
A logit model for shipment size choice with latent classes – empirical findings for Germany and discussions

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Abstract

Modelling the choice of shipment size is an essential and common aspect in developing freight transport models which also consider logistic choices. The implementation of the shipment size choice constitutes a high-level modelling approach of logistical decisions made by freight traffic demanders. The consideration of logistic choices in a freight transport context is accompanied with a huge diversity of involved actors and commodity characteristics which are influencing the choice behavior.

In this study we model the behavior of shipment size choice as a discrete outcome of the total logistic costs. Further we used a Latent Class Analysis approach that reduces the dimensions of influencing factors coming up in context with the shipment size choice. Dimension reduction in the sense of groups with similar behavior improves the application in freight transport models as well as analyzing and predicting policy impacts. The study also reveals possible starting points for further research dealing with homogenous groups of logistical behavior.

Keywords: freight transport; shipment size; Economic-Order-Quantity; discrete choice; latent class.

1. Motivation

Besides vehicles, shipments are the most visible objects in freight transport. Moreover, all involved actors in a certain transport process have to deal with the concerned shipment. Thus it is an interesting object for the examination of the behavioral foundations of freight transport. Although the single shipment is the unit that has to be dealt with in operative freight transport, it cannot be seen as being independent from the context in which it was prepared and will be routed on the way from the shipper to the recipient. This context comprises the relationship between shipper and recipient, the properties of the shipment itself and the supply of transport space that is available. Changes in the interaction patterns between shippers, recipients and carriers are likely to have consequences on freight transport operations and hence on the traffic of heavy goods vehicles. In the opposite direction, infrastructural and regulatory policy measures are taken with the intention to foster or alter the behavior that causes demand for freight transport. Thus the consideration of logistics is crucial in the development of behavior sensitive freight transport models. As every company organizes her logistic operations in a different way, it is not possible to reflect the behavior of each of these companies in a freight transport model. This holds especially in the case of large scale models that try to explain freight transport demand on an interregional or national level. Nevertheless, behavioral drivers of freight transport demand have to be accounted for in predictive models. Thus, simplifications have to be found that relieve the modeler from considering the details of trade and transport relationships. At the same time, these workarounds should not narrow the explanatory power of the model too much. If more complicated logistics aspects, like planning of networks and supply chain management strategies are set aside, the formation of shipment sizes remains as a proxy decision for the logistic interaction and optimization of all involved actors.

Shipment formation can thus be seen as one of the simplest links between commodity flows formed from the economic interactions on the transport demand side and the vehicle movements that take place on the various transport ways.

In developing a shipment size model, two questions arise:

- 1.) Which influences have to be considered as crucial and how can they be incorporated in an operational transport model?
- 2.) How can the heterogeneity of actors and decision situations be addressed in a manageable yet realistic way?

The purpose of this paper is to suggest answers to these two questions. Econometric models are the means of choice to deal with the variety of actors' choices under seemingly identical conditions. We propose a discrete shipment size choice model, as it has turned out that certain shipment size categories can be delimited due to given vehicle and bundle sizes. A case study is undertaken in which the choice of shipment size is modeled for shipments that were transported by truck in interregional trips with an emphasis on domestic trips in Germany. Discrete choice models are usually estimated from samples that cover a wide variety of flows and thus relationships. The consequence is that the obtained model instances suffer from aggregation bias when it comes to model the behavioral change of single actors on outside effects. A way to alleviate this problem is to delimit segments of freight transport demand that exhibit similar behavior. In our case study, decision makers are segmented according to logistics aspects by means of a latent class logit model.

The rest of the paper is organized as follows: Section 2 contains a brief review of the literature dealing with shipment size choice models and the formation of homogenous groups. In section 3 the conceptual framework which consists of the theoretical background of shipment size choice, the development of a discretized total logistic costs model and the essentials of determining latent classes. The source data and descriptive analysis are central in section 4 whereas section 5 presents the estimation results. Finally a summary and an outlook on further purposes of investigation will be given in section 6.

2. Literature review

The formation of a shipment ready for transport is usually influenced from two directions. First, logistics requirements of the shipper and recipient determine the size of a shipment. Second, transport services that are available determine costs and transport time. In cases where the underlying order is not a one-off business, a shipment will be ordered in the frame of a long-established business relationship. From a transport analyst's point of view, these relationships are only of interest as far as they influence transport services and vice versa. The first model considering the flow of goods which results from such a relationship in a certain period of time was presented by Baumol and Vinod (1970). They emphasize the tradeoff between fixed transport costs and the costs entailed by storing large quantities of the concerned good. Hall (1985) adds the aspect of transport mode choice when taking the lower envelope of the cost function of various transport modes as a decision criterion. By doing so, the inherently discrete choice of a transport mode is combined with the choice of a, continuous, shipment size. In the sequel, most transport models did not consider shipment size choice as isolated anymore but rather combined with the choice of mode, carrier or transport chain. However, Holguin-Veras (2002) points out that the choices on shipment size and transport mode are often not drawn by the same decision maker, although they are in many cases seen as belonging together. Transport models do not address the decisions of single shippers but rather consider the whole

population of them and their relevant behavior. As companies are different and can't be measured by a single quantitative model, regression has been prevailing in shipment size modelling. Within the group of econometric models, there are basically three different ways in which the problem has been addressed:

- 1.) Modeling of shipment sizes independently of transport mode choice (e.g. Combes (2009), Wisetjindawat et. al. (2005)).
- 2.) Modeling of continuous shipment sizes and discrete modes of transport (e.g. Holguin-Veras (2002), De Jong and Johnson (2009), Abate and de Jong (2014) and Abdelwahab and Sargiuos (1992))
- 3.) Modeling of discrete shipment sizes and discrete modes of transport (e.g. De Jong and Windisch (2009), De Jong and Ben Akiva (2007), Pourabdollahi et. al. (2013))

