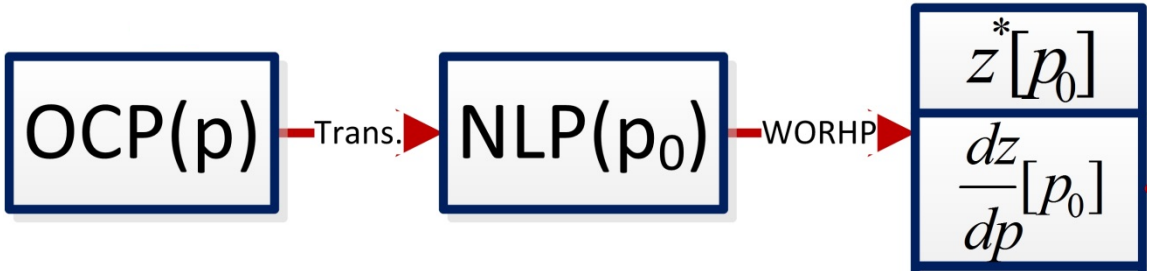

- Obtain nominal optimal solution $z_0 = z^*(p_0)$
- Obtain parametric sensitivity $\frac{dz}{dp}[p_0]$
- If z_0 fulfills strong second order sufficient conditions: Kuhn-Tucker matrix is invertible

$$\begin{bmatrix} \boldsymbol{\frac{dz}{dp}[p_0]} \\ \frac{d\eta^a}{dp}[p_0] \end{bmatrix} = -\begin{bmatrix} \nabla_z{}^2 L(z_0, \eta_0^a, p_0) & \nabla_z G^a(z_0, p_0)^T \\ \nabla_z G^a(z_0, p_0) & 0 \end{bmatrix}^{-1} \begin{bmatrix} \nabla_{zp}{}^2 L(z_0, \eta_0^a, p_0) \\ \nabla_p G^a(z_0, p_0) \end{bmatrix}$$

Online Solution Approximation: p-Step

- Approximation of optimal solution for disturbed parameters p using Taylor expansion

$$z^*(p) \approx z_1 := z_0 + \frac{dz}{dp}[p_0] \cdot (p - p_0)$$

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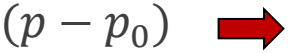
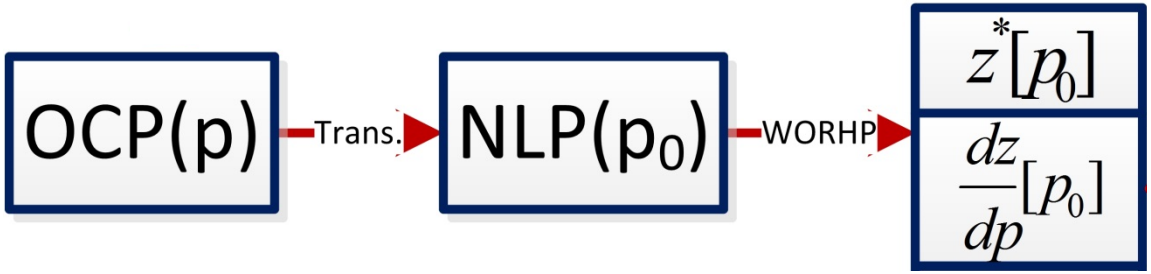


error in the active constraints

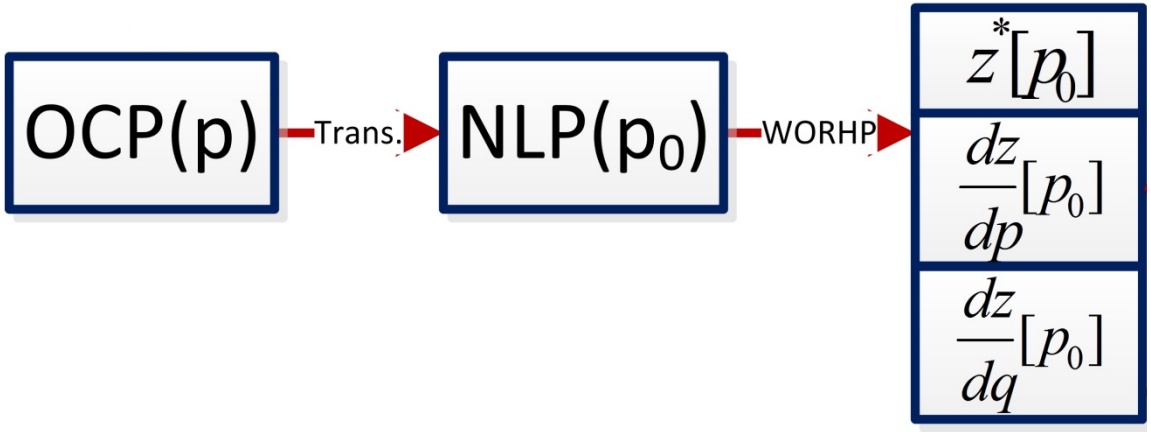
$$\|G^a(z_1, p)\| > 0$$

Offline Phase: Sensitivity Analysis II



$$\begin{aligned} \min_z \quad & F(z, p_0) \\ \text{w.r.t.} \quad & g_l \leq G(z, p_0) \leq g_u \end{aligned}$$

Offline Phase: Sensitivity Analysis II



- Additional parameter vector q with nominal value $q_0 = 0$

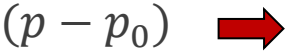
$$\begin{aligned} \min_z \quad & F(z, p_0) \\ \text{w.r.t.} \quad & g_l \leq G(z, p_0) - q_0 \leq g_u \end{aligned}$$

- $\frac{dz}{dq}$ can be computed analog to $\frac{dz}{dp}$

Online Solution Approximation: p-Step and q-Step

- Approximation of optimal solution for disturbed parameters p using Taylor expansion

$$z^*(p) \approx z_1 := z_0 + \frac{dz}{dp}[p_0] \cdot (p - p_0)$$



error in the active constraints

$$\|G^a(z_1, p)\| > 0$$

Online Solution Approximation: p-Step and q-Step

- Approximation of optimal solution for disturbed parameters p using Taylor expansion

$$z^*(p) \approx z_1 := z_0 + \frac{dz}{dp}[p_0] \cdot (p - p_0)$$

error in the active constraints

$$\|G^a(z_1, p)\| > 0$$

- $\frac{dz}{dq}$ is used to iteratively correct the constraint error and at the same time improve the optimality of the approximation

while $\|G^a(z_i, p)\| > \varepsilon$

$$q_i = G^a(z_i, p)$$

$$z_{i+1} := z_i + \frac{dz}{dq^a}[p_0] \cdot q_i$$

For $\|q - q_0\| < \delta$ iteration converges against a fixpoint z_∞ at which

$$\|G^a(z_\infty, p)\| = 0$$

[Büskens]

Example

- Scenario: Entry of a small capsule into the Martian atmosphere
- Control: Bank angle μ
- Assumptions: Constant AoA (capsule statically and dynamically stable)
- Dynamic model: Translational motion over a spherical, rotating planet (nonlinear, coupled, first order ODE system)

Example

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State	$x(t_0)$	$x(t_f)$
Altitude	$h_0 = 120000\ m$	$h_f = 10000\ m$
Longitude	$\lambda_0 = 0°$	$\lambda_f = 11.3°$
Latitude	$\varphi_0 = 25°$	$\varphi_f = 23.3°$
Velocity	$v_0 = 5440.8\ {}^{m}/_{s}$	$v_f \leq 450\ {}^{m}/_{s}$
FPA	$\gamma_0 = -14.5°$	$\gamma_f : free$
Heading	$\chi_0 = 97.4°$	$\chi_f : free$

Path Constraint	**Limit**
Heat flux	$\dot{Q} \leq 1600$ kW/m^2
Dynamic pressure	$q \leq 17$ kPa
Load factor	$n \leq 15$

Example

Parametrization

Initial condition: $\psi_0 = x(t_0) + p_I$

Vehicle mass: $m = m_0 + p_m$

Lift coefficient: $C_L = C_{L_0} + p_L C_{L_0}$

Drag coefficient: $C_D = C_{D_0} + p_D C_{D_0}$

Parameter: $p = (p_I, p_m, p_L, p_D)^T$

Nominal value: $p_0 = (0,0,0,0,0,0,0,0,0)^T = 0$

Example

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Parameter: $p = (p_I, p_m, p_L, p_D)^T$

Nominal value: $p_0 = (0,0,0,0,0,0,0,0)^T = 0$

Performance index

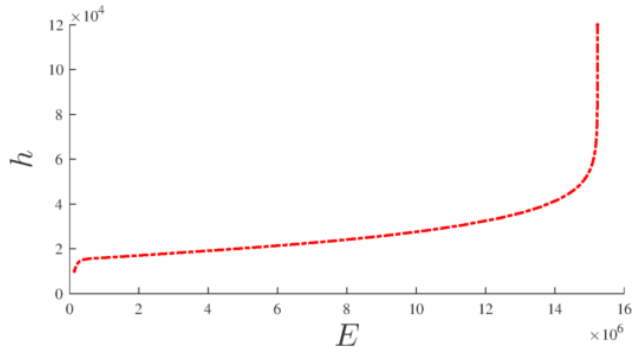
- Minimize terminal velocity v_f
- Avoid flight at max/min lift
- “Smooth” control function

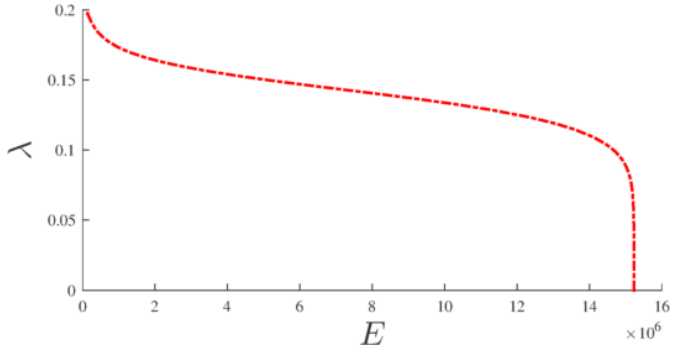
Objective function: $F(x,u,p) = w_1 v_f^2 + w_2 \int_{t_0}^{t_f} w_e(t)[w_3(\cos\mu)^2 + w_4\dot{\mu}^2]\,dt$

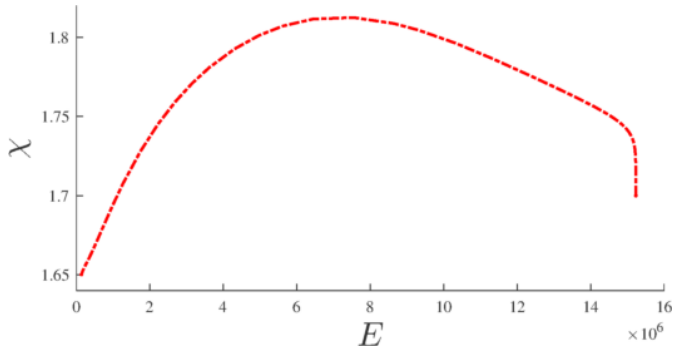
w_1, w_2, w_3, w_4: positive, constant

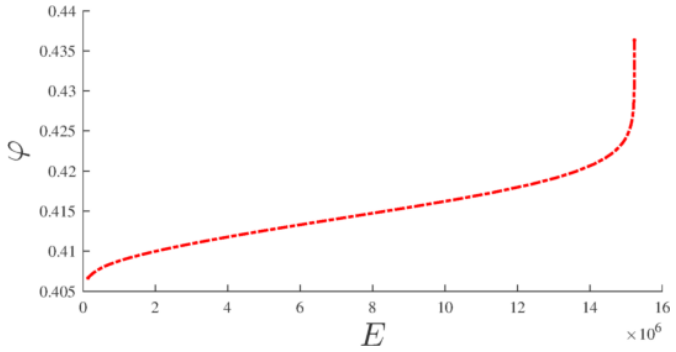
$w_e(t)$: positive function

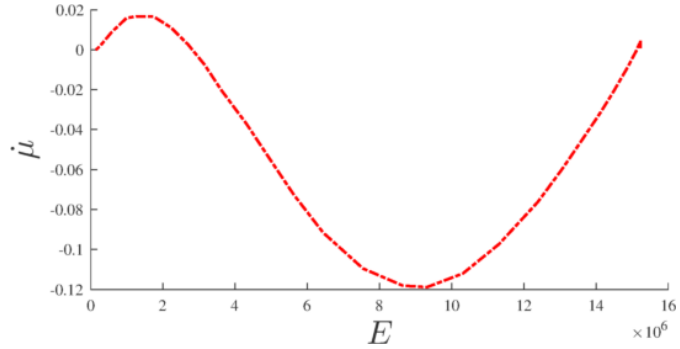
Nominal Solution ($p := p_0$)

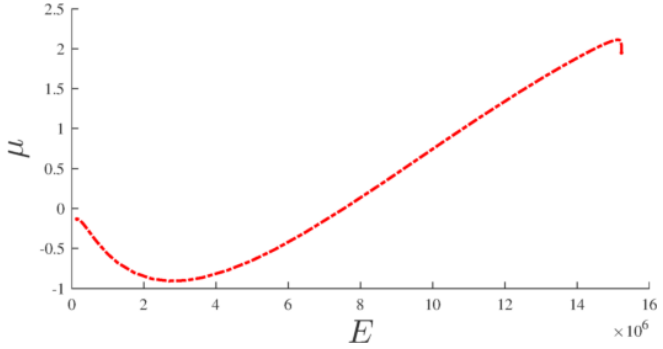












Perturbed Environment

Example: Estimated conditions at flight time t_0:

Perturbed initial conditions:

$\bar{x}_0(t_0)$	$\bar{p}_I$
$\overline{h_0} = h_0 +$	2000
$\overline{\lambda_0} = \lambda_0 -$	0.001
$\overline{\varphi_0} = \varphi_0 +$	0.005
$\overline{v_0} = v_0 -$	100
$\overline{\gamma_0} = \gamma_0 -$	0.0012
$\overline{\chi_0} = \chi_0 -$	0.0005

Perturbed model:

$\bar{p}_m = 9$

$\bar{p}_L = 0.03$

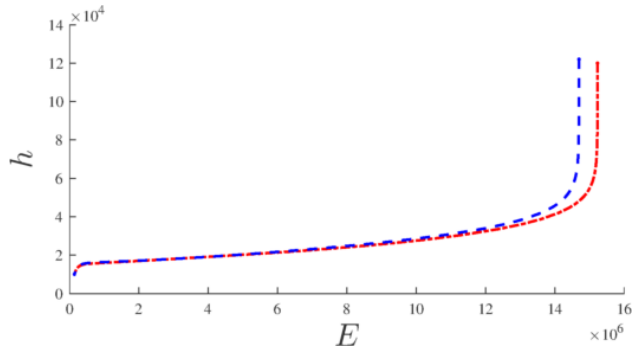
$\bar{p}_D = -0.04$

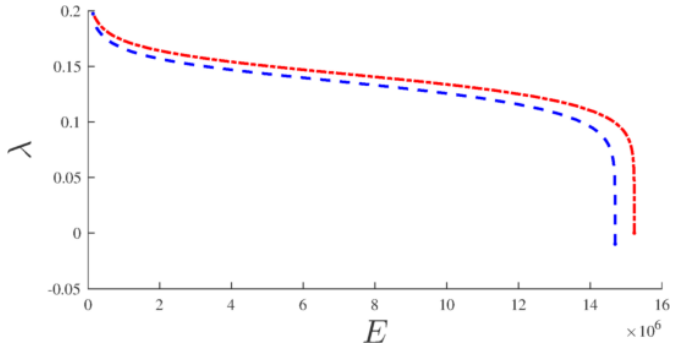
Perturbed parameter set:

