Probability distributions of link durations with *n* disruptive channels: application to ground-space optical communications

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ABSTRACT

We consider n independent disruptive channels with random availability periods and make use of channel diversity to maintain a link as often as possible. This situation arises when an optical link is to be maintained between a spacecraft and n ground stations affected by cloud coverage. Statistical independence between the stations and equal (un)availability duration distributions are assumed. Based on a given single-station (un)availability duration distribution, we derive analytically the network (un)availability duration distributions. Also derived is the distribution of the station operation duration within a network. Derived expressions allow evaluating the increase of the continuous operation duration of a station and the decrease of the network unavailability durations.

Keywords: MIMO, channel diversity, selection combining, random link duration, free-space optical communications, cloud coverage

1. Introduction

When a link suffers from random disruptions, several uncorrelated practicable channels provide diversity and selection combining increases the link availability. Free-space optical links are greatly blocked by clouds [1], so that a link between the Earth and space suffers from an intermittent availability. If n ground stations with a geographical separation large enough to have uncorrelated weathers are dedicated to this link, availability increases.

Many studies have investigated the availability of particular ground-station networks [2]-[5]. However, much less work has been carried out on the duration of the availability and unavailability periods which is the topic of this paper. We are interested in the influence of the number of stations on (un)availability periods. Also of interest is the frequency of the station switches within a given network. We thus propose a theoretical analysis of the link duration probabilities. Queuing theory and traffic models evaluate the availability and/or the congestion of communication networks [6]. Yet, these theoretical models do not apply in our context because we have no arrival rate of link requests. Instead, we consider a link service of infinite duration and the resource nodes (typically servers in traffic models, ground stations in our case) become unavailable due to an external perturbation (cloud) and not because resource nodes are already occupied with other link services.

The paper is organized as follows. In Section 2 called "general assumptions" we delineate the considered framework. Section 3 presents useful relations between the probability density functions (PDFs) of different durations depending on how the event outcome is selected. The following sections provide respectively distribution expressions for network unavailability (NU), station operation (SO) after a station switch, unconditioned SO and network availability (NA). We conclude with Section 8.

2. General assumptions

At ground stations, the alternation of availability and unavailability periods of a station is considered a stationary stochastic process. We thus neglect the variability of cloud statistics with e.g. seasons. The evolution of the link operation and the station switches within the network are also considered stationary. So,

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our analysis applies more to spacecraft with such an orbit that the ground-station network remains visible for a long time compared to the cloud variations.

The distributions of station availability (SA) duration D_{SA} and station unavailability (SU) duration D_{SU} are distinct and may be very different. Yet, for simplicity we will consider an identical (lognormal) model for both distributions. Also for simplicity, statistical independence between the stations and equal (un)availability duration distributions are assumed.

We define network unavailability (NU) as the unavailability of all stations. Figure 1 illustrates the possible evolution over time of the availability of 3 stations. The availability of the resultant network is also observable. The red straight line represents the optical link operated successively by the stations. Instances of the station operation (SO) duration D_{SO} and instances of the network unavailability (NU) duration D_{NU} are indicated on top of the figure.