Combes (2009) tries to verify the economic order quantity equation on the basis of the French shipment survey ECHO¹. He adds further dummy variables that indicate the chosen mode of transport for the sampled shipment at hand. Given the data from the survey, it is shown that the EOQ model is a good approximation for the choice of a continuous lot size regardless of what mode of transport was actually chosen. Moreover, further variables are added to explain the lot size formation such as the transport distance and the question if the shipment was transported directly or in a vehicle tour. As the model was estimated on a comprehensive sample of transport cases of all kinds, the theoretical EOQ-model, can be seen as feasible to insert in a more comprehensive freight transport model. Such an insertion was done by Wistejindawat et. al. (2005). The EOQ-formula was applied to create shipments in a commodity based model for urban freight transport to feed these shipments into vehicle tours. In both cases, the parameters were estimated by OLS regression.

Although the model of Wisetjindawat et. al. (2005) also incorporates a feedback on shipment size decisions from the transport system, parameters on the single model stages were estimated separately from the remaining parts of the model. This is different from models listed under 2.) and 3.). In these cases, shipment size choice is intertwined with other choices on the level of parameter estimation. The influence that multilevel decisions have on the parameter estimates depends on the model structure. In the case of type 2.) two different model types have been applied in the past. The more common one is the combination of a discrete choice model for the mode choice and a regression of a continuous lot size (e.g. Holguin-Veras (2002), De Jong and Johnson (2009), Abate and de Jong (2014)). In Abdelwahab and Sargiuos (1992) shipment and mode choice is modeled by a simultaneous equations model. In the models belonging to type 2.), two general problems occur. First, as shipment size often enters the mode or vehicle choice submodel as an independent variable, a simultaneous equation problem exists. Correlation between the submodels thus has to be accounted for. Further, in a model with revealed preference data, only such data records will exist in which the shipment size is conditional on the chosen mode or vehicle. This leads to selectivity bias which has been accounted for in several ways. The way pursued by Holguin-Veras (2002) and De Jong and Johnson (2009) is a two stage procedure that starts with estimating the parameters for the continuous shipment size model. To avoid selectivity bias and feedback between the submodels, shipment size is estimated

¹ Enquête ECHO Envoi-Chargeurs-Opérateurs de transport (Guilbault and Soppé (2009))

independently from the chosen mode or vehicle type on regressors that do not occur in the discrete choice submodel. The shipment size obtained by the OLS regression is entered into the discrete choice model for vehicle type or mode choice. Abate and de Jong (2014) also pursue a two-step approach but start with transport mode choice according to a MNL model. The obtained choice probabilities serve as arguments for a correction function that is added to the shipment choice equation. A completely different way of discrete/continuous choice is undertaken by Abdelwahab and Sargiuos (1992) who estimate a switching sequential equations model with a stage least squares regression and in one step with a maximum likelihood estimator.

Models of type 3.) assume that shipment sizes can be classified into discrete categories. This goes along with the finding of Hall (1985) that certain shipment sizes are unfavorable given the vehicles or transport modes to choose from. In the models of category 3.), random utility models are combined either by nesting (e.g. Windisch et. al. (2010), De Jong and Ben Akiva (2007)) or by copula functions (e.g. Pourabdallah et. al. (2013)). The central problem of nested discrete choice models is the determination of the nesting structure between lot sizes and transport modes. According to Windisch et. al. (2010), shipment sizes are better placed in the lower level of a nested logit model, indicating that a switch between shipments of various sizes is more usual than between transport chains.

Pourabdallah et. al. (2013) refrain from nesting discrete choices, but rather use various copula functions to link multinomial logit choices of shipment size and mode choice. By the choice of an appropriate copula function, dependency structures between the two decision problems can be modeled in greater flexibility than with the hierarchical nested logit model.

Regardless of the context in which the formation of shipment sizes is modeled, often three aspects are determined as important: Shippers' preferences, transport costs and properties of the good to be transported. Especially the first two aspects result from logistic or technical restrictions to which the actors have adapted to. If the choice of a shipment size also serves as a low resolution model for addressing aspects of logistics, adaptations have to be made. Drivers for the lot size choice can be addressed in econometric models as the ones mentioned above basically in two ways. The first one is the consideration of corresponding variables and the second one is the model structure. The latter refers to nesting structures in models of type 3.) and to the considerations of variables on the various stages of the models.

In several cases, logistics components have been mirrored by proxy variables such as characteristics of shipper and/or recipient (Holguin-Veras (2002), Pourabdallah et. al. (2013), De Jong and Johnson (2009)), the way that a shipment was packaged or handled (De Jong and Abate (2014), Abdelwahab and Sargiuos (1992), Windisch et. al. (2010)). Transport logistics aspects were addressed by Combes (2009) who added information indicating if the shipment was part of a tour or delivered directly. Windisch et. al. (2010) added transport logistics via the structure of the choice models. Shipment sizes are nested given certain intermodal transport chains in which they are transported.

Often preferences and logistics requirements are distributed very heterogeneously within the population of all transport cases at hand. Choice models that incorporate these motivations have to account for this heterogeneity to reach an increased explanatory value. Segmentation of the demand population is a possibility to enhance the behavioral foundation of econometric models. This is due to the fact that most econometric models are not really microscopic due to the way they are estimated. Although they display the behavior of single actors and the data sets used for estimation are derived from decisions on single transport cases, the estimation sample covers a range of more or less heterogeneous choice situations in which actors found themselves. Segmentation of demand can be endogenous to a model to various extents. A way

to group decision makers into classes in discrete choice situations are latent class models. Latent class models have their origin in marketing. They were picked up in transport research mainly to model mode choice decisions. In passenger transport, Bhat (1997) endogenously delimited several traveler groups according to socioeconomic characteristics and estimated the model with an Expectation-Maximization-Algorithm. Demand segmentation for mode choice in freight transport is done by Gopinath (1995) who classified shippers according to attitudes towards various modes of transport which in turn were derived from logistics figures such as maximum acceptable delay. Arunotayanun (2009) segmented mode choice according to logistic properties of the relationship between shipper and recipient. To our knowledge, latent class models have not been applied to shipment size decisions up to now.