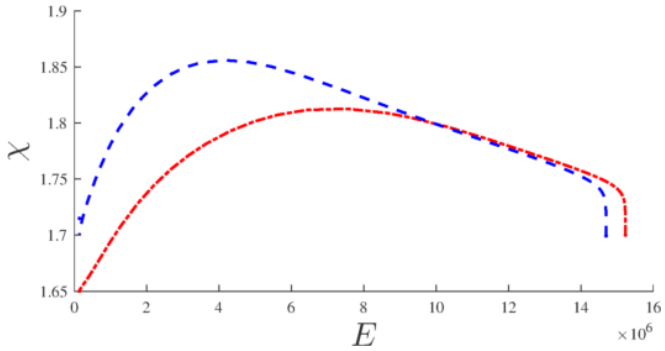
$$\bar{p} = (\bar{p}_I, \bar{p}_m, \bar{p}_L, \bar{p}_D)^T$$

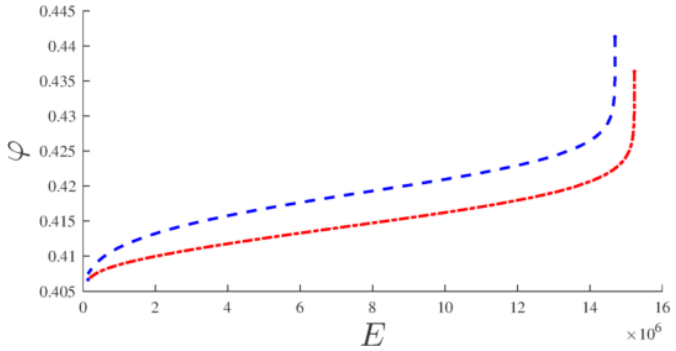
Approximate optimal control sequence and trajectory for perturbed parameters $\bar{p}$

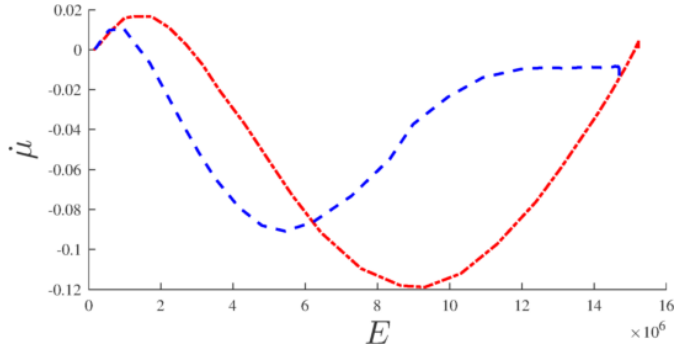
Solution Approximation for $p := \overline{p}$ (p-Step)

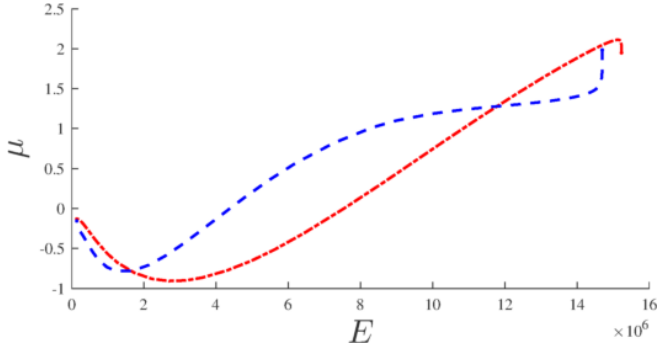




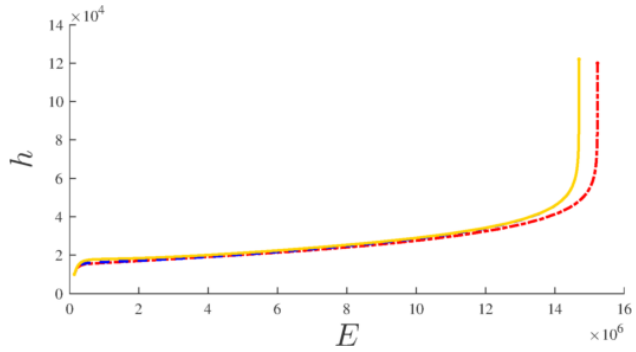


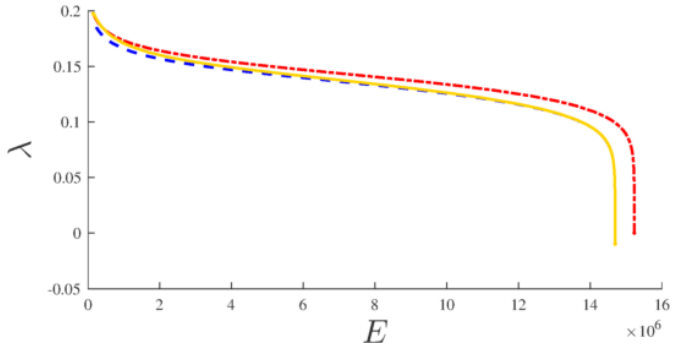


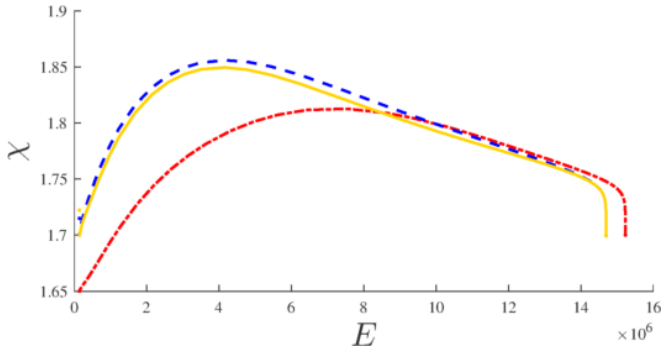


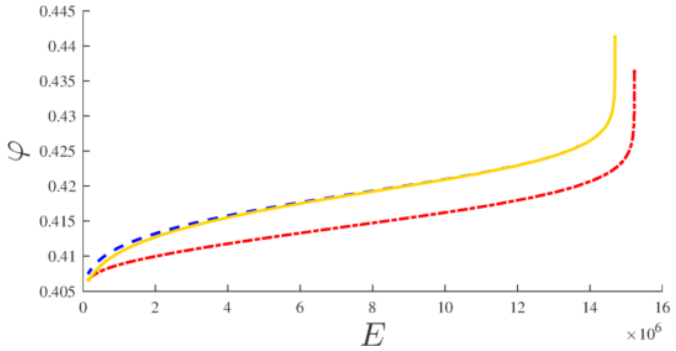


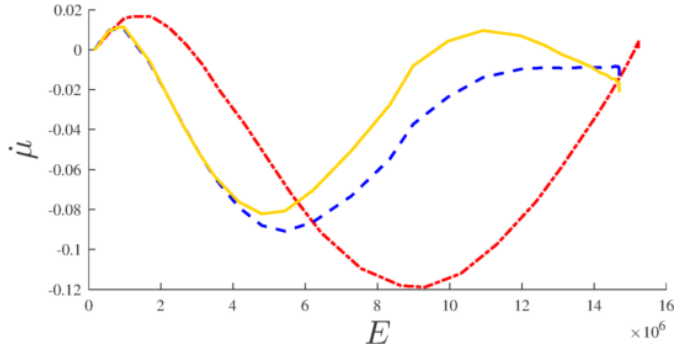
Solution Approximation for $p := \overline{p}$ (q-Step 1)

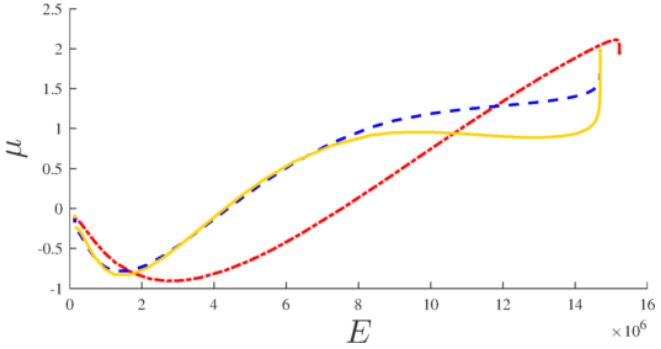




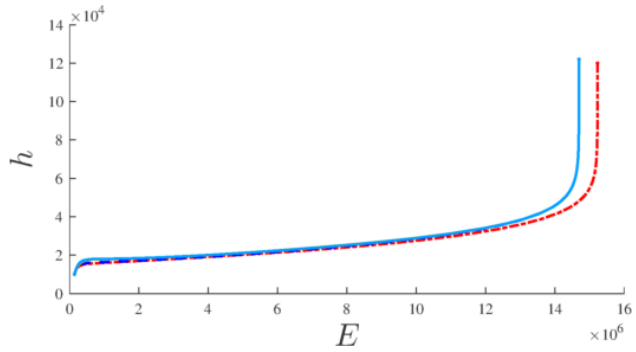


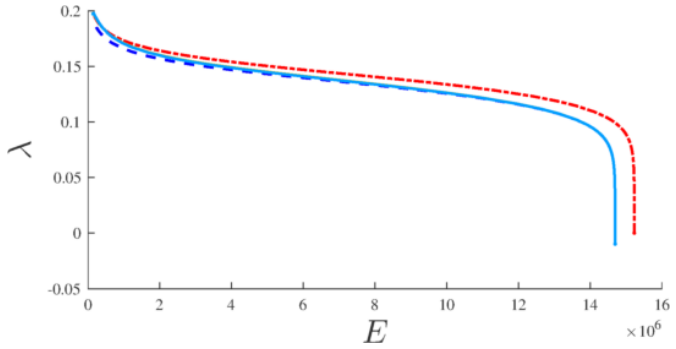


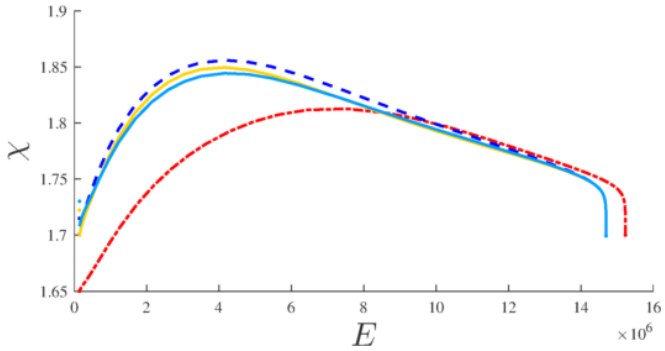


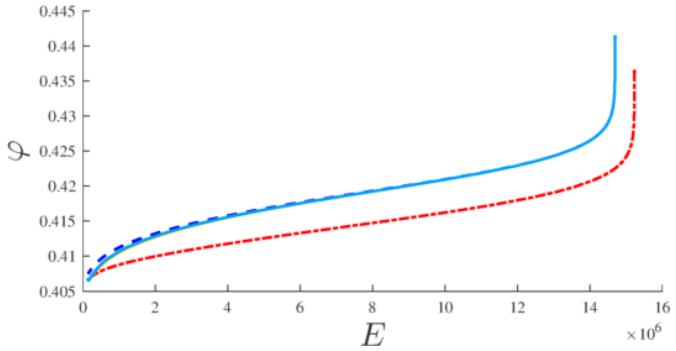


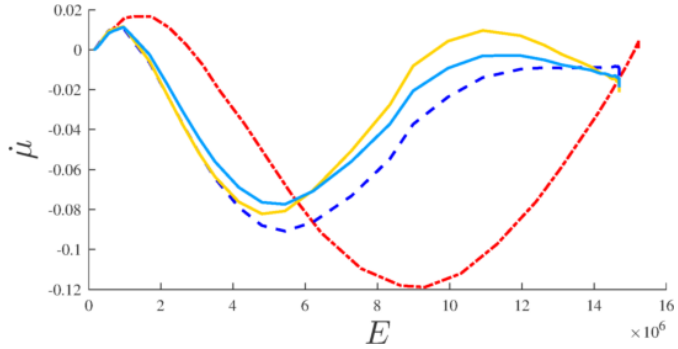
Solution Approximation for $p := \overline{p}$ (q-Step 2)

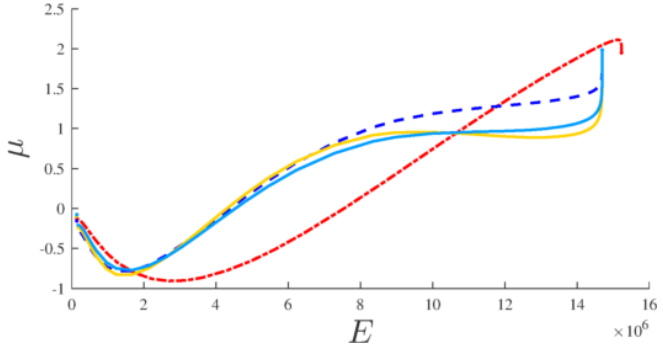




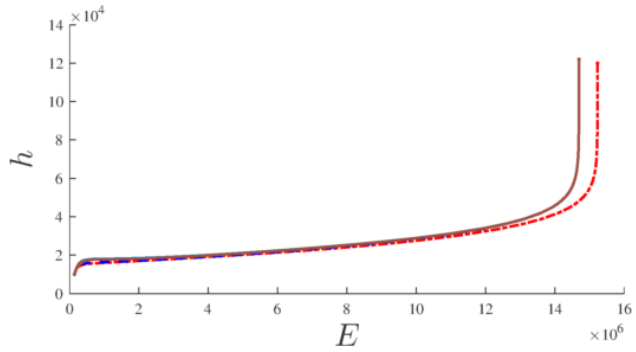


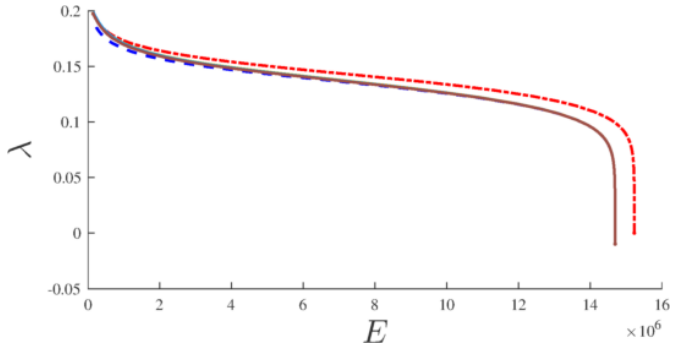


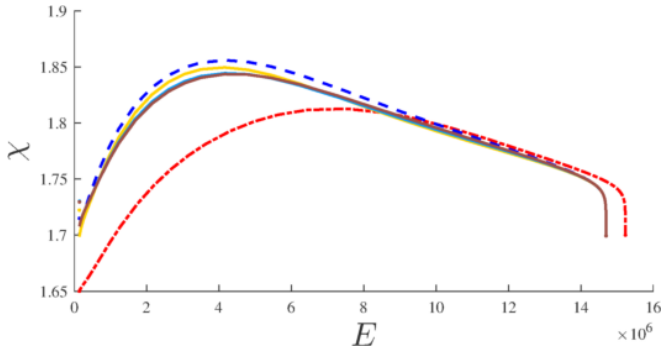


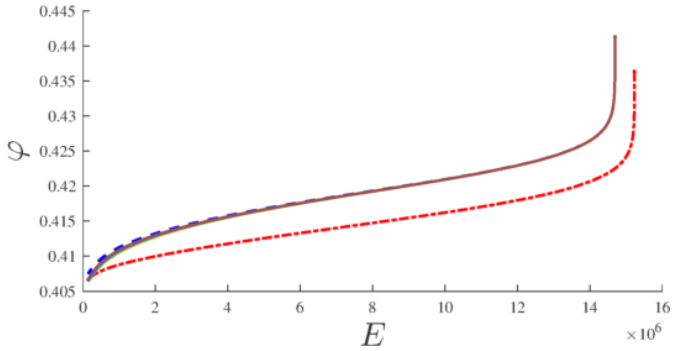


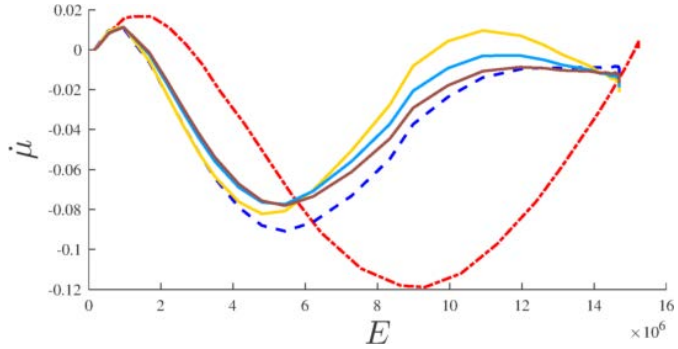
Solution Approximation for $p := \overline{p}$ (q-Step 3)