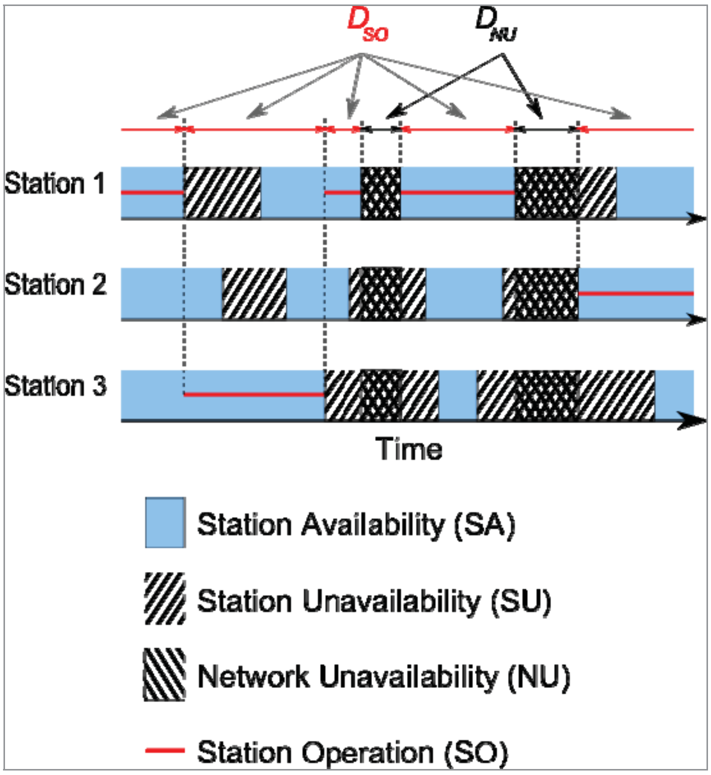

Figure 1: Illustration of the availability of a network consisting of *n* = 3 stations. An example of how the stations are successively operated is shown by a red line.

3. Distribution of state durations

The considered states are those of a ground station: 'available' and 'unavailable'. But the statistics of a state duration depend on how the state event is selected as a random outcome. Under this section, we establish relations between the probability distributions of three state durations with different event selection procedures. For a given station and a given state, we consider the three following random variables:

D: duration of a state event *selected arbitrarily among other state events*
T: duration of a state event *selected at an arbitrary time*
R: remaining time of a state event *selected at an arbitrary time*

The probability density function (PDF) $f_D(t)$ of D is given by the frequency of occurrence of the states with duration D, whereas the PDF $f_T(t)$ of T is given by the temporal proportion of the states with duration T. The relation between the two PDFs is thus:

$$f_T(t) = \langle D \rangle^{-1} f_D(t) \times t \tag{1}$$

where $\langle D \rangle$ is the mean value of D. When arbitrarily selecting a time within an event of duration T, the probability density of the remaining time R for that event is $1/T$ (uniform distribution). The density f_R conditioned to T is thus

$$f_R(t_R \mid T = t) = \frac{1}{t}, \qquad 0 \le t_R \le t \tag{2}$$

And the unconditional density can be written as

$$\begin{aligned} f_R(t_R) &= \int_0^\infty f_R(t_R \mid T = t) f_T(t)\, dt \\ &= \int_{t_R}^\infty \frac{1}{t} f_T(t)\, dt \\ &= \langle D \rangle^{-1} \left[1 - F_D(t_R)\right] \end{aligned} \tag{3}$$

We conventionally write F_X the cumulative function of f_X so that $F_D(t)$ is the CDF of D. Figure 2 shows the densities $f_D(t)$, $f_T(t)$ and $f_D(t)$ when D is modelled (in a quite arbitrarily way, not validated by measurements) as a lognormal variable with mean and standard deviation of 10 hours.

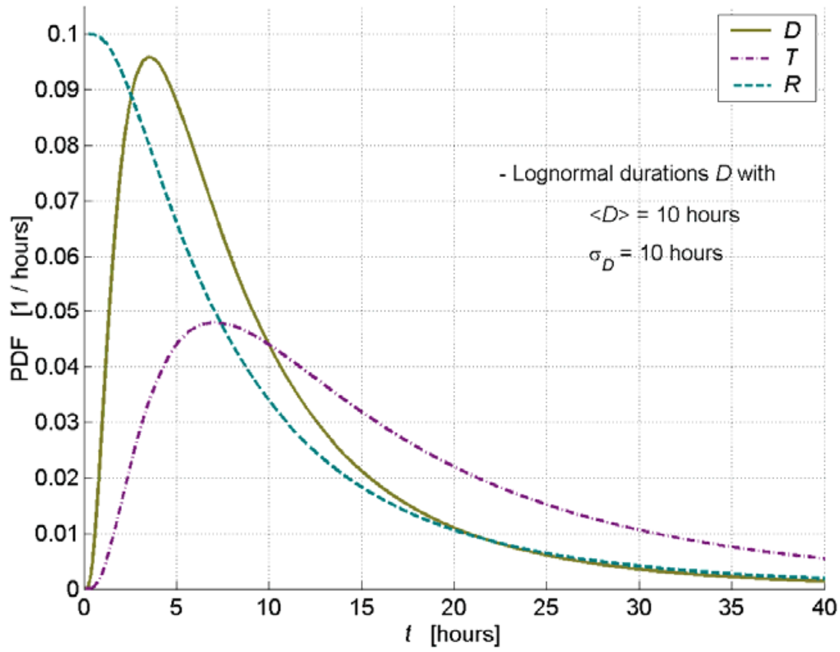

Figure 2: Probability density functions $f_D(t)$, $f_T(t)$ and $f_D(t)$ when D is a lognormal variable with a mean and a standard deviation of 10 hours.