3. Modelling approach

The empirical validation of the EOQ-model for long-distance haulage conducted by Combes (2009) offers the opportunity to model the choice of shipment size in the context of freight transportation. As the freight transport markets are characterized by a huge variety of decision makers and transported commodities, the utilization of the EOQ-model enables the application of a decision rule which is suitable for the generality of logistical actors. Also the high explanatory power, the necessary high-level perspective and the relatively strong robustness of optimal shipment sizes substantiate the appropriateness of the EOQ-model (Combes (2009)).

A discrete shipment size choice model for road transports on the basis of minimal total logistic costs provides a promising basis to establish a consistent conjunction between logistics choices and the discrete mode choice. In general, the total amount of transported goods per period which represents the firm's commodity flows empirically contributes a big portion to the explanatory power of the shipment size choice (Abate and de Jong (2014), Combes (2009)). In other words the variance of the periodic demand mainly causes the heterogeneity regarding the shipment size choice. To account for the exceeding heterogeneity which isn't captured by the EOQ-model itself, the models in the literature use variables like commodity type and/or commodity characteristics (Abate and de Jong (2014), Pourabdollahi et al. (2013), Windisch et al. (2010), de Jong and Johnson (2009), de Jong and Ben-Akiva (2007), Holguín-Veras (2002)). Due to the amount of possible characteristics, the concomitant absence of simplicity and the tendency of an inflation of the estimated models in the statistical sense we use an approach that groups decision makers into clusters with similar behavior and therefore simplify the model.

3.1 The EOQ-model as core of the shipment size choice

According to the microeconomic theory a consumer with transitive, reflexive and complete preferences chooses a bundle of continuous and positive quantities of goods and services – while satisfying prevalent constraints - for which he receives the maximum utility. The utility itself is a dependency function of the goods and services which mathematically represents the preferences and is except an order preserving transformation unique. Transferred to the logistical context, the decision maker chooses the shipment size for which he receives the maximum utility (minimum costs). This is represented by the optimal distribution of the annual flow of goods on various shipments leading to an optimal shipment size as the result of the minimization of a firm's logistic costs based on the tradeoff between inventory costs and fixed transport costs. The total logistic costs per year for a decision maker n are given by:

$$C_n(q_n) = \frac{Q_n}{q_n} F_n + Q_n c_n(q_n) + \frac{q_n}{2} (w_n + r v_n), \quad (1)$$

whereby the individual parameters are described as follows:

Q_n : Constant and continuous flow of goods regarding individual n per period (ton/year).

q_n : Shipment size per transport of individual n to satisfy the total demand Q_n (ton/shipment).

F_n : Fixed transport costs per shipment for individual n independent of the shipment size q_n (cost/shipment).

$c_n(q_n)$: Variable transport costs for individual n dependent on the shipment size q_n (€/ton).

w_n : Warehousing costs per unit of commodity per year for individual n (€/ton).

r : Interest rate valuing the bounded capital in form of inventory holding costs

v_n : Value density of the transported commodities (€/ton)

The total logistic costs are not a typical utility function in the sense of the microeconomic consumption theory which increases with a higher amount of the consumed good and which is marked by the consideration of multiple goods or/and services. The continuous cost function in contrast is one-dimensional with respect to the decision space as well as regarding the image space and reaches its optimum for q_n in the interval $(0, Q_n]$ depending on the values of the parameters representing the costs for transport and inventory holding.

Assuming a linear nondecreasing function representing proportional variable transport costs $c_n(q_n)$ the minimization of $C_n(q_n)$ leads to the optimal shipment size for individual n :

$$q_n^* = \sqrt{\frac{2F_n Q_n}{w_n + r v_n}}. \quad (2)$$

Equation (2) reveals the relationship between the optimal shipment size for individual n and its influencing parameters. An increase of the fixed transport costs like order, handling and set up costs are leading to a higher optimal shipment size as well as an increasing flow of goods Q_n . As the inventory holding costs increase the optimal shipment size will decrease due to the relatively higher costs for capital commitment and storage.

3.2 Choice sets and applied model

The model presented in this publication is based on the discrete choice theory using a random utility approach. A prevalent discrete space of alternatives changes the conditions of decision making: Choosing only one alternative and therefore not realizing the remaining ones leads to corner solutions and thus to a not applicable marginal calculus which necessitates the consideration of the utility functions of each alternative directly. The decision rule is given by the

selection of the alternative which spends the highest utility among all reachable alternatives. The utility functions are assumed to be a summation of attributes describing the alternatives and the characteristics of the decision maker. The attributes themselves are weighted with parameters representing the influence on the utility functions as linear in parameter specifications.