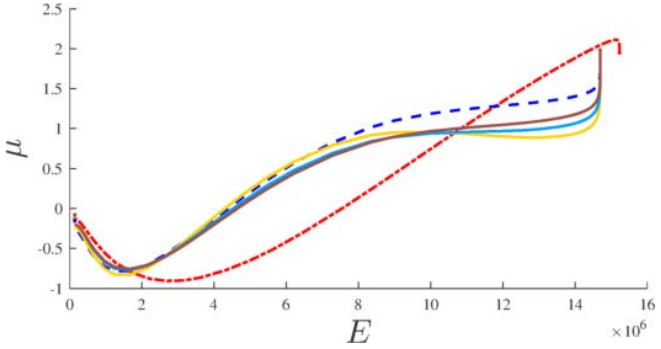




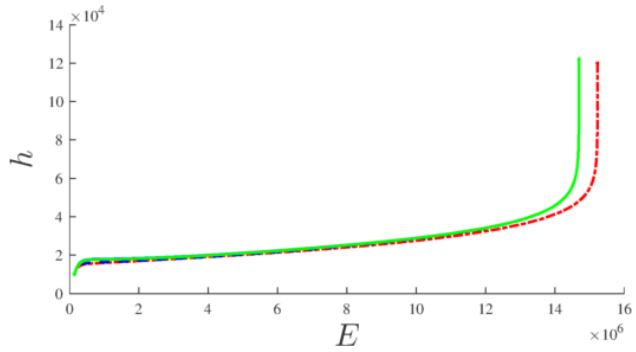


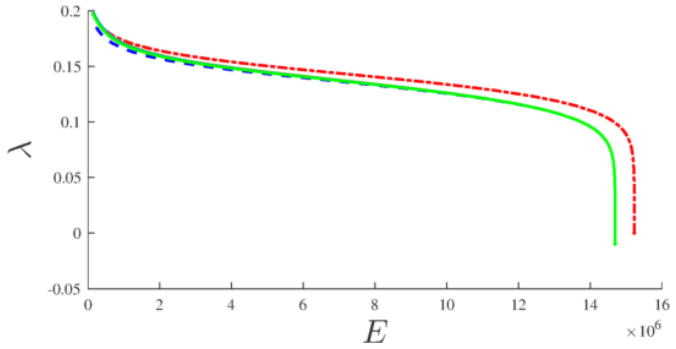


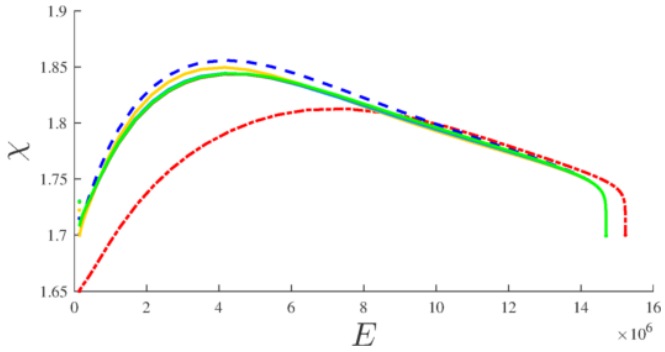


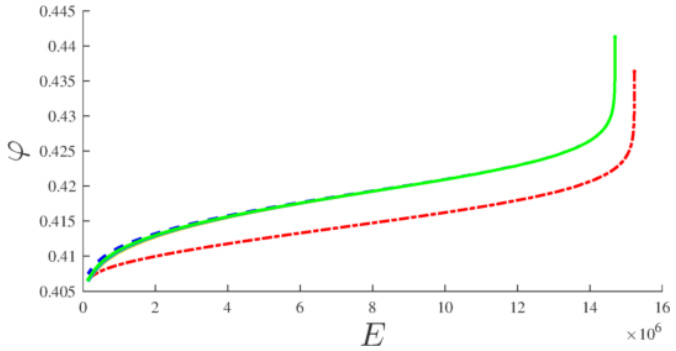


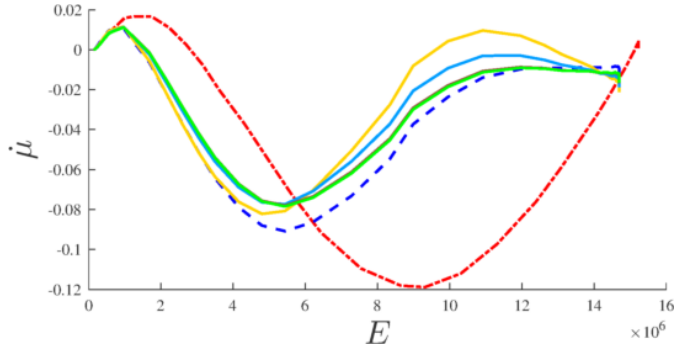
Solution Approximation for $p := \overline{p}$ (q-Step 4) (Termination)

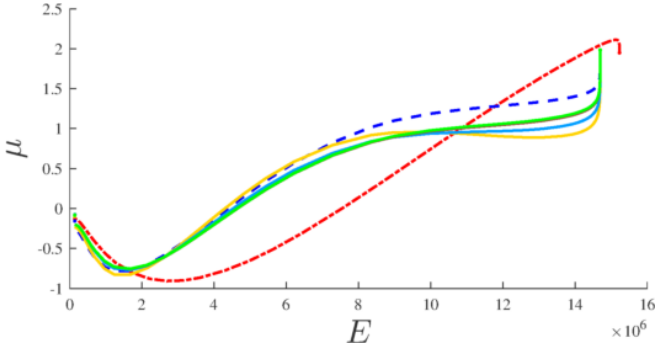






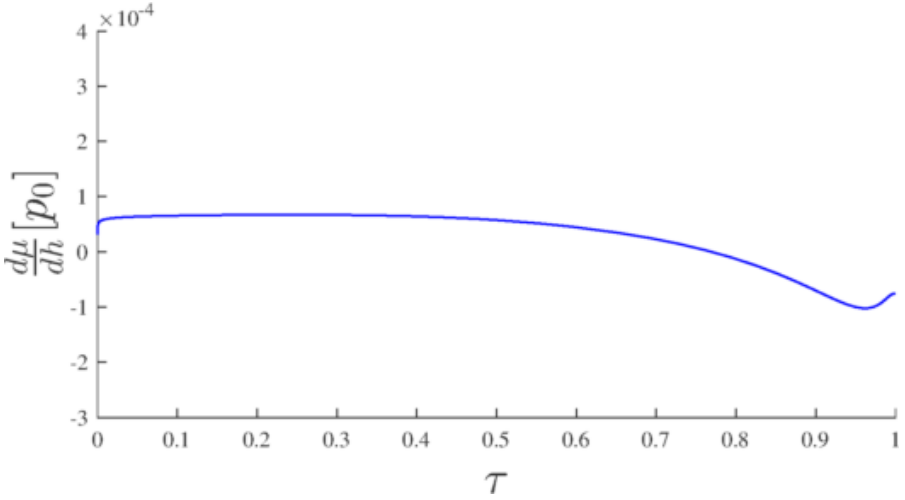






Parametric Sensitivities on Discrete Points of the Nominal Trajectory

Image 1: Sensitivity of μ against perturbations in h at $\psi_0^0 = x^*(t_0)$



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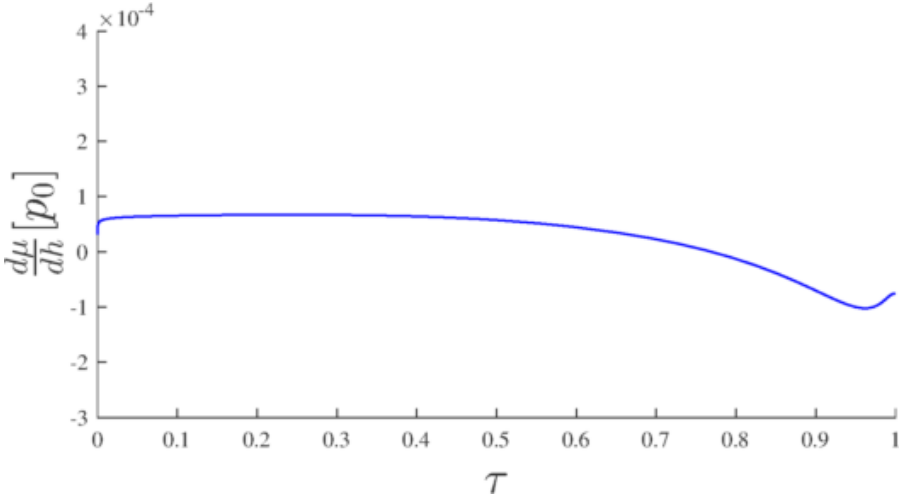
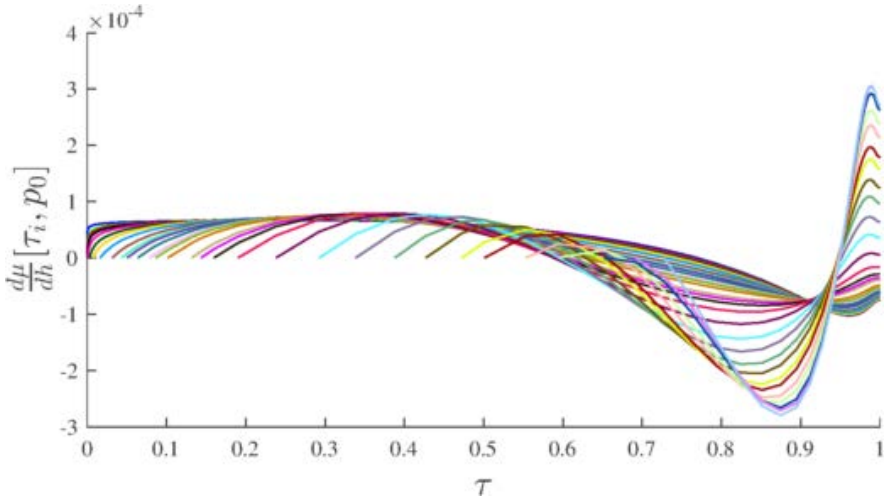
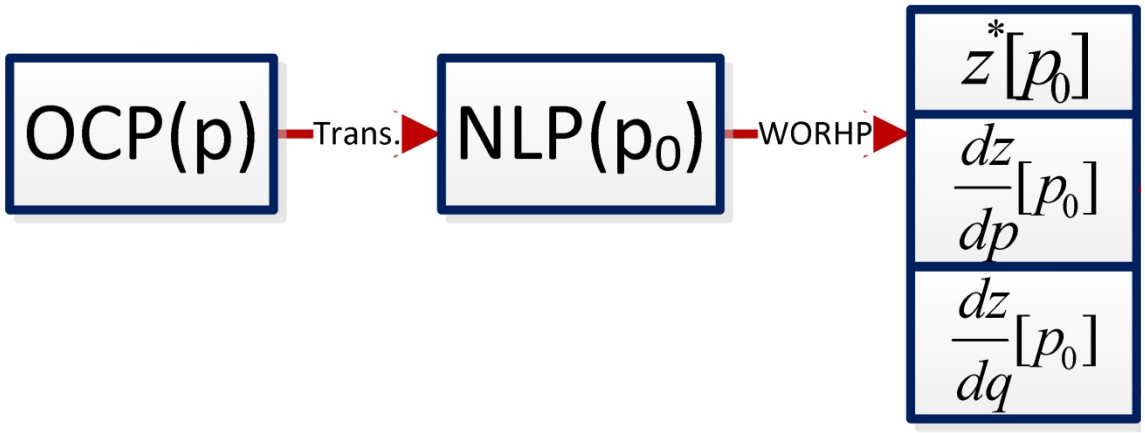


Image 2: Sensitivity of μ against perturbations in h at $\psi_0^{t_i} = x^*(t_i),\ 0 \leq i \leq l$

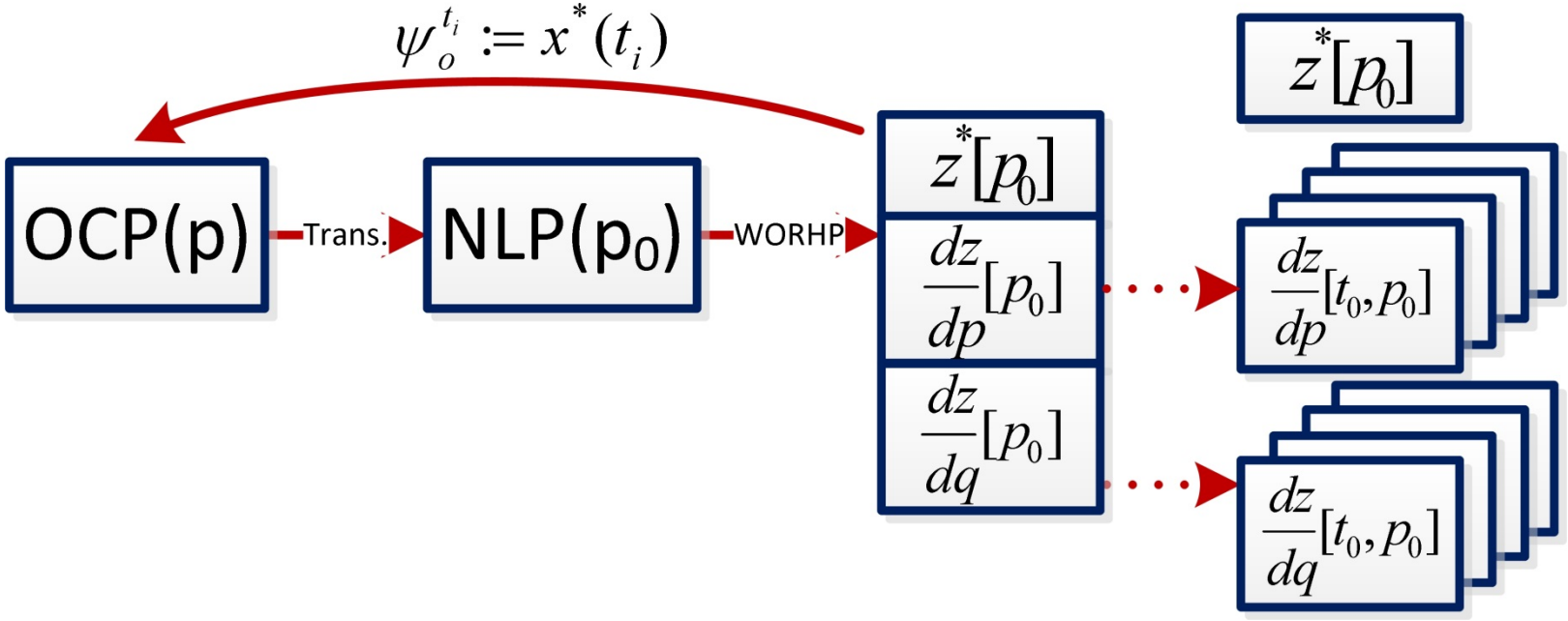


Offline Phase: Sensitivity Catalog



- Trajectory computation at a fixed time t requires sensitivity differentials for initial condition $\psi_0^t := x^*(t),\ t \in [t_0, t_f)$