4. NU duration

A NU may possibly start when an operative station becomes unavailable. Let D_{SU_1} be the SU duration at this last operative station. When this operative station becomes unavailable, let $R_{SU_2}, R_{SU_3}, \cdots, R_{SU_n}$ be the remaining unavailability durations at the other stations. For these other stations, the remaining durations $R_{SU_2}, R_{SU_3}, \cdots, R_{SU_n}$ are considered to start at an arbitrary time and, at that time, we may have a SA state instead of a SU one. To assure a NU, we condition each remaining duration to a SU state and append the condition $|\mathrm{SU}$ to the variables $R_{SU_2}, R_{SU_3}, \cdots, R_{SU_n}$. The NU duration, noted D_{NU}, is given by the minimum of all SU durations:

$$D_{NU} = \min\left(D_{SU_1}, R_{SU_2}\,|\mathrm{SU}, R_{SU_3}\,|\mathrm{SU}, \cdots, R_{SU_n}\,|\mathrm{SU}\right). \tag{4}$$

Assuming equal statistics for all stations, we drop the subscript number identifying the stations. Applying the assumption of statistically independent stations, the CDF of D_{NU} is given by [7]

$$\begin{aligned} F_{D_{NU}}(t) &= 1 - \left[1 - F_{D_{SU}}(t)\right]\left[1 - F_{R_{SU}}(t|\mathrm{SU})\right]^{n-1} \\ &= 1 - \langle D_{SU}\rangle f_{R_{SU}}(t)\left[1 - F_{R_{SU}}(t|\mathrm{SU})\right]^{n-1} \end{aligned}. \tag{5}$$

where in the second line we have used Eq. (3). Figure 3 shows the PDF $f_{D_{NU}}(t)$ of D_{NU} for networks with 4 different numbers n of stations. For D_{SU}, the same distribution as for D in Figure 2 was assumed.

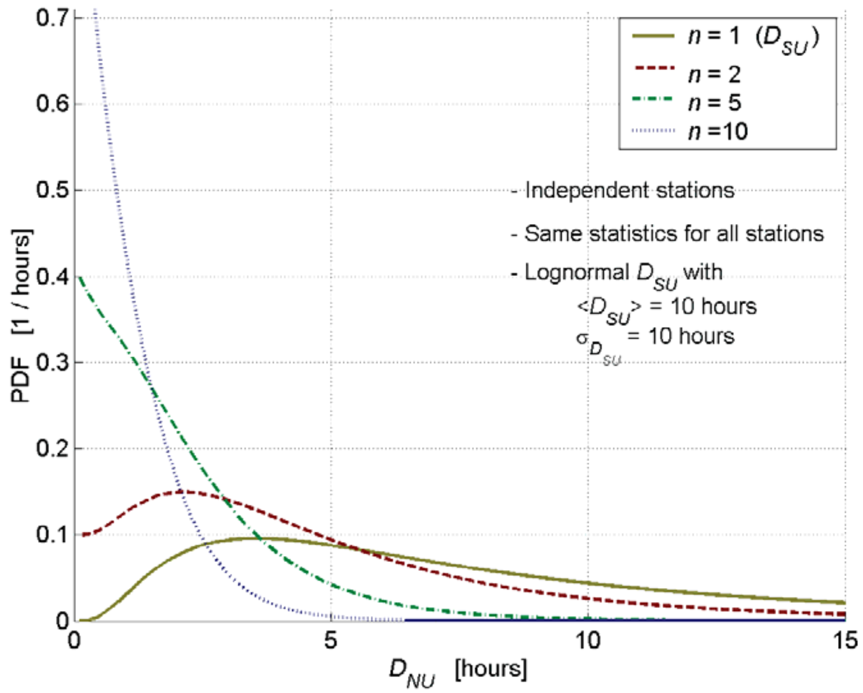

Figure 3: Probability density function $f_{D_{NU}}(t)$ of the network unavailability durations for networks with 3 different numbers n of stations. For D_{SU}, the same PDF as for D in Figure 2 was assumed.