The development of the discrete shipment size choice model includes the categorization of the continuous shipment size into different shipment size classes. For this procedure we used a systematic approach: Let $q_i = 1, 2, \dots, I$ denote the i -th shipment size class which is characterized by the interval $(\underline{q}_i, \dots, \bar{q}_i]$ where \underline{q}_i and \bar{q}_i represent the respective class bounds. Then the width b_{q_i} of class q_i is given by

$$b_{q_i} = \bar{q}_i - \underline{q}_i, \quad i = 1, 2, \dots, I. \quad (3)$$

We applied a growth factor τ which determines – by choosing \underline{q}_i arbitrarily – the bounds and therefore the widths by the following way:

$$\underline{q}_i = \begin{cases} \underline{q}_i, & \text{if } i = 1, ; \underline{q}_i > 0 \\ \tau \underline{q}_{i-1}, & \text{if } i = 2, 3, \dots, I. \end{cases} \quad (4)$$

$$\bar{q}_i = \begin{cases} \tau \underline{q}_i, & \text{if } i = 1, ; \underline{q}_i > 0 \\ \tau \bar{q}_{i-1}, & \text{if } i = 2, 3, \dots, I. \end{cases} \quad (5)$$

Referring to the classification of piece goods, partial loads and (multiple) full loads on road transports we divided the shipment sizes by choosing $\tau = 4$ and $\underline{q}_1 = 0.75t$ into three classes:

$$q_1 = \begin{cases} 1, & \text{if } 0.75t < q_n^* \leq 3t \text{ and} \\ 0, & \text{otherwise.} \end{cases}$$

$$q_2 = \begin{cases} 1, & \text{if } 3t < q_n^* \leq 12t \text{ and} \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

$$q_3 = \begin{cases} 1, & \text{if } 12t < q_n^* \leq 48t \text{ and} \\ 0, & \text{otherwise.} \end{cases}$$

This yields to the global choice set $S = \{q_1, q_2, q_3\} = S_n, \forall n$ assuming all alternatives are accessible for every decision maker n .

The model in this case is restricted to road transports which are not distinguished by different vehicles or vessels. The logistic costs $C_n(q_i)$ – adapted from equation (1) – for actor n choosing shipment size class q_i can now be expressed by:

$$C_n(q_i) = \left(\frac{F_n}{q_i} + c_n(q_i) \right) Q_n + \frac{q_i}{2} (w_n + rv_n). \quad (7)$$

Due to the exclusive consideration of road transports we assume the fixed transport costs F_n to be constant for all logistical actors n . This assumption can be relaxed in further enhancements with multiple means of transport or varying fixed costs within each of the modes. The variable transport costs $c_n(q_n)$ from equation (1) are originally represented by an increasing function dependent on the shipment size. As the shipment size in our model is discretized the variable transport costs $c_n(q_i)$ are dependent on the shipment size categories. Out of that reason we consider them to be constant within each shipment size class and not varying between the decision makers which also can be replaced by detailed tariff functions in further research.

The warehousing costs mainly depend on the handling of the good (heating, cooling, packaging etc.) and the respective space requirement (sizing of warehouses, opportunity costs of space consumption etc.) during the storage processes. The handling and the used space strongly go along with the characteristics, the weight and the respective concentration of the commodity. Due to data constraints regarding the concentration of the transported goods and as a useful simplification without loss of model validity, we suppose the warehousing costs w in this model only to vary between the different shipment size classes and not between each decision maker n . This leads to the following specification:

$$C_n(q_i) = \left(\frac{F}{q_i} + c(q_i) \right) Q_n + \frac{q_i}{2} (w_{q_i} + rv_n). \quad (8)$$

The division of expression (8) by the constant and continuous flow of goods Q_n resulting in the total costs per ton represents a model formulation, which can be on the one hand empirically estimated and is on the other hand properly interpretable with respect to the alternative-specific constants:

$$\frac{C_n(q_i)}{Q_n} = \frac{F}{q_i} + c(q_i) + \frac{q_i w_{q_i}}{2} \cdot \frac{1}{Q_n} + \frac{q_i r}{2} \cdot \frac{v_n}{Q_n}. \quad (9)$$

Choosing the alternative with maximum utility, which is in our case equivalent to minimum costs, requires the formulation of a utility function $U_{q_i,n}$ of decision maker n for the alternatives in S_n . Adding a stochastic component $\varepsilon_{q_i,n}$, a matrix with further influencing factors \mathbf{X}_n and interpreting increasing costs per ton for shipment size class q_i as negative utility results in the following parametrized function:

$$-U_{q_i,n} = \frac{C_n(q_i)}{Q_n} + \varepsilon_{q_i,n} = \alpha_{q_i} + \beta_{q_i,1} \cdot \frac{1}{Q_n} + \beta_{q_i,2} \cdot \frac{v_n}{Q_n} + \boldsymbol{\beta}_{q_i,X} \cdot \mathbf{X}_n + \varepsilon_{q_i,n}. \quad (10)$$

According to the discrete choice theory in association with the random utility approach a shipment size class q_i gets chosen by an individual n if $U_{q_i,n} \geq U_{q_k,n} \quad \forall k \in S_n, i \neq k$. This means that the sum of the observable and unobservable part of the costs of a chosen alternative has to be lower than the costs of all other alternatives in the choice set S_n .

Due to the huge variety of shippers and the diversity of the transported commodities, the categorization of homogeneous subgroups offers a possibility to reduce the behavioral heterogeneity and therefore improves the model. To enhance the core of the shipment size choice with homogenous cluster we applied a so-called “exogenous segmentation” approach. The integration of these classes is realized by adding supplements and deductions to the utility of each alternative depending on the membership in the respective class of each individual n . These supplements and deductions also capture the already addressed warehousing costs which strongly depend on the characteristics of the commodities. This is formally represented by the vector \mathbf{X}_n in this model which includes the classes with the respectively assigned transports. Although exogenously derived clusters don't guarantee the minimization of heterogeneity in the population, they lead in our model to an intuitive interpretation of the segments itself and to a meaningful enrichment to the analysis of shipment size decisions.