Offline Phase: Sensitivity Catalog

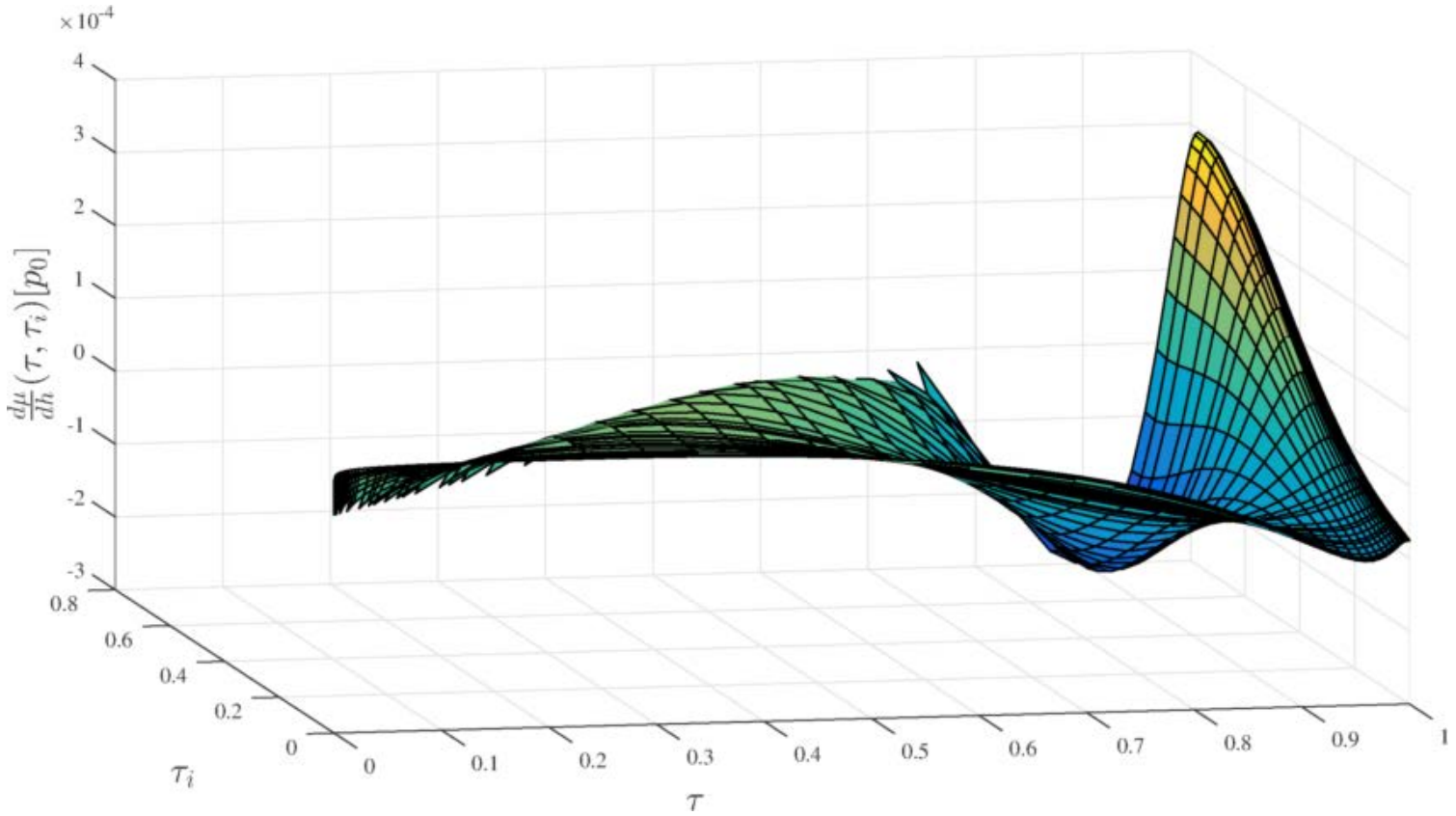


- Trajectory computation at a fixed time t requires sensitivity differentials for initial condition $\psi_0^t := x^*(t)$, $t \in [t_0, t_f)$

- Sensitivity analysis is repeated at discrete points $t_i \in [t_0, t_f)$, $0 < i \leq l$ of the nominal trajectory $x^*(t)$

Interpolation of Parametric Sensitivity Differentials

- Interpolate parametric sensitivities between sufficiently close initial conditions (required for all combinations of states/controls and perturbation parameters)



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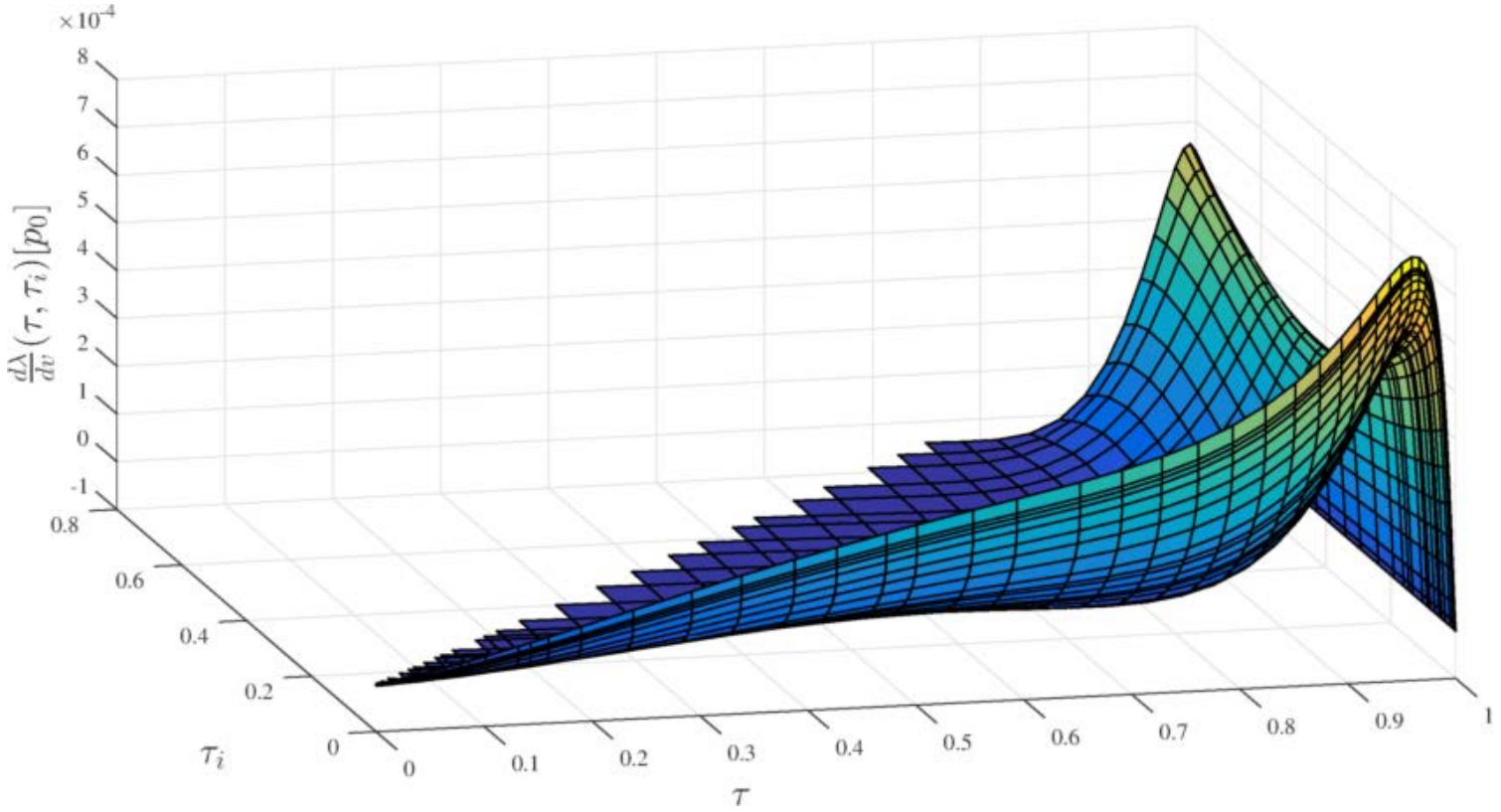
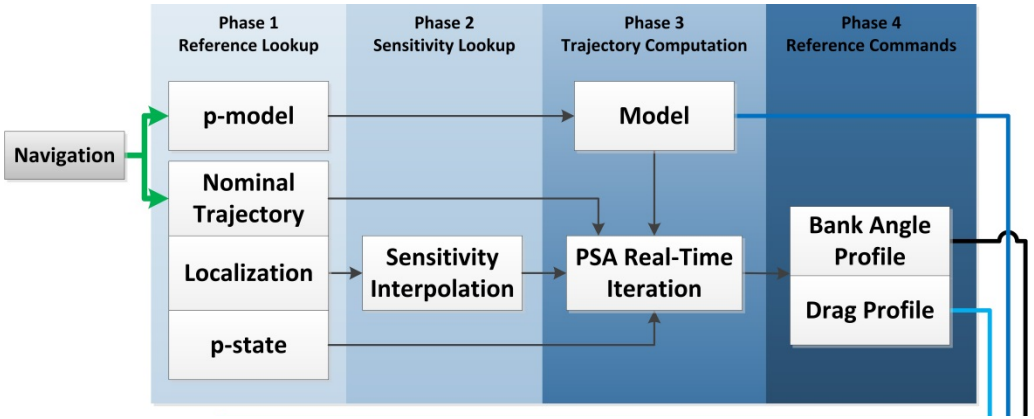


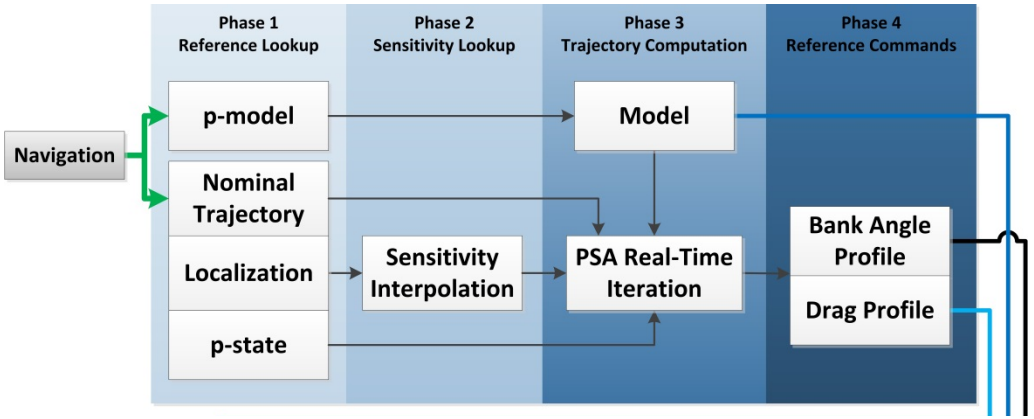
Procedure: Repeated Online Trajectory Computation

1. Estimate current state $\bar{x}$ and model perturbations $\bar{p}_m, \bar{p}_L, \bar{p}_D$



Procedure: Repeated Online Trajectory Computation

1. Estimate current state $\bar{x}$ and model perturbations $\bar{p}_m, \bar{p}_L, \bar{p}_D$
2. Determine $\bar{t}$ such that $\|\bar{p}_I\|$ is sufficiently small with $\bar{p}_I = \bar{x} - x^*(\bar{t}, p_0)$



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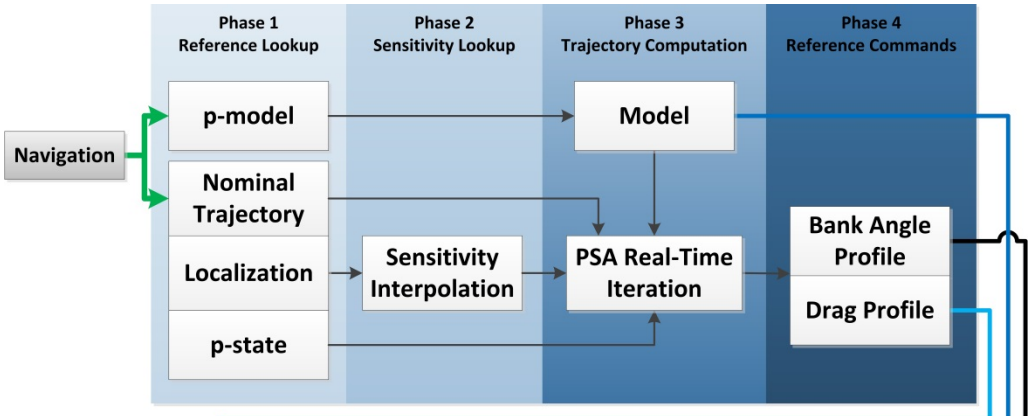
3. Set $p := (\bar{p}_I, \bar{p}_m, \bar{p}_L, \bar{p}_D)^T$



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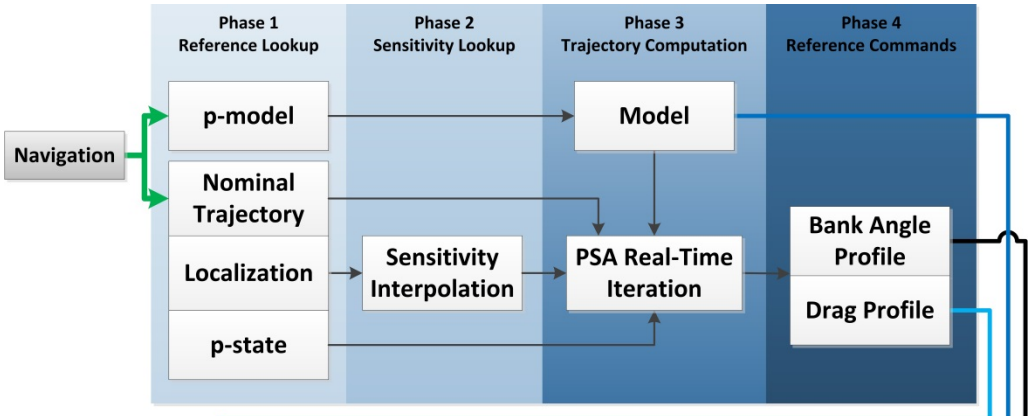

4. Evaluate sensitivity surfaces at $\bar{t}$ to obtain parametric sensitivities approximations $\frac{\widetilde{dz}}{dp}[\bar{t}, p_0]$, $\frac{\widetilde{dz}}{dq}[\bar{t}, q_0]$ (corresponding to initial condition $\psi_0^{\bar{t}} := x^*(\bar{t}, p_0)$)

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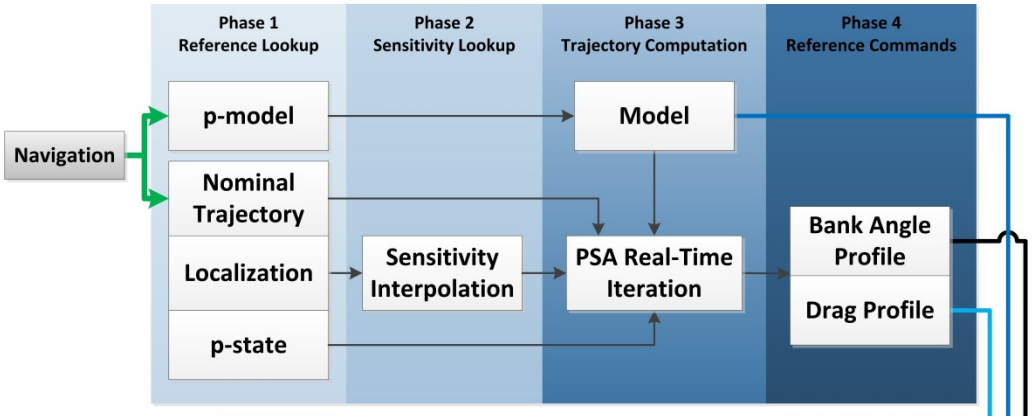

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5. Execute real-time iteration scheme with arguments
   a. $x^*(t, p_0), t \in [\bar{t}, t_f]$ (remaining nominal trajectory)
   b. $\frac{\widetilde{dz}}{dp}[\bar{t}, p_0], \frac{\widetilde{dz}}{dq}[\bar{t}, q_0]$ (interpolated parametric sensitivities)
   c. $p := (\bar{p}_I, \bar{p}_m, \bar{p}_L, \bar{p}_D)^T$ (perturbation parameters)

Convergence Region

What are the maximal perturbations that can be compensated? ⟺ Convergence region of the real-time iteration scheme?