5. SO duration after a station switch

We wish to derive the distribution of D_{switch}, the SO duration after a station switch (handover). In a network of n stations, the switching process consists in choosing among n-1 stations the next operative station. If we can predict the remaining durations of the current availabilities at each station, we may decide to switch to the station providing the longest remaining availability duration in order to minimize the number of switches. Like the R_{SU} variables in Section 4, the remaining availability durations $R_{SA_2}, R_{SA_3}, \cdots, R_{SA_n}$ are considered to start at an arbitrary time and, at that time, we may have a SU state instead of a SA one.

To assure a NA after a switch, we append the condition $|\mathrm{NA}$ to the remaining availability duration of the selected station. We thus define D_{switch} according to:

$$D_{switch} = \max\left(R_{SA_2}, R_{SA_3}, \cdots, R_{SA_n}\right)\big|\mathrm{NA}\,. \tag{6}$$

To take into account the probability that a station is available, we write the duration distribution as the sum of 2 distributions, one for each state: SA and SU. In case of SU, we set by convention the availability durations to 0 (a negative value would also be possible) so that the conditional CDF $F_{R_{SA}}\left(t\,|\,\mathrm{SU}\right)$ of the SU duration in case of SU is modelled as a Heaviside step function $H(t)$.

$$F_{R_{SA}}\left(t\right) = F_{R_{SA}}\left(t\,|\,\mathrm{SA}\right)\left(1-p_{SU}\right) + H\left(t\right)p_{SU} \tag{7}$$

where p_{SU} is the probability of station unavailability *at an arbitrary time*.

Assuming stations with equal and independent statistics, the CDF of D_{switch} is found by multiplying the CDFs of the clear-sky durations of all stations [7]. Because D_{switch} is by definition conditioned to a NA, we remove the Heaviside step function artefact which is now weighted by p_{SU}^{n-1}, i.e. the probability that n-1 stations are unavailable at an arbitrary time. Finally, rescaling the function by $1/(1-p_{SU}^{n-1})$ to obtain a probability distribution, we obtain:

$$F_{D_{switch}}\left(t\right) = \frac{\left[F_{R_{SA}}\left(t\right)\right]^{n-1} - H\left(t\right)p_{SU}^{n-1}}{1-p_{SU}^{n-1}}\,. \tag{8}$$

If we arbitrarily switch to one of the available stations, the CDF of D_{switch} is

$$F_{D_{switch}}\left(t\right) = F_{R_{SA}}\left(t\big|\mathrm{SA}\right). \tag{9}$$

Figure 4 shows the PDF $f_{D_{switch}}\left(t\right)$ of D_{switch} for networks with 3 different numbers n of stations and to 2 different SU probability p_{SU}. It is assumed the longest remaining availability period was chosen after a switch. For D_{SA}, the same distribution as for D in Figure 2 was assumed. Note that the $f_{D_{switch}}\left(t\right)$ curve for n = 2 is identical to the PDF of R displayed in Figure 2.

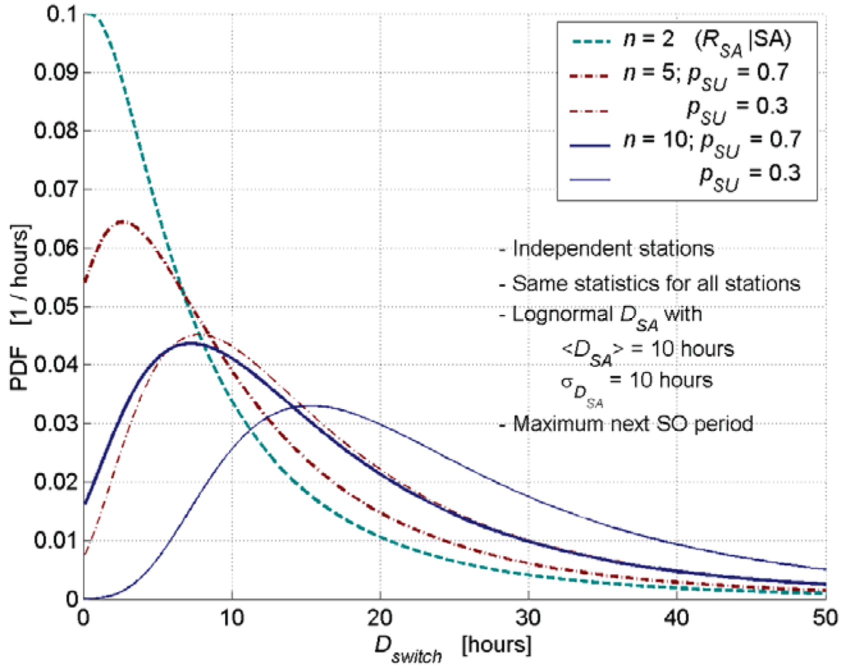

Figure 4: Probability density function $f_{D_{switch}}(t)$ for networks with 3 different numbers n of stations and to 2 different station unavailability probability p_{SU}. For D_{SA}, the same PDF as for D in Figure 2 was assumed.