One way to determine segments exogenously is the application of the Latent Class Analysis (LCA). Latent classes are characterized as an unobservable and categorical variable which has a nominal level of measurement and is measured by categorical indicators. The notation in this paper is associated with the one from Collins and Lanza (2013). The basis of the LCA is a contingency table, which contains the response categories $r_j = 1, \dots, R_j$ of the indicator variables $j = 1, \dots, J$ and their absolute frequencies. Each cell of the multidimensional contingency table represents a specific response pattern $\mathbf{y} = (r_1, \dots, r_J)$ recording the answer to each of the J indicator variables. The LCA clusters individuals with similar response patterns and is essentially determined by two sets of parameters: the latent class prevalences γ describing the proportion of individuals in the respective class and the item-response probabilities ρ . The item-response probabilities express the relation between each indicator variable and the latent classes and provide out of that reason the basis for the interpretation of the latent classes. Let \mathbf{Y} be the matrix with all possible response patterns \mathbf{y} and L be the set of all latent classes $c = 1, \dots, C$. The core of the LCA is the given by the probability of a specific response pattern

$$P(\mathbf{Y} = \mathbf{y}) = \sum_{c=1}^C \gamma_c \prod_{j=1}^J \prod_{r_j=1}^{R_j} \rho_{j,r_j|c}^{I(y_j=r_j)} \quad (11)$$

where γ_c is the latent class prevalence of class c , $\rho_{j,r_j|c}$ is the item-response probability for responding r_j to indicator variable j conditional on the membership to class c and $I(y_j = r_j)$ represents an indicator variable being one if the response y_j on variable j is given by r_j and zero otherwise. The parameters are estimated by maximizing the Log-Likelihood function of equation (11) via an Expectation-Maximization-Algorithm. Based on the results of the LCA each individual is classified into the group for which it reaches the maximum membership probability

$$\begin{aligned}
P(L = c | Y = \mathbf{y}) &= \frac{P(Y = \mathbf{y} | L = c)P(L = c)}{P(Y = \mathbf{y})} \\
&= \frac{\left(\prod_{j=1}^J \prod_{r_j=1}^{R_j} \rho_{j,r_j|c}^{I(y_j=r_j)}\right) \gamma_c}{\sum_{c=1}^C \gamma_c \prod_{j=1}^J \prod_{r_j=1}^{R_j} \rho_{j,r_j|c}^{I(y_j=r_j)}}.
\end{aligned} \tag{12}$$

The results of the LCA are subsequently integrated into the choice model. Let $L_{c,n}$ be a binary variable indicating if shipper n belongs to the estimated latent classes $c = 1, \dots, C$ then are the different variables $L_{c,n}, c = 1, \dots, C$ part of the attribute matrix \mathbf{X}_n . For each actor n and each class the matrix \mathbf{X}_n therefore contains inter alia the information about the membership which is mathematically expressed by zeros and ones. The already mentioned supplements and deductions to the utility are then added in reference to a specific base class if and only if the entry in the matrix of shipper n is given by one.

4. Data and descriptive analysis

The data which is used for estimating the model was gathered within the scope of the research project "Development of a model for the calculation of freight traffics' modal shifting to derive consistent evaluation approaches for the German federal infrastructure planning (BVWP)" (BVU – Beratergruppe Verkehr + Umwelt, TNS Infratest (2014)). Revealed preference (RP) data provide the base frame of both the survey and the model presented in this paper.² The observations of the used dataset were ascertained via computer assisted personal interviews (CAPI) with responsible logistics employees of companies from all areas of processing and manufacturing trade. The chosen enterprises of the quota sample were drawn from a German-wide business directory with about 10000 addresses whereby unsuitable members of the sample were excluded through a multicriteria screening.

Table 1: Summary statistics of used variables for 487 observations

Variable	Definition	Mean/Freq	Std. dev.
q	Shipment size (t)	13.47	9.75
Q	Flow of goods (t/year)	1845.45	2369.30
v	Value density (€/t)	11311.84	55958.44
1/Q	Inverse of Flow of goods	0.0038	0.0076
v/Q	Relation of value density and flow of goods	69.53	414.67
d	Haulage distance (km)	447.81	363.40
Fragile	1 if commodity is fragile	76	
Valuable	1 if commodity is extraordinary valuable	175	
Awkward	1 if commodity is awkward	150	
Temperature	1 if commodity has to be handled temperature-controlled	48	
Food	1 if commodity is a food product	58	
Dangerous	1 if commodity is inflammable, explosive, poisonous, caustic etc.	62	
Grabbable/Bulk	1 if commodity is unpacked grabbable or bulk cargo	35	
Liquid	1 if commodity is unpacked liquid good	5	
Standard	1 if commodity is transported on standard unit loads	213	
Custom	1 if commodity is a custom-made item	84	
Accumulation	1 if commodity is an accumulation of several articles	170	

Note: Regarding the categorical attributes no responses as well as multiple responses were possible.

² Within this survey also stated preference (SP) experiments were performed which based on the information obtained from the revealed preference part.

In each interview, two representative transports and the corresponding attributes were recorded. The attributes contained information about the type, the weight, the value and the properties of the commodity. Relevant properties of the transported goods described the handling during the transport processes as well as other characteristics of the commodities which potentially influenced the execution of the transport. Further, the frequency of the transports and the position within the logistic chain was logged where the last-mentioned characteristics neither supported the formation of clusters nor improved the explanatory power of the models by individual integration. In addition to this, also the duration, the distance and the costs of each transport with the respective mode were ascertained. In total an amount of 926 transport cases gained from 474 interviews are available. Restricting the model to road transports below 48 tons decreases the amount of applicable observations to 794.

Table 1 presents the summary statistics of the main variables of interest which are initially revealed by the EOQ-model. The constant and continuous flow of goods Q_n was not directly inquired in the datasets. It was instead calculated from the shipment size of the representative transports and their frequency of occurrence per year. The value density v_n was calculated by dividing the value of the transported goods by the shipment size. Because of the occurrence of missing values, the calculation of the flow of goods per year and the value density finally narrows the data basis to 487 useable observations. Due to the use of RP data and the concomitant unavailability of attributes, our model is estimated only on the basis of individual-specific variables. Additionally used variables in the econometric estimation of the model are the transport distance d_n and the attributes regarding the properties of the transported goods presented in table 1.