- If the functions F and G are three times continuously differentiable and strong sufficient conditions of optimality hold:

  $\exists\, U(p_0)$ of p_0 such that $\forall\, p = p_0 + \Delta p \,\epsilon\, U$ it holds that $\|G^a(z_\infty, p)\| = 0$

- Extend of U: No analytic answer for nonlinear problems
- U is defined for a fixed set of active constraints G^a
- If G^a changes for a perturbation $p \quad \Rightarrow \quad p \notin U$

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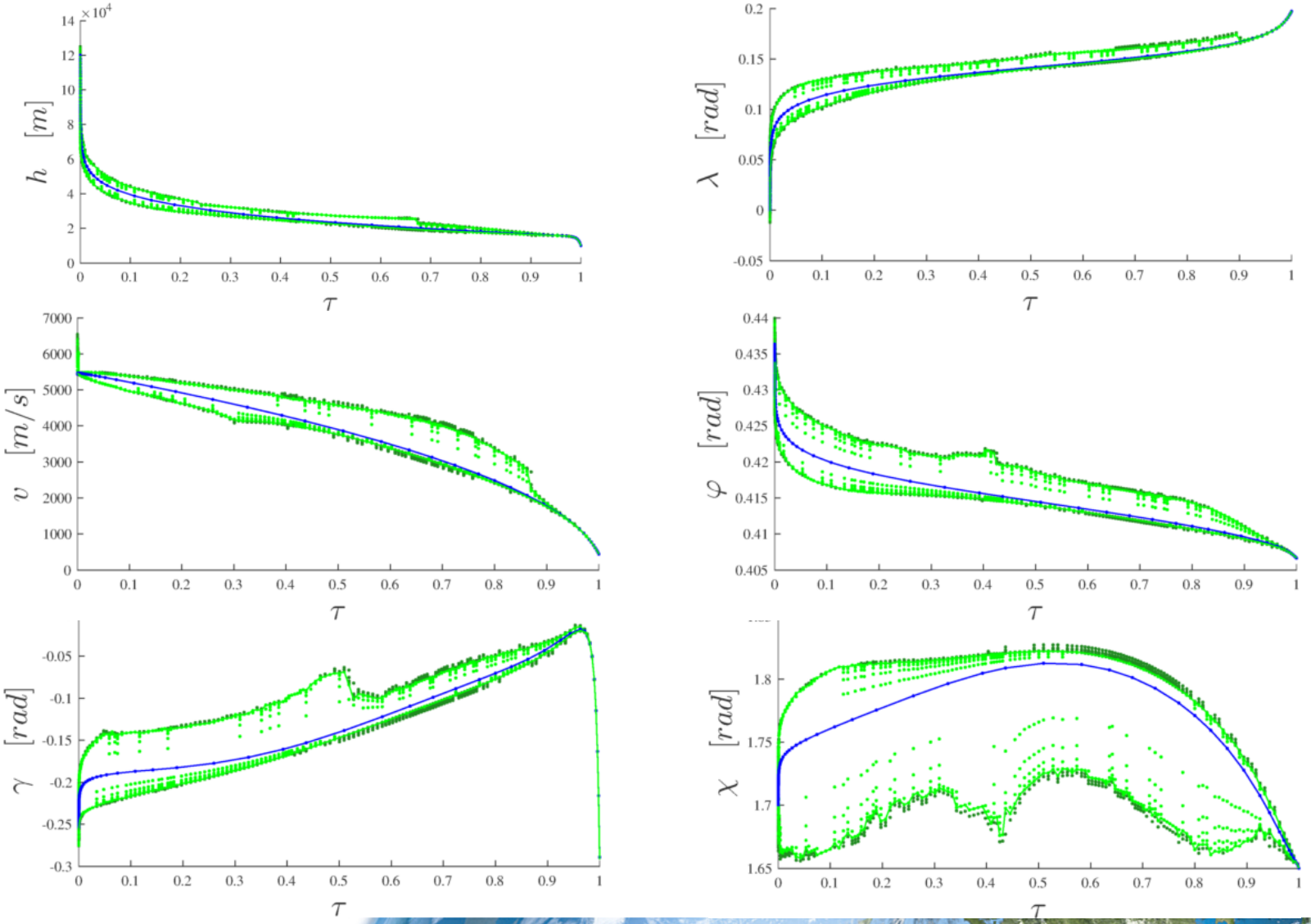
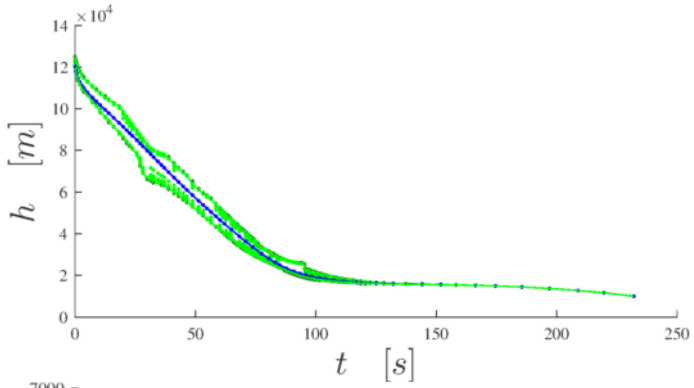
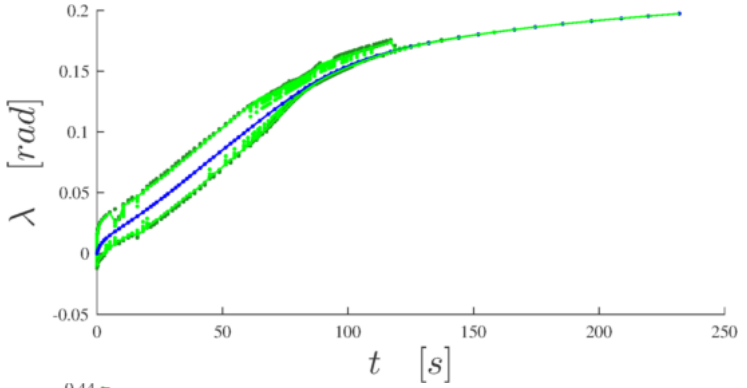
Investigate convergence region!
(for selected perturbations)

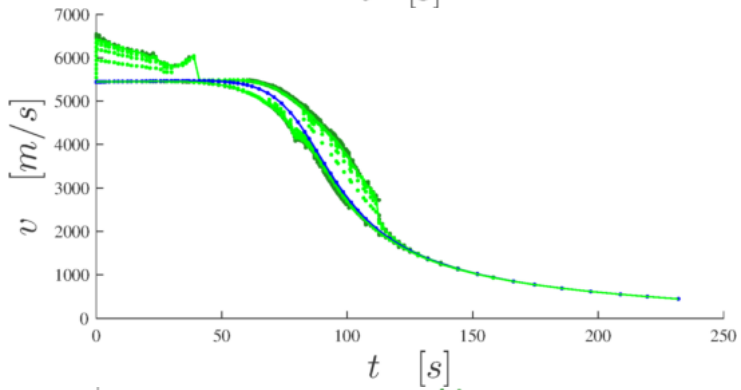
Convergence Region : Single State Error (vs. Norm. Energy)

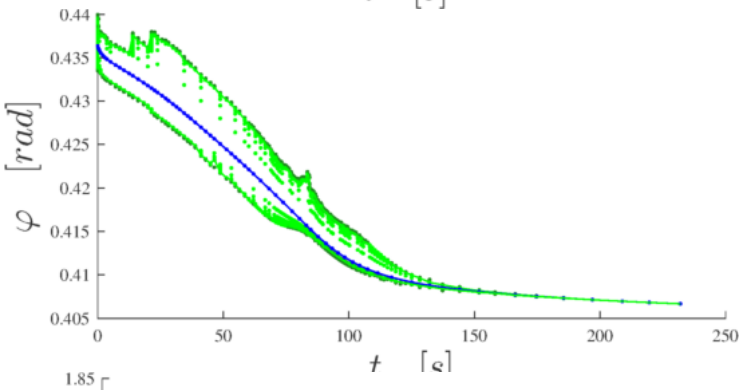


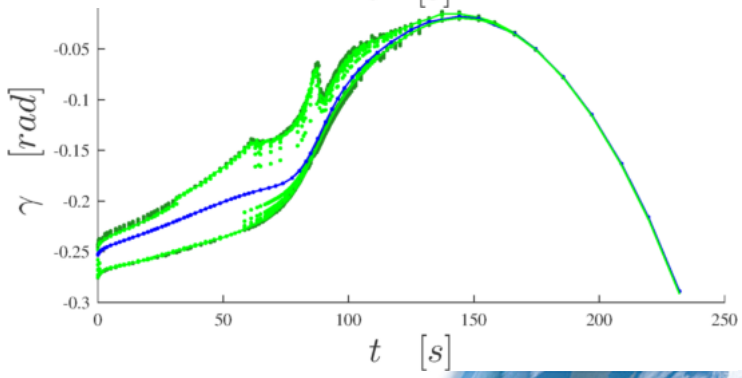
Convergence Region : Single State Error (vs. Time)

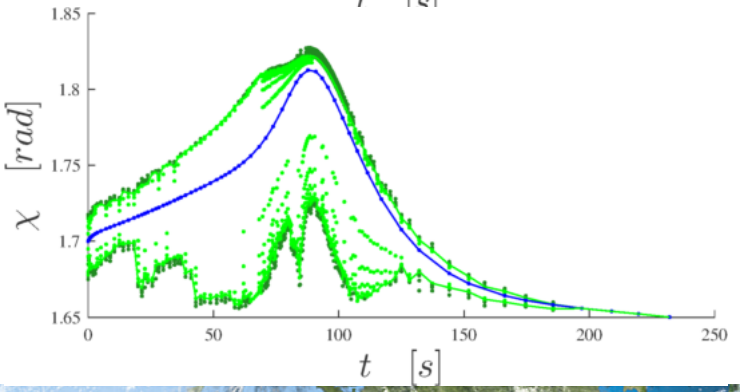












Convergence Region : Some Conclusions

- The convergence region $U(p_0)$ is large enough to cover expected errors at the entry interface point

- The choice of the reference point $x^*(\bar{t}, p_0)$ is important! Metric $|.|$ desirable such that:
  - $\bar{p}_I = min_{\bar{t}\in[t_0,t_f]} \; |\bar{x} - x^*(\bar{t}, p_0)| \;\; \Rightarrow \;\; \bar{p}_I \in U$

- U shrinks after peak deceleration such that successful trajectory computation cannot be guaranteed

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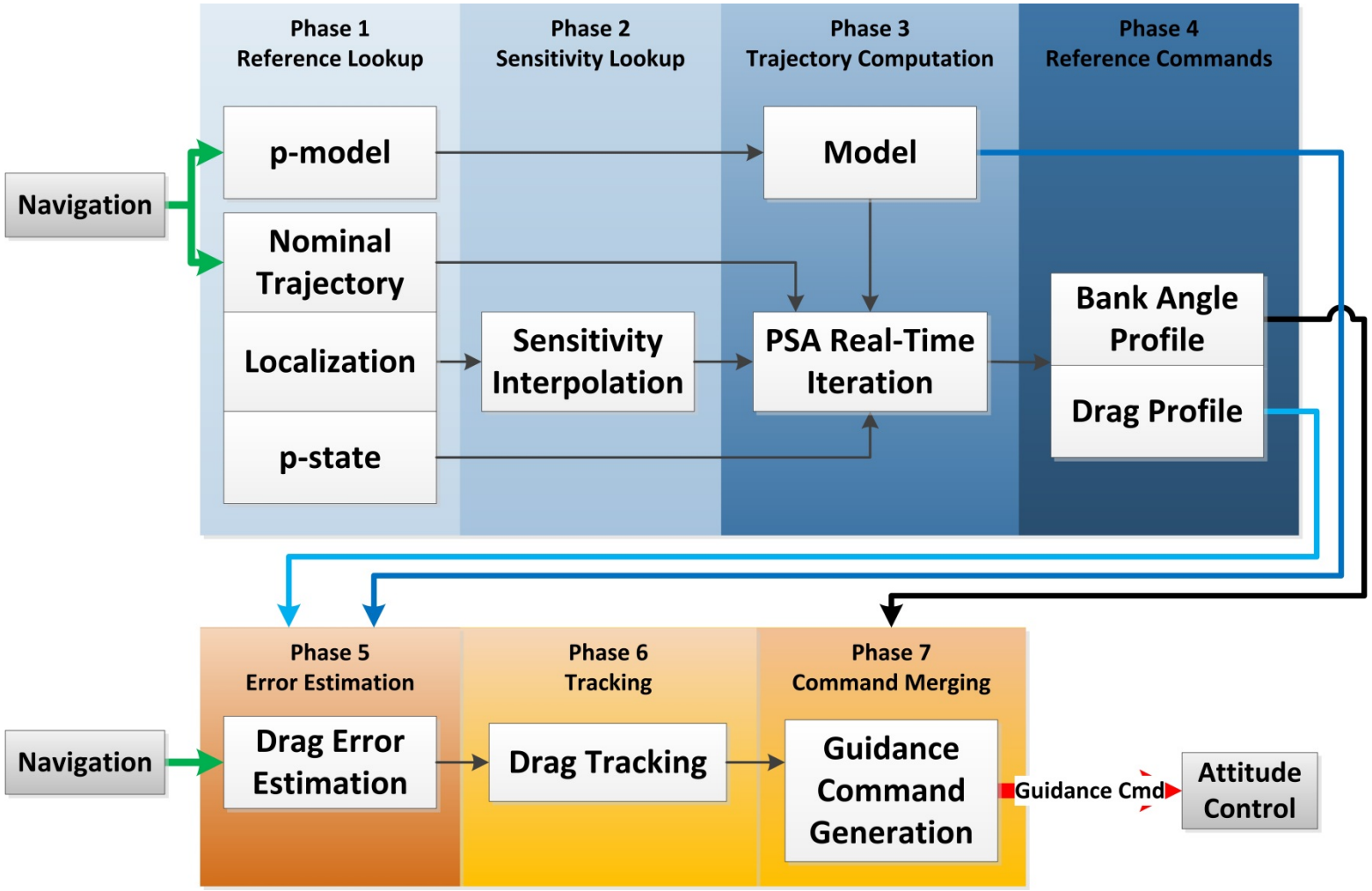
Additional guidance strategy required for “low” velocity flight

Trajectory tracking

- Feedback linearization
- Drag-energy dynamics

Guidance Stages



Feedback Linearization 1

Feedback linearization can be applied to nonlinear systems of the form

$\dot{x} = f(x) + g(x)u$ with state vector $x \in \mathbb{R}^n$, control $u \in \mathbb{R}^p$, output $y \in \mathbb{R}^m$

$y = h(x)$

Goal: Input-Output linearization

Method: Transform $\dot{x}$ into a new system whose states are the output $y \in \mathbb{R}^m$ and its first (*n*-1) 'time' derivatives.

$$
\begin{aligned}
y &= h(x) \\
\dot{y} &= L_f h(x) \\
\ddot{y} &= L_f^2 h(x) \\
&\vdots \\
y^{(n-1)} &= L_f^{n-1} h(x) \\
y^{(n)} &= L_f^n h(x) + L_g L_f^{n-1} h(x) u
\end{aligned}
$$