6. SO duration

In order to find the PDF of the SO duration D_{SO}, we first provide an approximate probability distribution of the number k of switches within a NA period. The approximation resides in the assumption of statistical independence between the availabilities at a switch time and the availabilities at the next switch time. It is reasonable to state that the correlation between availabilities decreases with their temporal separation. Therefore, to minimize approximation errors, we will consider only the longest possible D_{SO} after a switch as expressed in Eq. (6).

After a NU period, a station becomes operative. When this station becomes unavailable, we have a probability p_{SU}^{n-1} that the n-1 remaining stations are also unavailable, i.e. that there has been k = 0 switch before another network unavailability. We have a probability $\left(1-p_{SU}^{n-1}\right)p_{SU}^{n-1}$ that the link is switched once (k = 1) to another station before being interrupted by a network unavailability. And so on, so that the probability function of k is $\left(1-p_{SU}^{n-1}\right)^k p_{SU}^{n-1}$. It is worth mentioning that, although p_{SU} is the SU probability at an arbitrary time, this probability function of having k switches is meant for an arbitrary NA event, not for an arbitrary time of the NA state.

Among the k+1 SO periods contained in a NA period, there are one SA period of full duration (variable D_{SA}) and k SO periods starting after a switch (variable D_{switch}). Thus, the PDF $f_{D_{SO}}(t)$ of SO durations is the weighted sum of the distributions $f_{D_{SA}}(t)$ and $f_{D_{switch}}(t)$, where $f_{D_{SA}}(t)$ is weighted k times more than $f_{D_{switch}}(t)$. After averaging over the k distribution, we obtain:

$$f_{D_{SO}}(t) = \sum_{k=0}^{\infty} \left(1 - p_{SU}^{n-1}\right)^k p_{SU}^{n-1} \left[\frac{1}{k+1} f_{D_{SA}}(t) + \frac{k}{k+1} f_{D_{switch}}(t) \right]. \tag{10}$$

Recognizing the power series of the ln() function, Eq. (10) can be put in the form

$$f_{D_{SO}}(t) = q f_{D_{SA}}(t) + (1-q) f_{D_{switch}}(t) \tag{11}$$

with

$$q = \frac{\ln\left(1/p_{SU}^{n-1}\right)}{1/p_{SU}^{n-1} - 1}. \tag{12}$$

$f_{D_{SO}}(t)$ is dominated by $f_{D_{SA}}(t)$ when $q > 0.9$, i.e. when $p_{SU}^{n-1} > 0.8$. Whereas $f_{D_{SO}}(t)$ is dominated by $f_{D_{switch}}(t)$ when $q < 0.1$, i.e. when $p_{SU}^{n-1} < 0.03$.

7. NA duration

To calculate the distribution of the NA duration, we distinguish, like in Section 6, the NA periods according to the number k of switches they contain. Because the NA duration is the sum of SO durations, $f_{D_{NA}}(t)$ is found by convolving the duration distributions of the various SO contained in the NA period [7]:

$$f_{D_{NA}}(t) = \sum_{k=0}^{\infty} \left(1 - p_{SU}^{n-1}\right)^k p_{SU}^{n-1} \left[f_{D_{SA}}(t) \overbrace{\otimes f_{D_{switch}}(t) \otimes \ldots \otimes f_{D_{switch}}(t)}^{k \text{ convolutions}} \right]. \tag{13}$$

Again, the assumption of statistical independence between two consecutive SO periods makes the formula an approximation only.

8. Conclusion

Derived expressions of duration distributions provide insights in the benefit of expanding a network of ground stations for space-ground links. Increasing the number n of stations makes the SO and NA durations increase and makes the NU durations decrease. Derived expressions rely on (un)availability duration distributions. Experimental data are required to find suitable distribution models for both station availability and unavailability. Lognormal models have only been used as examples, as they seemed realistic to the authors.

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