Table 2: Shipment size categories and distribution of variables

q_i		Q_n	v_n	$1/Q_n$	v_n/Q_n	$\ln 1/Q_n$	$\ln v_n/Q_n$	d	No. obs.
0.75t – 3t	Min	12	366.7	0.00133	0.815	-6.620	-0.2048	40	104
	Q1	100	2 250	0.00333	9.944	-5.704	2.2970	200	
	Med.	150	5 000	0.00667	33.333	-5.011	3.5066	302.5	
	Mean	237.9	18 544.2	0.01045	250.543	-5.055	3.5663	377.8	
	Q3	300	10 000	0.01000	101.042	-4.605	4.6154	500	
	Max	750	500 000	0.08333	5 000	-2.485	8.5172	1 480	
3t – 12t	Min	30	43.6	0.00033	0.0159	-8.006	-4.1434	5	154
	Q1	250	1 531.2	0.00080	2	-7.131	0.6931	205	
	Med.	600	3 660.7	0.00167	5.7060	-6.397	1.7414	397.5	
	Mean	892.3	14 020.7	0.00358	42.602	-6.303	1.9492	450	
	Q3	1 250	7 500	0.00400	21.276	-5.521	3.0570	550	
	Max	3 000	625 000	0.03333	1 562.5	-3.401	7.3540	3 000	
12t – 48t	Min	90	3.7	0.00004	0.0005	-10.127	-7.5090	20	229
	Q1	1 100	500	0.00018	0.1736	-8.613	-1.7509	220	
	Med.	2 400	1 250	0.00042	0.6250	-7.783	-0.4700	380	
	Mean	3 217	6 205.6	0.00094	5.4366	-7.648	-0.6090	478.1	
	Q3	5 500	3 000	0.00091	1.8333	-7.003	0.6061	600	
	Max	25 000	752 380.9	0.01111	358.28	-4.500	5.8813	3 000	

Table 2 contains information about the shipment size classification and the distribution of the continuous variables in each of the classes. The positive influence of the total flow of goods and negative impact of the value density revealed by equation (2) can be already perceived considering the mean values in each class. It is obvious that the distributions of the variables in

each class are right-skewed. On top of that, the variances of Q_n are increasing with the class-width whereas the statistical scatter regarding value density v_n of the commodities is decreasing with the shipment size classes. Also the inverse of Q_n and the ratio v_n/Q_n show a varying dispersion whereby the relative variance of the value density is different to the variance of the annual flow of goods. These insights are also quite intuitively as the growing class widths can cause stochastic dependency of the error terms $\varepsilon_{q_i,n}$ and the used variables which is associated with the possible violation of a main assumption of the Logit-Model: the error terms $\varepsilon_{q_i,n}$ are independent and identically Gumbel distributed. This problem can be attended to by taking the natural logarithm of the variables cushioning the distributions nearly to a bell-shaped curve and also approaches the variance between the different shipment size classes.

5. Model estimation

In this section several multinomial logit models based on the specification of equation (10) in have been estimated. As a first step, models without latent classes, which represent the core of the shipment size choice, will be presented whereas the enhanced models with latent classes are part of the second section. We used the statistical software “R” with its supplemental packages “mlogit” and “poLCA” for the estimation of the models.

5.1 Basic model

Table 3: Shipment size choice model 1 – non-standardized variables.

	$q_2: 3t - 12t$	$q_3: 12t - 48t$
Constant	1.0838*** (0.1837)	2.4482*** (0.2015)
$1/Q_n$	-110.99*** (25.076)	-631.48*** (84.136)
v_n/Q_n	-0.0005 (0.0005)	-0.0071 (0.0044)
Log-Likelihood: -403.43		
McFadden R^2 (ρ^2): 0.21		
Adj. McFadden \bar{R}^2 ($\bar{\rho}^2$): 0.1982		
Likelihood ratio test: $\chi^2 = 214.46$ (p.value = < 2.22e-16)		

Note: Significance levels: . p<0.1; * p<0.05; ** p<0.01; *** p<0.001; 487 observations.

Table 3 shows the estimation results with $\frac{1}{Q_n}$ and $\frac{v_n}{Q_n}$ as independent variables. The value 0.21 of ρ^2 indicates – despite the menace of heterogeneity – a well-performing shipment size choice model which was estimated in relation to the smallest shipment size class with its referencing parameter values equal to zero. Every coefficient has the expected sign respective the expected order. The ratio of value density and annual flow of goods doesn’t have a statistically significant impact whereas the other estimated coefficients are highly significant. As can be seen, the constants have a positive sign and an increasing order regarding the shipment size classes which describes the average influences of not considered attributes which are in our model $\frac{F}{q_i} + c_n(q_i)$. The positive sign and the order of the constants show on average decreasing costs per unit choosing higher shipment size classes in reference to the smallest one. A decreasing annual flow of goods leads to an increase of the inverse $\frac{1}{Q_n}$ which induces higher costs for higher

shipment size classes. This goes along with the theoretically positive impact of the annual flow revealed by equation (2). The high negative values of the estimated coefficients regarding the inverse of the annual flow of commodities are due to the low values of $\frac{1}{Q_n}$ which are all positive but smaller than one and shown in table (2).

Table 4: Shipment size choice model 1 – logarithmized variables.