Taking Lie derivatives of the output

$$L_f h(x) = \frac{\mathrm{d}\, h(x)}{\mathrm{d}\, x} f(x),$$

$$L_g h(x) = \frac{\mathrm{d}\, h(x)}{\mathrm{d}\, x} g(x).$$

Feedback Linearization 2

Derivatives give the transformation T and the transformed system z

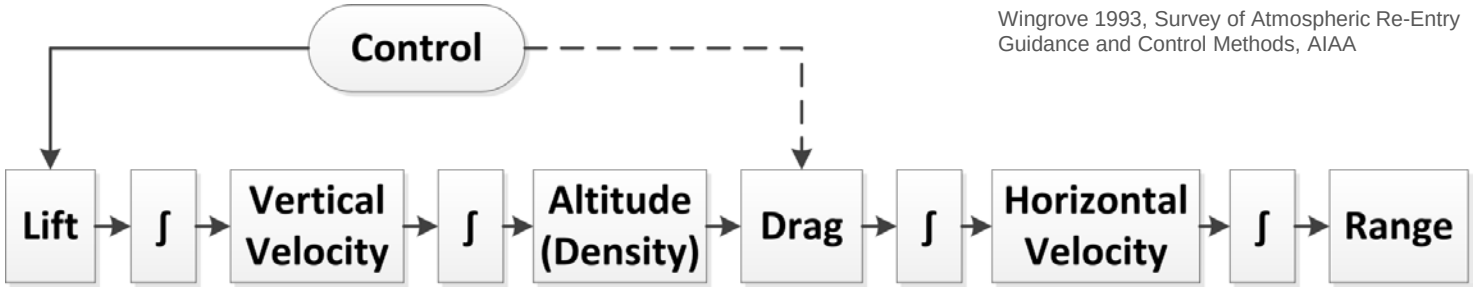
$$z = T(x) = \begin{bmatrix} z_1(x) \\ z_2(x) \\ \vdots \\ z_n(x) \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(n-1)} \end{bmatrix} = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{bmatrix}$$

$$\begin{aligned} \dot{z}_1 &= L_f h(x) = z_2(x) \\ \dot{z}_2 &= L_f^2 h(x) = z_3(x) \\ &\vdots \\ \dot{z}_n &= L_f^n h(x) + L_g L_f^{n-1} h(x) u \end{aligned} .$$

The feedback law $$u = \frac{1}{L_g L_f^{n-1} h(x)} (-L_f^n h(x) + v)$$

- Creates a linear input-output map from v to $z_1 = y$
- System z is a cascade of n integrators
- Outer-loop control v can be chosen using linear system methods

Downrange Control

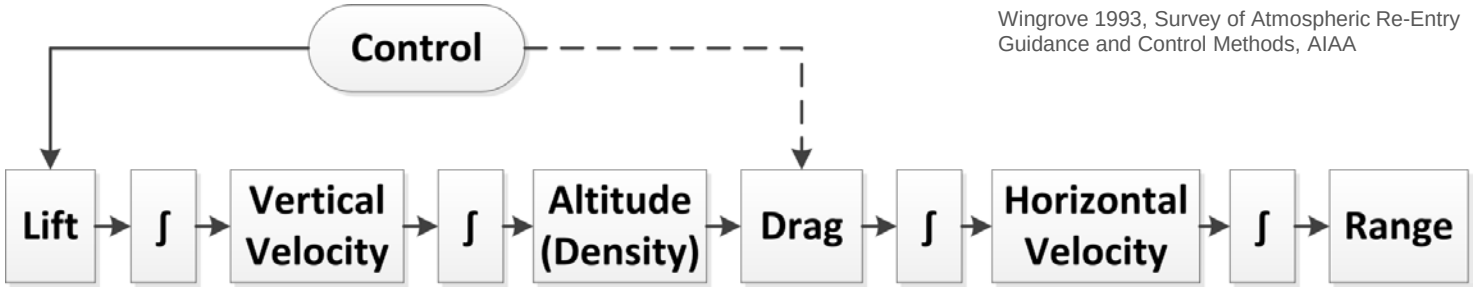


Wingrove 1993, Survey of Atmospheric Re-Entry Guidance and Control Methods, AIAA

Drag is the deciding factor!

- Entry capsule cannot control drag directly
- Control lift via bank angle rotation

Downrange Control



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$$D[\alpha, r, V_{rel}] = \frac{1}{2} \cdot \rho[r] \cdot V_{rel}^2 \cdot C_D\,[\alpha, M[V, c[r]]] \cdot S$$

Chose drag as system output.

ρ atmospheric density
C_D drag coefficient
α angle of attack
M Mach number
c speed of sound
S aerodynamic reference area

Application of FBL to Entry Guidance

- Controller design in energy domain
- Separation of dynamics: Consider only longitudinal motion in v-r plane!

$$h' = -\frac{V \sin\gamma}{D}$$

$$v' = \frac{D + g\sin\gamma}{Dv}$$

$$\gamma' = -\frac{L}{Dv^2}\cos\boldsymbol{\mu} + \left(\frac{g}{Dv^2} - \frac{1}{h + r_p D}\right)\cos\gamma$$

Assumptions:
- Spherical, nonrotating planet
- No side slip
- Constant angle of attack

Independent variable:

$$E = \frac{1}{2}v^2 - \left(\frac{GM}{r} - \frac{GM}{r_P}\right)$$

Energy as independent variable causes a system order reduction compared to the time domain: A minimal energy domain representation needs only retain either h or v!

Drag-Energy Derivatives

- Take output derivatives and apply feedback linearization mechanism

$$D(r, V_{rel}) = \frac{1}{2} \cdot \rho(r) \cdot V_{rel}^2 \cdot C_D\,(M(V, c(r))) \cdot S$$

$$D' = \frac{1}{2} S v \left(v\, \rho(h)\, \mathrm{Cd}'(M) \frac{dM}{d\mathrm{En}} + \mathrm{Cd}(M) \left(v \frac{dh}{d\mathrm{En}} \rho'(h) + 2\, \rho(h) \frac{dv}{d\mathrm{En}} \right) \right)$$

$$D'' = \frac{1}{2} S \left(v \left(v\, \rho(h)\, \mathrm{Cd}''(M) \left(\frac{dM}{d\mathrm{En}} \right)^2 + \mathrm{Cd}'(M) \left(v\, \rho(h) \frac{d^2 M}{d\mathrm{En}^2} + 2 \frac{dM}{d\mathrm{En}} \left(v \frac{dh}{d\mathrm{En}} \rho'(h) + 2\, \rho(h) \frac{dv}{d\mathrm{En}} \right) \right) \right) + \right.$$
$$\left. \mathrm{Cd}(M) \left(v \left(v \frac{d^2 h}{d\mathrm{En}^2} \rho'(h) + 2\, \rho(h) \frac{d^2 v}{d\mathrm{En}^2} + v \left(\frac{dh}{d\mathrm{En}} \right)^2 \rho''(h) \right) + 4\, v \frac{dh}{d\mathrm{En}} \frac{dv}{d\mathrm{En}} \rho'(h) + 2\, \rho(h) \left(\frac{dv}{d\mathrm{En}} \right)^2 \right) \right)$$

$$D'' = a + bu$$

2nd Drag-Energy Derivative

$$D'' = \boldsymbol{a}(D, \dot{D}, E) + \boldsymbol{b}(D, \dot{D}, E)\boldsymbol{u}(D, \dot{D}, E)$$

$$\begin{aligned}
\boldsymbol{a} = \; & \frac{1}{4\,D^2\,v\,c(h)^4\,(h+\mathrm{rM})}\,S\left(2\,D^2\,v^5\,\rho(h)\,(h+\mathrm{rM})\,c'(h)^2\left(\frac{dh}{d\mathrm{En}}\right)^2\mathrm{Cd}''\left(\frac{v}{c(h)}\right)+4\,D^2\,v^4\,c(h)\,\rho(h)\,(h+\mathrm{rM})\,c'(h)\,\frac{dh}{d\mathrm{En}}\left(c'(h)\,\frac{dh}{d\mathrm{En}}\,\mathrm{Cd}'\left(\frac{v}{c(h)}\right)-\frac{dv}{d\mathrm{En}}\,\mathrm{Cd}''\left(\frac{v}{c(h)}\right)\right)-\right.\\
& 2\,v^3\,c(h)^2\left(\mathrm{Cd}'\left(\frac{v}{c(h)}\right)\left(D^2\,v\,\rho(h)\,(h+\mathrm{rM})\,c''(h)\left(\frac{dh}{d\mathrm{En}}\right)^2+c'(h)\left(\rho(h)\left((h+\mathrm{rM})\left(6\,D^2\,\frac{dh}{d\mathrm{En}}\,\frac{dv}{d\mathrm{En}}+v\sin(y)\,\frac{dD}{d\mathrm{En}}\right)+D\cos^2(y)\left(g\,(h+\mathrm{rM})-v^2\right)\right)+2\,D^2\,v\,(h+\mathrm{rM})\left(\frac{dh}{d\mathrm{En}}\right)^2\rho'(h)\right)\right)-\\
& D^2\,\rho(h)\,(h+\mathrm{rM})\left(\frac{dv}{d\mathrm{En}}\right)^2\mathrm{Cd}''\left(\frac{v}{c(h)}\right)\Bigg)+\\
& v\,c(h)^3\,\mathrm{Cd}'\left(\frac{v}{c(h)}\right)\left(\rho(h)\left(2\,(h+\mathrm{rM})\left(D^2\,\frac{dv}{d\mathrm{En}}\left(4\,v\,\frac{dv}{d\mathrm{En}}-1\right)-g\sin(y)\left(v\,\frac{dD}{d\mathrm{En}}+D\,\frac{dv}{d\mathrm{En}}\right)\right)-2\,D\,g\cos^2(y)\left(g\,(h+\mathrm{rM})-v^2\right)\right)+4\,D^2\,v^2\,(h+\mathrm{rM})\,\frac{dh}{d\mathrm{En}}\,\frac{dv}{d\mathrm{En}}\,\rho'(h)\right)+\\
& c(h)^4\,\mathrm{Cd}\left(\frac{v}{c(h)}\right)\left(v^2\left(2\,\rho'(h)\left((h+\mathrm{rM})\left(4\,D^2\,\frac{dh}{d\mathrm{En}}\,\frac{dv}{d\mathrm{En}}+v\sin(y)\,\frac{dD}{d\mathrm{En}}\right)+D\cos^2(y)\left(g\,(h+\mathrm{rM})-v^2\right)\right)+2\,D^2\,v\,(h+\mathrm{rM})\left(\frac{dh}{d\mathrm{En}}\right)^2\rho''(h)\right)+\\
& \left.4\,\rho(h)\left((h+\mathrm{rM})\left(D^2\,\frac{dv}{d\mathrm{En}}\left(v\,\frac{dv}{d\mathrm{En}}-1\right)-g\sin(y)\left(v\,\frac{dD}{d\mathrm{En}}+D\,\frac{dv}{d\mathrm{En}}\right)\right)+D\,g\cos^2(y)\left(v^2-g\,(h+\mathrm{rM})\right)\right)\right)\Bigg)
\end{aligned}$$

$$\boldsymbol{b} = \frac{S\,\mathrm{uT}\cos(y)\left(v^3\,\rho(h)\,c'(h)\,\mathrm{Cd}'\left(\frac{v}{c(h)}\right)+g\,v\,c(h)\,\rho(h)\,\mathrm{Cd}'\left(\frac{v}{c(h)}\right)+c(h)^2\,\mathrm{Cd}\left(\frac{v}{c(h)}\right)\left(2\,g\,\rho(h)-v^2\,\rho'(h)\right)\right)}{2\,v\,c(h)^2}$$

$\boldsymbol{u} = \dfrac{\boldsymbol{L}}{\boldsymbol{D}}\cos\boldsymbol{\beta}$ State dependent control transformation!

Drag Tracking Law

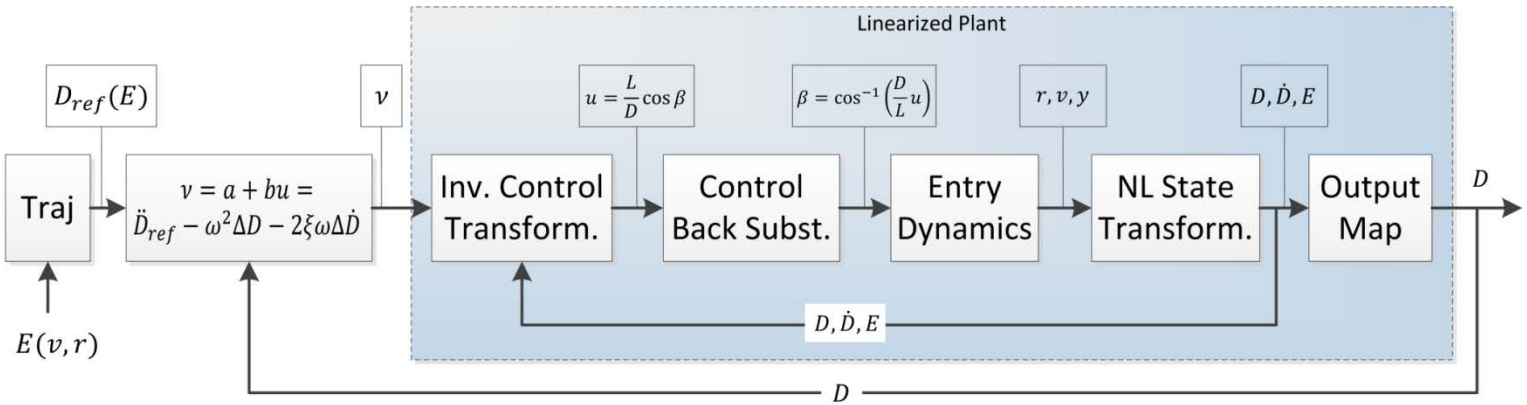
Bijective transformation $T_E: [r, v, y] \rightarrow [D, \dot{D}, E]$ *(Drag – Energy Space)*

System order: 2
Output relative degree: 2 ➔ No internal dynamics!

Linearized plant should have error dynamics of 2nd order linear system.

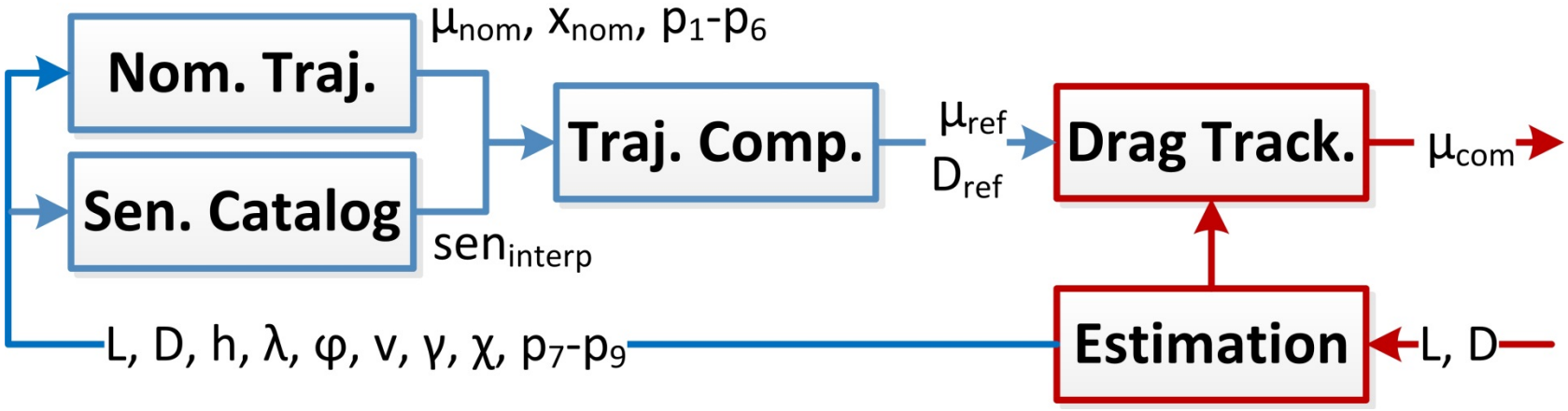
Obtain control law:

$$\mathbf{u} = \frac{1}{b}(-a + \ddot{D}_{ref} - \omega^2 \Delta D - 2\xi\omega\Delta\dot{D})$$



Two-Degree of Freedom Guidance System

Guidance System Overview



- Two-degree-of-freedom design
  - Fast inner tracking loop (20 Hz)
  - Slow outer trajectory loop (0.05 Hz)

- Trajectory computation outputs
  - near optimal discrete u^*, x^* for the entire remaining process
  - optimal bank angle profile μ_{ref} and drag profile D_{ref} obtained from u^*, x^*

- Drag tracking controller based on Mease et. al.

PSA + Drag Tracking

Drag Tracking

✓ Strength

- Fast feedback based directly on physical measurement
- Robust against atmospheric disturbances
- Performant during high-drag flight

o Weakness

- No planning / prediction
- Based on separated and simplified dynamics
- Over-sensitive during low-drag flight

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- Not applicable after peak deceleration

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- Large initial correction space
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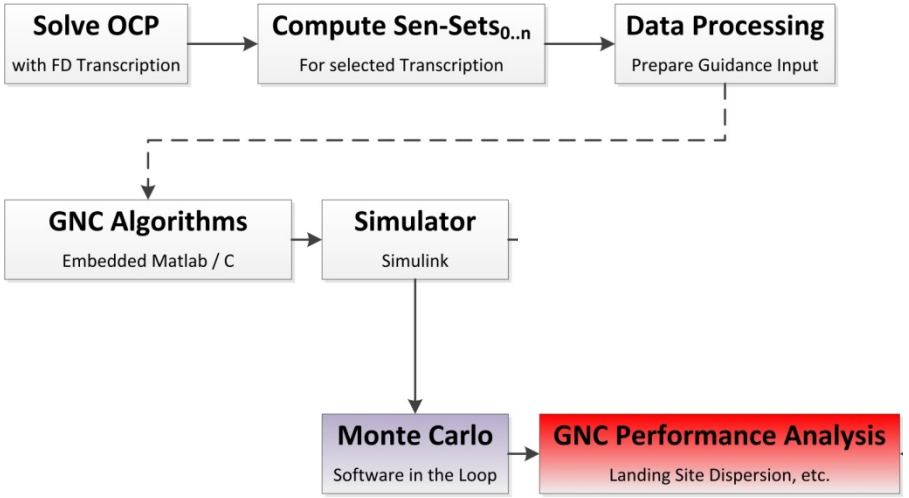
PSA update and drag tracking compliment each other well!

Monte Carlo Campaign

Monte Carlo Campaign

- Large EIP state errors (uniform error distribution)

State	Pert.
h_0	+- 3 km
λ_0	+- 0.3265°
φ_0	+- 0.1632°
v_0	+- 200 m/s
γ_0	+- 1°
χ_0	+- 1°

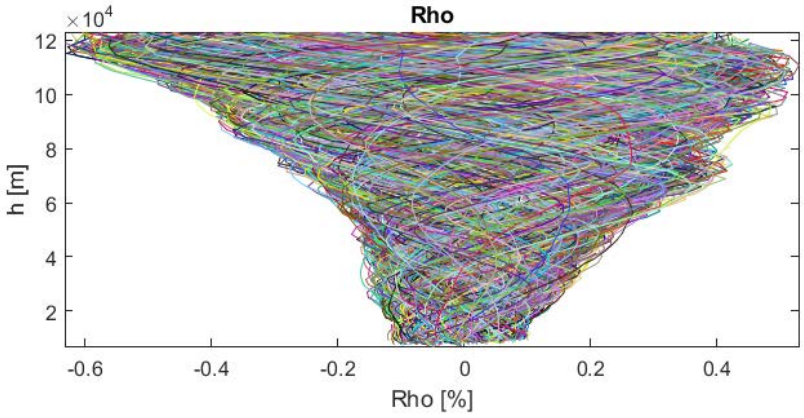


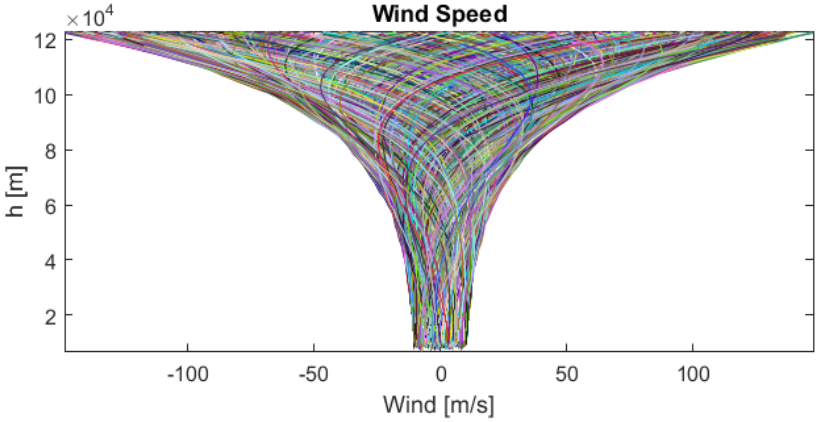
- Guidance input
  - true state, lift and drag falsified with white noise
  - Perturbation parameters are estimated using an extended Kalman filter

Monte Carlo Campaign: Perturbed Environment

- Atmosphere perturbations
  - Random temperature profile between warm and cold conditions
  - Random sinusoidal density perturbations of up to 50% amplitude
- Aerodynamic coefficients perturbed by up to 10%

Param	Pert.
ρ	Temp. +- 50%
c_L	+- 10 %
c_D	+- 10 %
wind	+- 200 m/s
mass	+- 20 kg





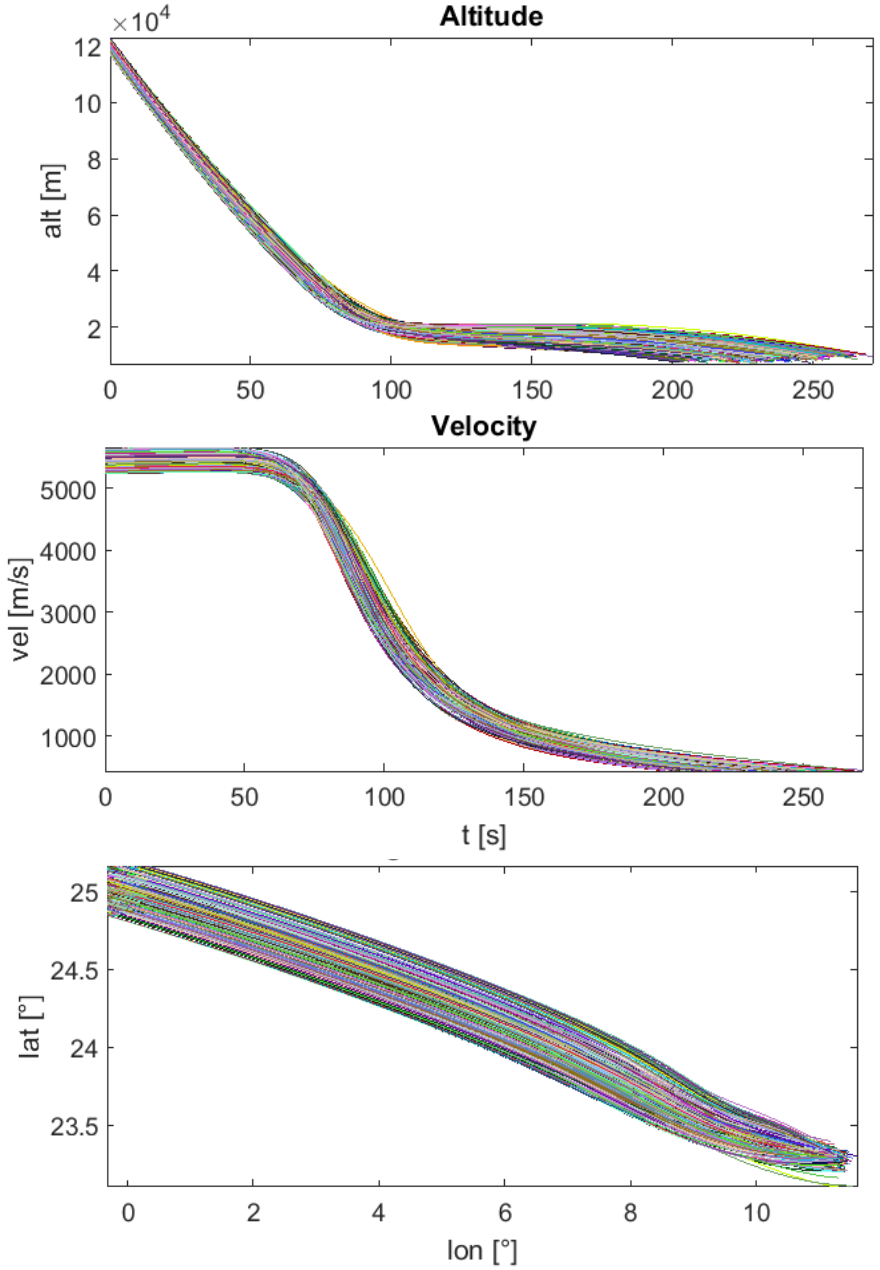
Monte Carlo Campaign: Results

($|\mu| + 3\sigma$) hori. dist.: **12.3 km**

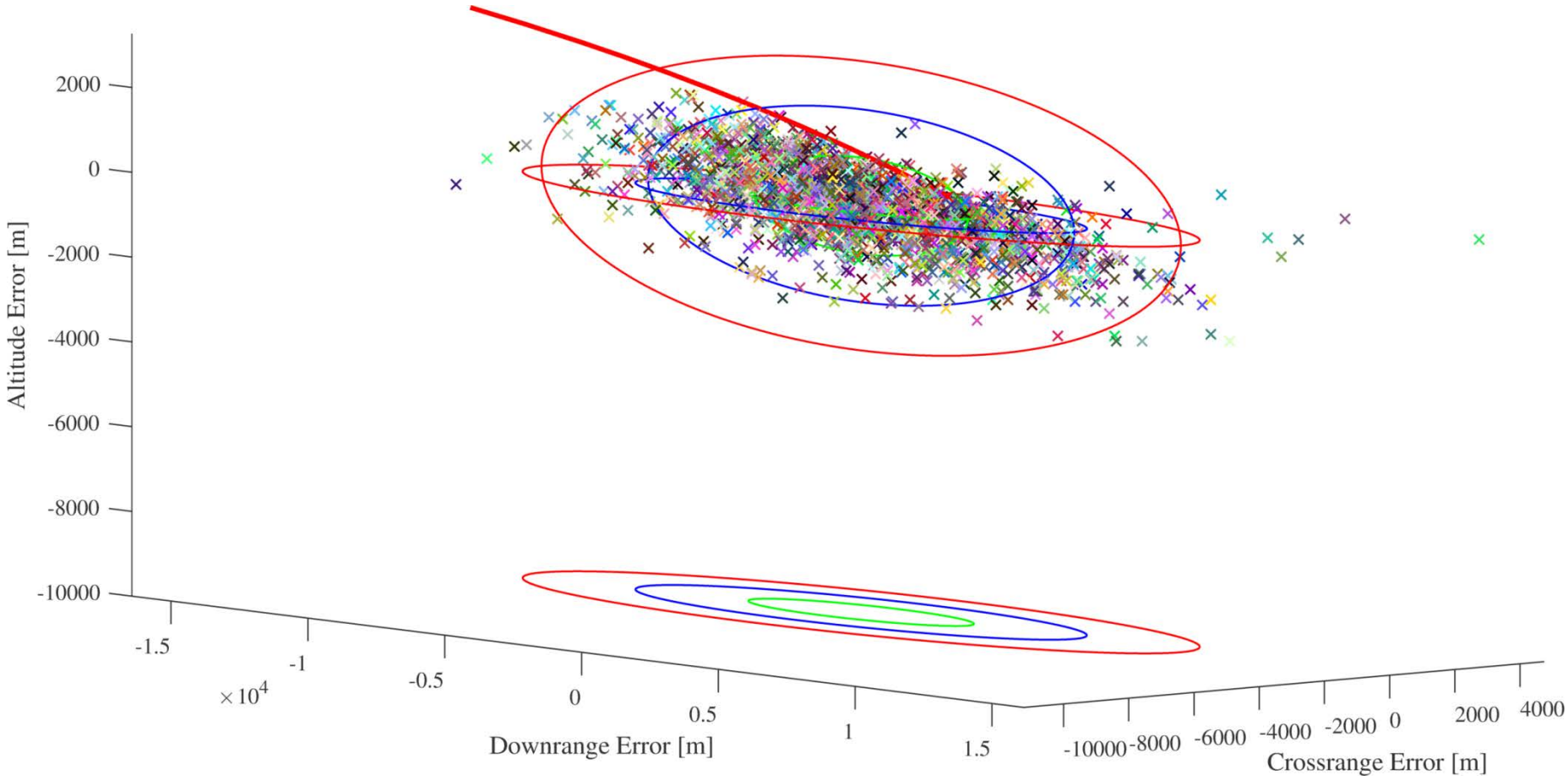
($|\mu| + 3\sigma$) alt. error: **2.4 km**

(3.5 DoF, 2500 MC cases)

Result	Mean (μ)	Std. Dev. (σ)
Eucl. dist.	4.1 km	2.6 km
Hori. dist.	4 km	2.7 km
DR error	- 2.6 km	3.8 km
CR error	- 0.4 km	1.1 km
Alt. error	- 0.4 km	0.7 km
Vel. error	4 m/s	6 m/s