	$q_2: 3t - 12t$	$q_3: 12t - 48t$
Constant	-4.5762*** (1.2542)	-7.9800*** (1.5676)
$\ln(1/Q_n)$	-0.9143*** (0.1855)	-1.5013*** (0.2272)
$\ln(v_n/Q_n)$	-0.0775 (0.1041)	-0.5267*** (0.1295)
Log-Likelihood: -344.41		
McFadden R^2 (ρ^2): 0.3256		
Adj. McFadden \bar{R}^2 ($\bar{\rho}^2$): 0.3138		
Likelihood ratio test: $\chi^2 = 332.49$ (p.value = < 2.22e-16)		

Note: Significance levels: . p<0.1; * p<0.05; ** p<0.01; *** p<0.001; 487 observations.

As mentioned before, the danger of a misspecification concerning the possible heterogeneity is omnipresent. Out of that reason we estimated the model again transforming the independent variables by taking the natural logarithm. Table 4 contains the results which show an improved performance, although we're moving away from core of the model represented by the total logistics cost per ton. An additional feature of this representation lies in the interpretability and therefore in the comparison of the parameters. The use of logarithmized independent variables in standard linear regression models admits – assuming small changes of the regressors – the interpretation of the coefficients approximately as semi-elasticities (Stock and Watson (2007)). Due to the comparison to the smallest shipment size class and the specification of its utility to zero in our case this practice can also be applied. More precisely: an increase for example of $\frac{1}{Q_n}$ by 1% changes the total logistic costs per ton of shipment size class q_2 by $0,01 \cdot \beta_{q_2,1}$ compared to q_1 . The alternative-specific constants are becoming negative with relatively high values. This is induced by the transformation of the variables which decreases small values (< 1) strongly to negative values and flattens out the slope with increasing values. Again all signs and orders of the coefficients are as expected although we also must point out that the coefficient for the $\frac{v_n}{Q_n}$ -relationship regarding the second shipment size class has no significant influence. The coefficient of $\frac{v_n}{Q_n}$ explain the variation of the costs per ton which is not already covered by the reciprocal annual flow of goods. The impact is negative but has a relatively weak influence on the decision of the shipment size choice. This result supports the conclusion that the costs for tied-up capital are not going to pervade completely the shipment size choice and have therefore less explanatory power than the annual flow of goods. A reason is given by the fact that the value density may not describe the importance of the good in the supply chain and therefore underestimates the effect. This was also observed by Combes (2009).

5.2 Enhancement of the discretized EOQ-model with latent classes of shipments' attributes

The reasonable explanatory power of the discretized EOQ-model enables the enhancement of the model with further characteristics influencing the shipment size systemically. In light of the already acknowledged influences of commodity characteristics on the shipment size choice behavior the model will be expanded in this section. We therefore use an approach which has to our knowledge not been undertaken up to now in the modelling of shipment size choice behavior: clustering the logistical actors respectively the realized transports to model similar behavior more accurate and reduce the dimensions of possible heterogeneity which also leads to an improvement of the models in the statistical sense by bypassing the possibility of insignificant control parameters of commodity types and characteristics.

Table 5: Information criteria of LCA's.

No. classes	Log-Likelihood	BIC
2	-2284.309	4710.949
3	-2232.398	4681.386
4	-2191.574	4673.995
5	-2175.075	4715.257
6	-2158.490	4756.347
7	-2146.076	4805.778

To group the individuals, we used the LCA based on the characteristics of the transported goods which joins the actors by searching for similar response patterns. Due to the propensity of the Expectation-Maximization-Algorithm to get stuck in local optima, we repeated the estimation 1000 times for each number of classes. The determination of the proper number of classes is not endogenously. Out of that reason we had to decide by reference to the values of the Bayes-Information-Criterion (BIC) which are displayed in table 5 for several counts of classes. We picked the classification with four classes for which the results of the LCA can be obtained in table 6. The conditional probabilities of answering "Yes" to a specific indicator denote the probability to answer "Yes" if an individual is assigned to the respective class. High or low values – also in comparison to the other classes - are therefore characteristic and affect the interpretation of it. We marked in bold notation the conditional probabilities being characteristic for the class. This means that they are on the one hand relatively high compared to other probabilities in the same class and on the other hand quite different regarding the same indicators across the other estimated classes. The bold values are therefore used to apply the necessary subjective interpretation of each class.

We called class 1 "(Temperature-controlled) Food Products" as every individual belonging to this class will answer assuredly "Yes" to the Food item and with more than 60% to Temperature item. Also the transported goods in this class are never dangerous or awkward and are transported with probability 0.555 on standard unit loads. Class 2 is mainly characterized by the items Standard and Dangerous wherefore we interpreted it as "Miscellaneous Standard Cargo" including dangerous commodities transported on the same. This class inherits also the main proportion of the population with 36%. The next class has high loads on the items Fragile, Valuable, Awkward and Custom which can be interpreted as "Special Goods" and describes mostly the mechanical engineering sector. The proportion of this class with about 35% seems relatively high and indicates a general overrepresentation of it in the sample. At least we named

class 4 “Unpacked Bulk Goods” as it has on the one side relatively high probabilities for the items Grabbable/Bulk and Liquid and on the other side low probabilities on Custom, Standard, Accumulation and Fragile.

Table 6: LCA result with four classes.

	<i>Class 1</i>	<i>Class 2</i>	<i>Class 3</i>	<i>Class 4</i>
γ_c	0.1070	0.3600	0.3475	0.1854
Conditional probability of answering “Yes”				
Fragile	0.1727	0.0634	0.3302	0.0000
Valuable	0.2631	0.2395	0.6373	0.1266
Awkward	0.0000	0.2376	0.4942	0.2734
Temperature	0.6331	0.0855	0.0000	0.0000
Food	1.0000	0.0000	0.0000	0.0651
Dangerous	0.0000	0.2079	0.0599	0.1707
Grabbable/Bulk	0.0398	0.0000	0.0000	0.3647
Liquid	0.0555	0.0000	0.0000	0.1800
Standard	0.5550	0.8113	0.2308	0.0305
Custom	0.0384	0.0841	0.3974	0.0000
Accumulation	0.4781	0.4134	0.3992	0.0557
Observations: 487				
Parameter estimated:47				

Note: Bold parameters are characteristic for the class with respect to the class itself and across classes.