Parachute Opening Zone



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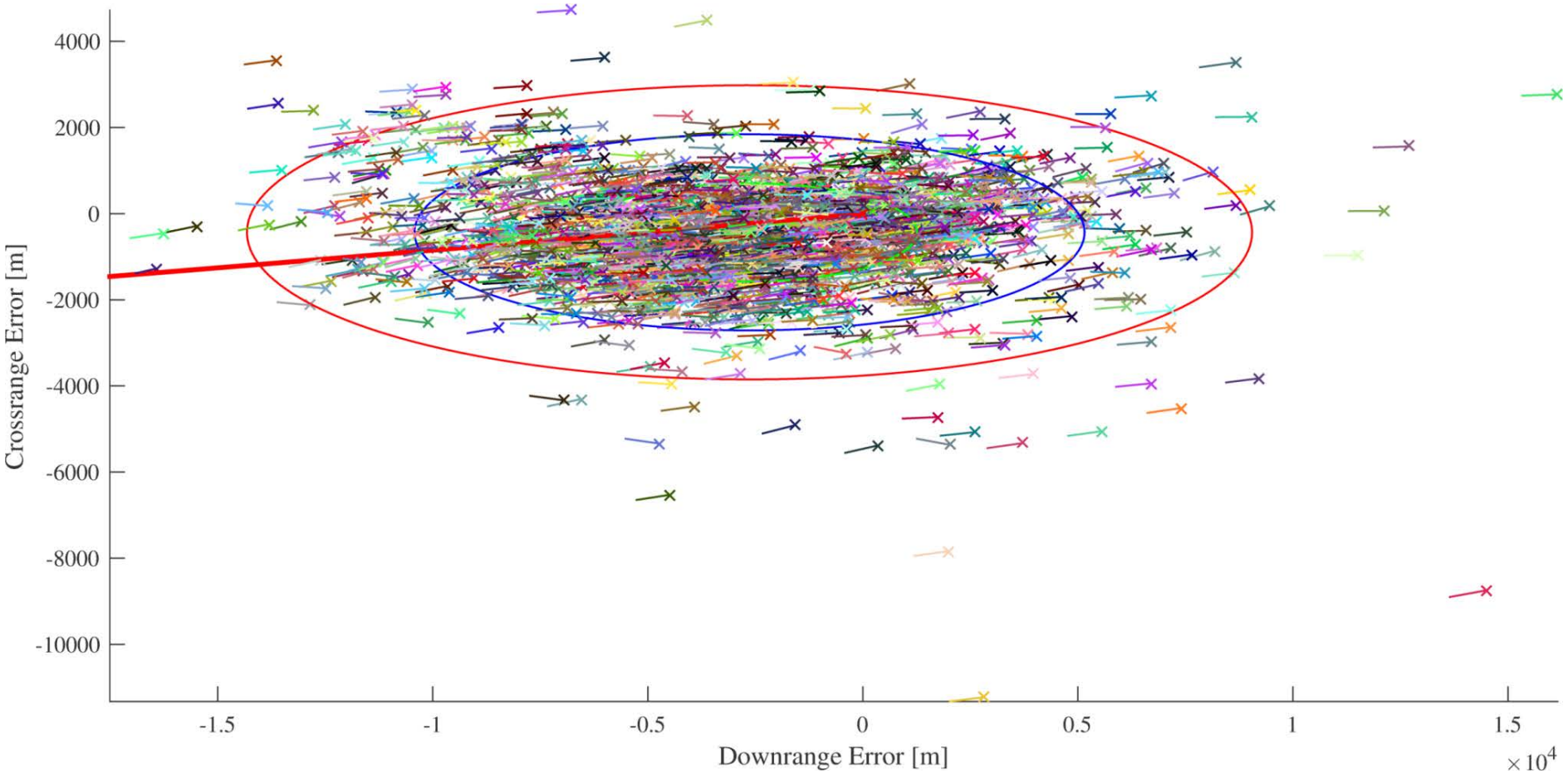
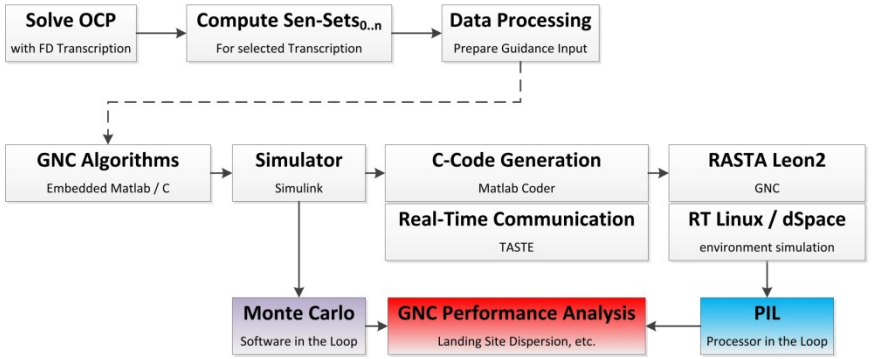


LEON2 Processor-in-the-Loop

Processor-in-the-loop

- Test on RASTA-101 with 80 MHz LEON2 processor
- GNC c-code from autocoding from Embedded Matlab
- TASTE toolset: Onboard SW interface definition, communication setup and target compilation
- Problem sizing
  - dense NLP formulation
  - grid length 70
  - ~25 MB sensitivity data



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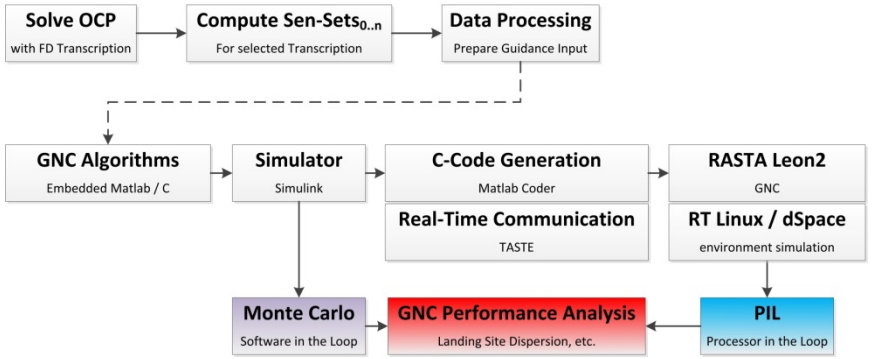
Trajectory computation time: **< 1 sec.**



Developed Software Tools

Sensitivity Analysis Framework (SAF)

Software tools to enable or support:

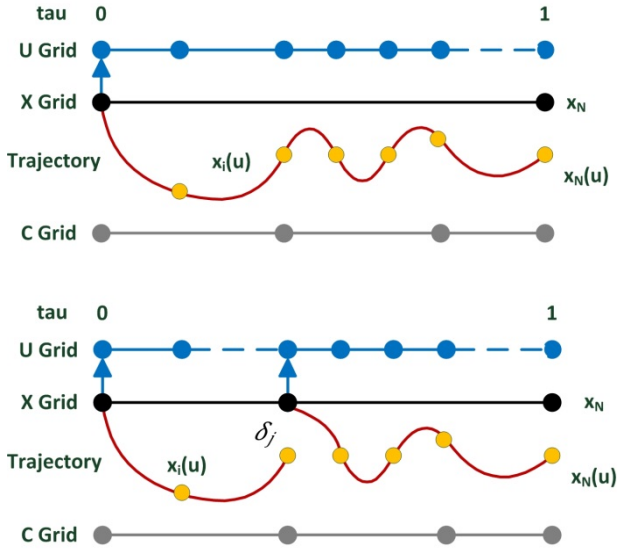
- Transcription of the OCP(p) into a NLP(p)
- Solution of multiphase OCP(p) (WORHP, IPOPT)
- High precision derivative computation (ADOL-C)
- Analysis, visualization and processing of the optimal solution and the sensitivity differentials
- Analysis and test of the real-time iteration scheme

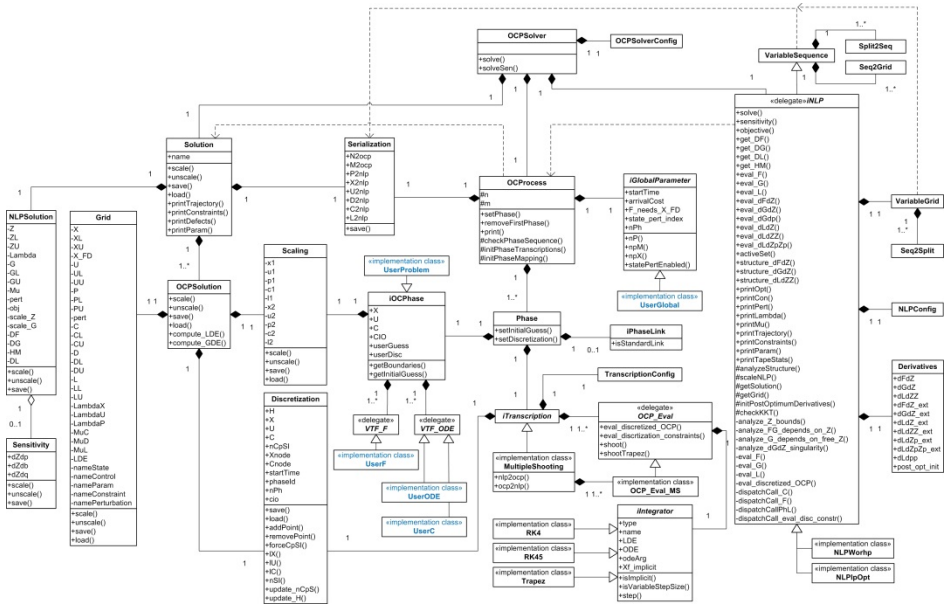
SAF components:

- OCP transcription layer (C++)
- Analysis toolbox (Matlab)
- Mars entry simulator (Embedded Matlab/Simulink)
- Mars guided entry GNC (Embedded Matlab, autocoding capable)

SAF Transcription Layer

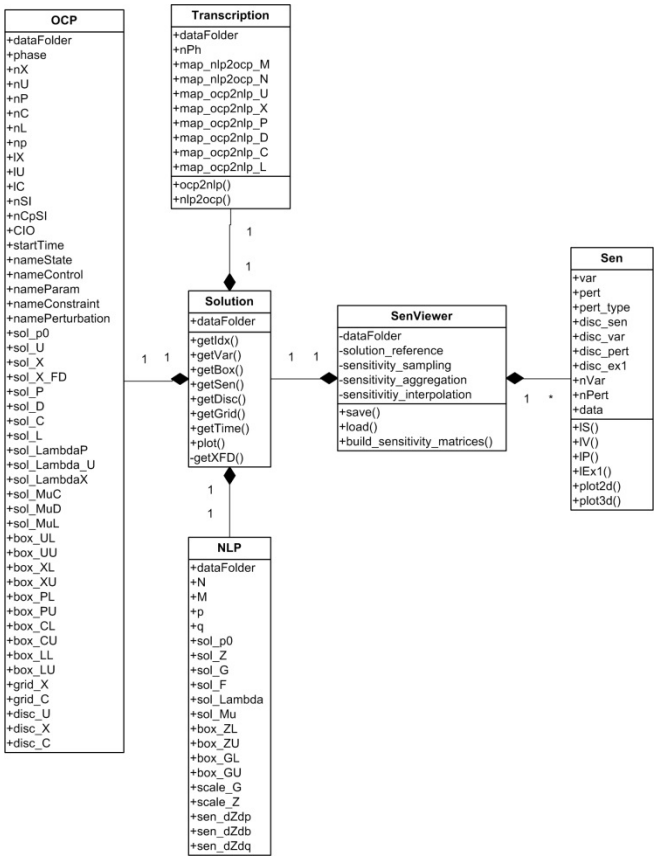
- Supports formulation of multi-phase parametric optimal control problems
- Uses automatic derivative computation with ADOL-C
- Enables parametric sensitivity computation using WORHP
- Generic multiple shooting transcription
- Automatic grid adaption
- Automatic problem scaling
- Generic interface is adaptable to most NLP solvers (interfaces existing for WORHP and IPOPT)

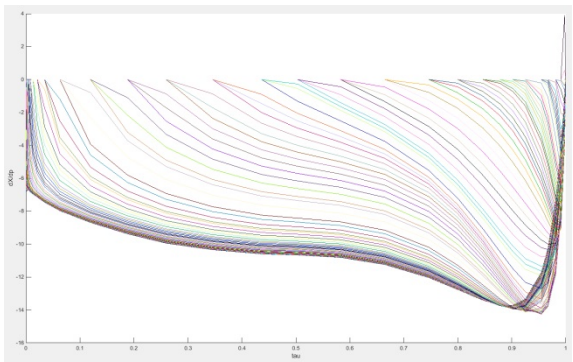


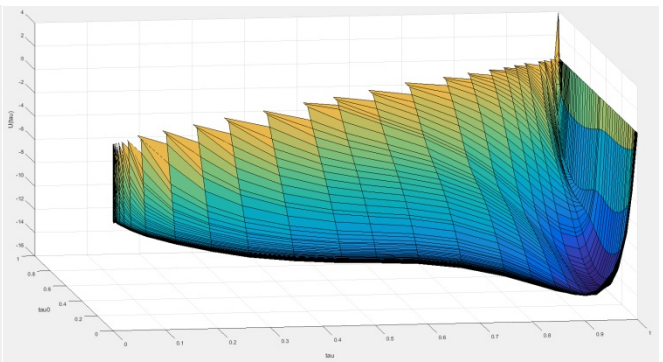


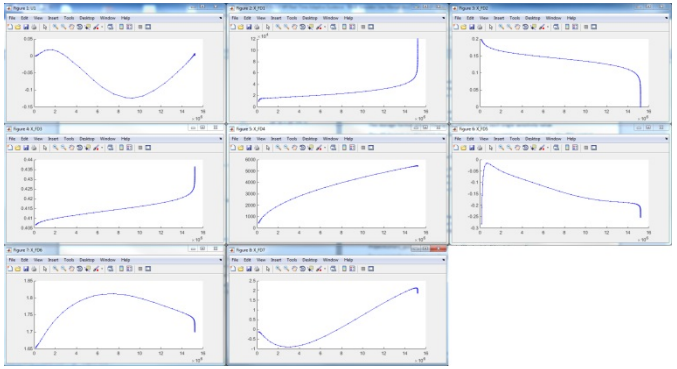
Sensitivity Analysis Toolbox

- Modular, object oriented design
- No commercial software quality, but
- Fairly tested and documented R&D tools for expert users
- User manual and architectural design guide provided







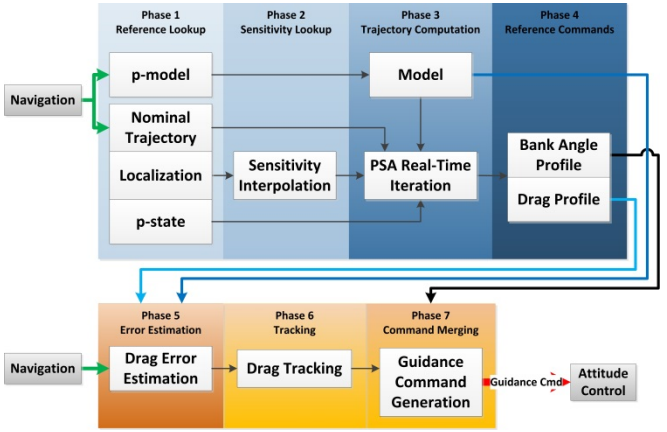


Summary and Outlook

Summary

- Sensitivity analysis of discretized optimal control process
- Parametrization of dynamic model and initial conditions
- Repeated online trajectory computation
  - Parametric sensitivities + interpolation
  - Taylor expansion and iterative constraint correction
- Two degree of freedom guidance system
  - PSA real-time iteration
  - Drag tracking
- Promising results in 3.5 DoF Monte Carlo campaign
- Real-time capability proven by PIL test on LEON2 processor

Tackling the Difficult Problems



Non-convexity and path constraints

- ✓ Offline analysis and trajectory design
- ✓ Reference trajectories and sensitivity catalog
- ✓ Path constr. can be represented in drag domain

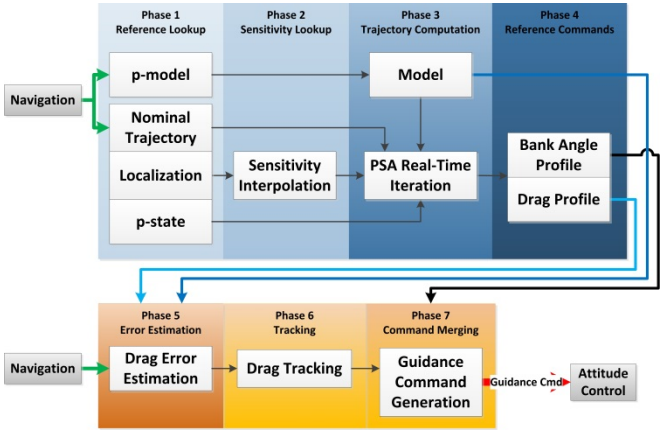
Computational load

- ✓ Cascaded loop structure schedules and staggers expensive tasks
- ✓ PSA: fast online adaption relies on precomputed optimal solution derivative
- ✓ Drag feedback law

Model dependency

- ✓ Disturbance estimation and online model adaption
- ✓ Drag guidance law based directly on physical measurement

Tackling the Difficult Problems



Non-convexity and path constraints

- ✓ Offline analysis and trajectory design
- ✓ Reference trajectories and sensitivity catalog
- ✓ Path constr. can be represented in drag domain

Computational load

- ✓ Cascaded loop structure schedules and staggers expensive tasks
- ✓ PSA: fast online adaption relies on precomputed optimal solution derivative
- ✓ Drag feedback law

Model dependency

- ✓ Disturbance estimation and online model adaption
- ✓ Drag guidance law based directly on physical measurement

What could be done next?

- ❑ Embedded NLP solver is critical enabling technology
- ❑ NLP solver as FPGA?
- ❑ Hybrid: Online optimization and sensitivity computation + PSA update

Thank you for your attention!

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