The assignment of the individuals to the classes is accomplished by calculating the membership probability showed in equation (12) and taking the maximum of the probabilities as allocation rule. This procedure denotes a probabilistic approach in contrast to a deterministic assignment and therefore needs a validation of applicability. One established indicator is the so called “Odds of Correct Classification” (OCC) which sets the average probability of the individuals assigned to a class in ratio to the general proportions of each class and has a ratio higher than five as threshold (Nagin (2005)). In our classification this is reached for every class which can be seen in table 7.

Table 7: Key figures of classification.

<i>Class</i>	<i>Mean</i>	<i>Variance</i>	<i>OCC</i>
1	0.93	0.02	109.45
2	0.85	0.02	10.01
3	0.84	0.02	10.12
4	0.75	0.04	13.11

Note: This table includes means and variances of the maximum probabilities from the assigned individuals.

The next step consists in the integration of the identified classes into the model. Additionally we incorporate the haulage distance d_n to capture possible influences which are empirically verified (Abate and de Jong (2014), Combes (2009), Jansson and Shneerson (1982)). Table 8 shows the estimation results for the comprehensive model. The performance of the enhanced model increased which can be deduced from the value of \bar{p}^2 . Also all coefficients become at least significant with respect to 0.1 level. The transport distance has a straight positive effect on the shipment size choice which goes along with the empirical findings. There is some dissent on the

interpretation of this effect. On the one hand there might be a deviation from of the assumed linear tariff function and the real tariff function which could be expressed by less than proportional increasing fuel/time cost per shipment for larger vehicles (Abate and de Jong (2014)). On the other hand this effect might be caused by cultural properties or upcoming safety needs which take the increasing risk of delays into account going along with longer haulage distances. Combes (2009) illustrates the effect with a potential decoupling between production location and regional retail center conducted through a regional distribution center. If in general the transports have larger shipment sizes from the production location to the distribution center and they are dispatched the positive influence of the transport distance could be explained.

Table 8: Shipment size choice model 2

	$q_2: 3t - 12t$	$q_3: 12t - 48t$
Constant	-5.5253*** (1.3670)	-10.027*** (1.7849)
$\ln(1/Q_n)$	-0.8900*** (0.1933)	-1.4964*** (0.2451)
$\ln(v_n/Q_n)$	-0.2239. (0.1175)	-0.7521*** (0.1516)
d_n	0.0013* (0.0005)	0.0027*** (0.0006)
Miscellaneous Standard Cargo	0.9168. (0.4884)	1.3603* (0.5799)
Special Goods	1.2750* (0.5004)	1.9700** (0.6004)
Unpacked Bulk Goods	1.1386* (0.5776)	2.4094*** (0.6741)
Log-Likelihood: -321.15		
McFadden R^2 (ρ^2): 0.3711		
Adj. McFadden \bar{R}^2 ($\bar{\rho}^2$): 0.3482		
Likelihood ratio test: $\chi^2 = 379$ (p.value = < 2.22e-16)		

Note: Significance levels: . p<0.1; * p<0.05; ** p<0.01; *** p<0.001; 487 observations.

The impacts of the latent classes have all been estimated in comparison to the first class of “(Temperature-controlled) Food Products”. As expected, all other classes tend to choose bigger shipment sizes as the warehousing of the goods is more expensive and the perishability doesn’t allow high order quantities. At first glance, the order of the coefficients for the classes is somehow contradictory. “Miscellaneous Standard Cargo” has in comparison to the group with “Special Goods” smaller coefficients which is from a superficial point of view unexpected. But the interviews showed that the members of the German mechanical engineering sector need to use higher shipment sizes because of the weight of their products. Finally the “Unpacked Bulk Goods” show by far the highest effect on the largest shipment size class. This behavior is reasonable as the warehousing costs should be relatively low and the production of the goods is performed in big batches. In general, the results reveal that the effects of the groups don’t vary much with respect to the medium shipment size class. Solely the food products are less likely to be sent in medium shipment sizes.

6. Conclusion and further research

In this paper, a discrete shipment size choice model based on a discrete formulation of the total logistic costs was developed. In order to reach an appropriate microscopic representation of the choice situations' heterogeneity, the choice behavior was addressed separately for clusters of homogeneous decision-makers. The decision makers respectively the conducted transports were classified by using a Latent Class Analysis approach on the basis of the commodity characteristics. The estimation of the models was performed on a database which was collected within the scope of the German federal infrastructure planning (BVWP).

From a transport analyst point of view, the interesting results are that the volumes of the underlying commodity flows can explain the choice of shipment sizes to a large extent. The integration of the latent classes improved the model and provided reasonable signs and orders of magnitude for the estimated coefficients. Being estimated on a sample of very heterogeneous transport cases, qualifies the latent class model for incorporation in operational large scale freight transport models. Moreover, the latent class part shows that a shipment size model is a way to incorporate logistics aspects into freight transport models on the required coarse grained level of detail. It remains to be examined whether the same categorization applies to other decisions drawn on the shipments such as transport mode choice. Moreover, the influence of distance alludes that spatial aspects play a role in the choice of shipment sizes.

A ubiquitous problem in the context of this model is the heteroscedasticity which is caused by the categorization of the shipment size classes and should be attended by future research. An additional aspect is represented by the possible variations of the estimated parameters across the groups of decision-makers which reveals different influences on the costs for different groups. One possible case would be the warehousing costs which strongly depend on the characteristics of the goods and therefore should have different influences with respect to the total logistic costs. At least should the shipment size choice model be extended to several modes of transports and in general be combined with the mode choice.

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