Discrete optimisation problems on an adiabatic quantum computer

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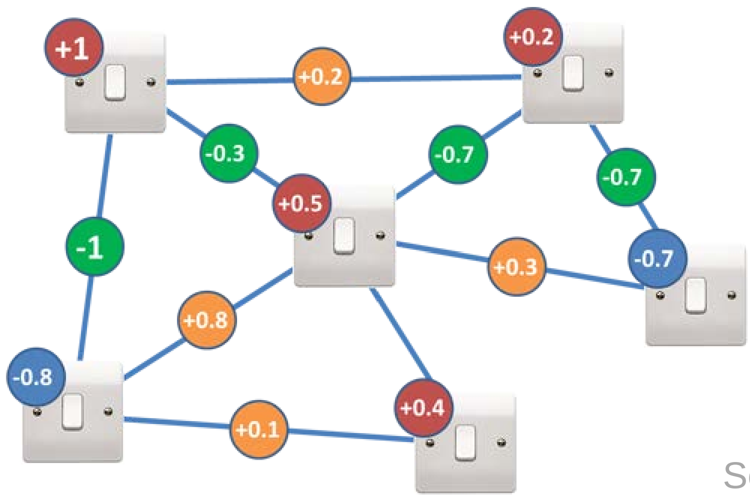
Outlook

- Introduction to Adiabatic Quantum Computing
- Hardware limitations
- Application: Clique Problem

What is an Adiabatic Quantum Computer?

- Solver for Quadratic Unconstraind Binary Optimisation Problems (QUBOs)

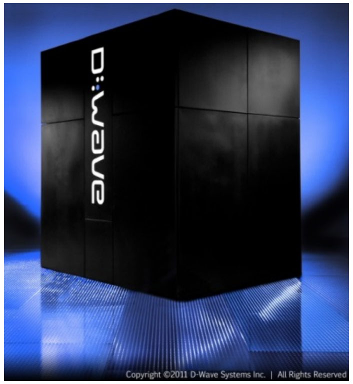
$$E(s_1, s_2, \ldots, s_n) = \sum_i H_i s_i + \sum_{ij} J_{ij} s_i s_j \quad \text{with } s_i \in \{0,1\}$$



$H_i \in \mathbb{R}$ On-site strength

$J_{ij} \in \mathbb{R}$ Coupling

Source: D-Wave Sys.



- Device by company D-Wave Systems commercially available
- We have access to hardware simulator / programming interface

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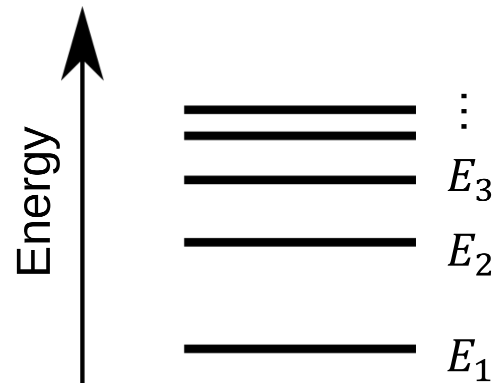
Quantum physics

- Eigenvalue equation determines physical state of a system

$$\hat{H}\,|\psi_i\rangle = E_i|\psi_i\rangle$$

where E_i is the energy of the state and $|\psi_i\rangle \in$ Hilbert space and $\hat{H}$ is the Hamilton operator.

- Discrete Eigenvalues lead to quantised energy levels.



Measurement in Quantum physics

- Let the system be in a superposition of energy Eigen states $|\psi_i\rangle$

$$|\chi\rangle = \sum_i a_i |\psi_i\rangle$$

- Measurement of energy E_j changes state

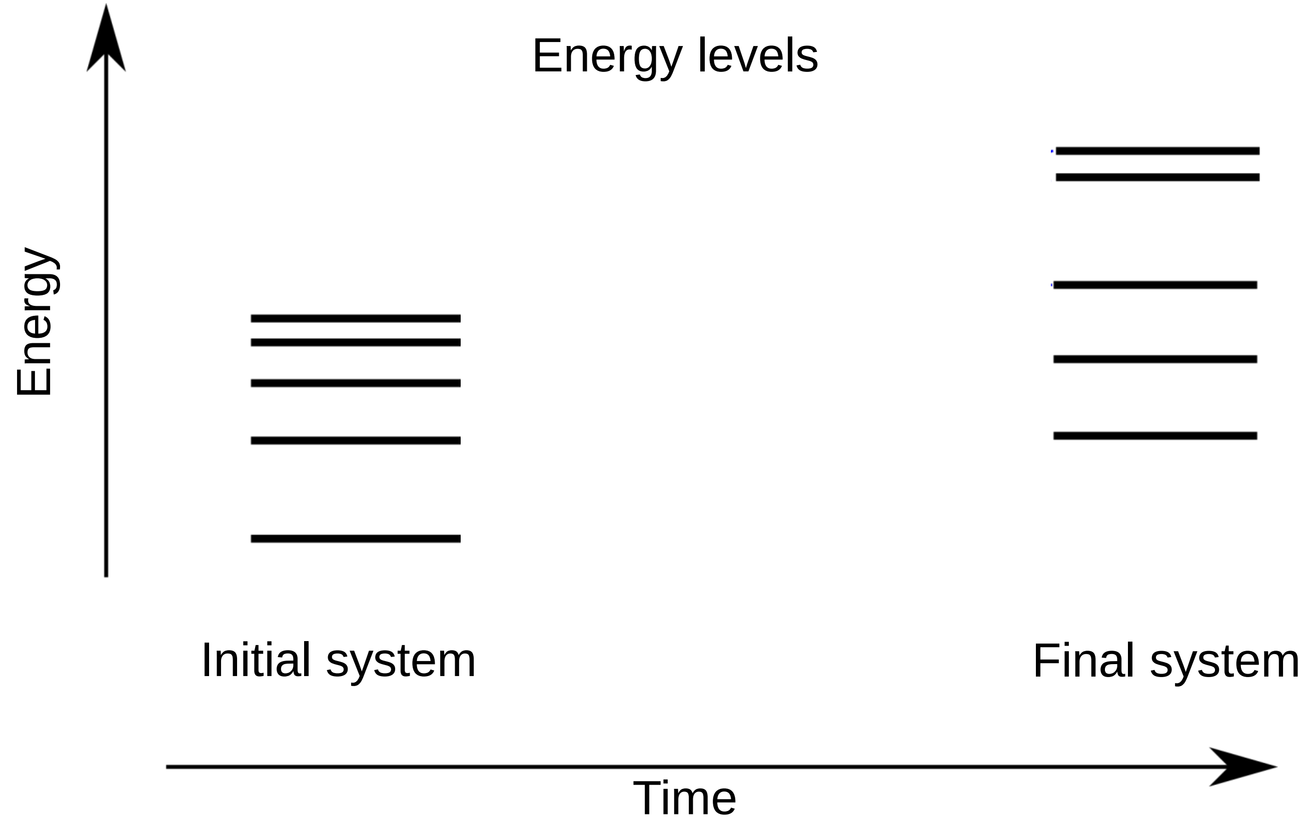
$$|\chi\rangle \rightarrow |\psi_j\rangle\langle\psi_j| \, |\chi\rangle = a_i \, |\psi_j\rangle$$

- Probability P_{j} to measure state $|\psi_j\rangle$

$$P_j = |a_i|^2$$

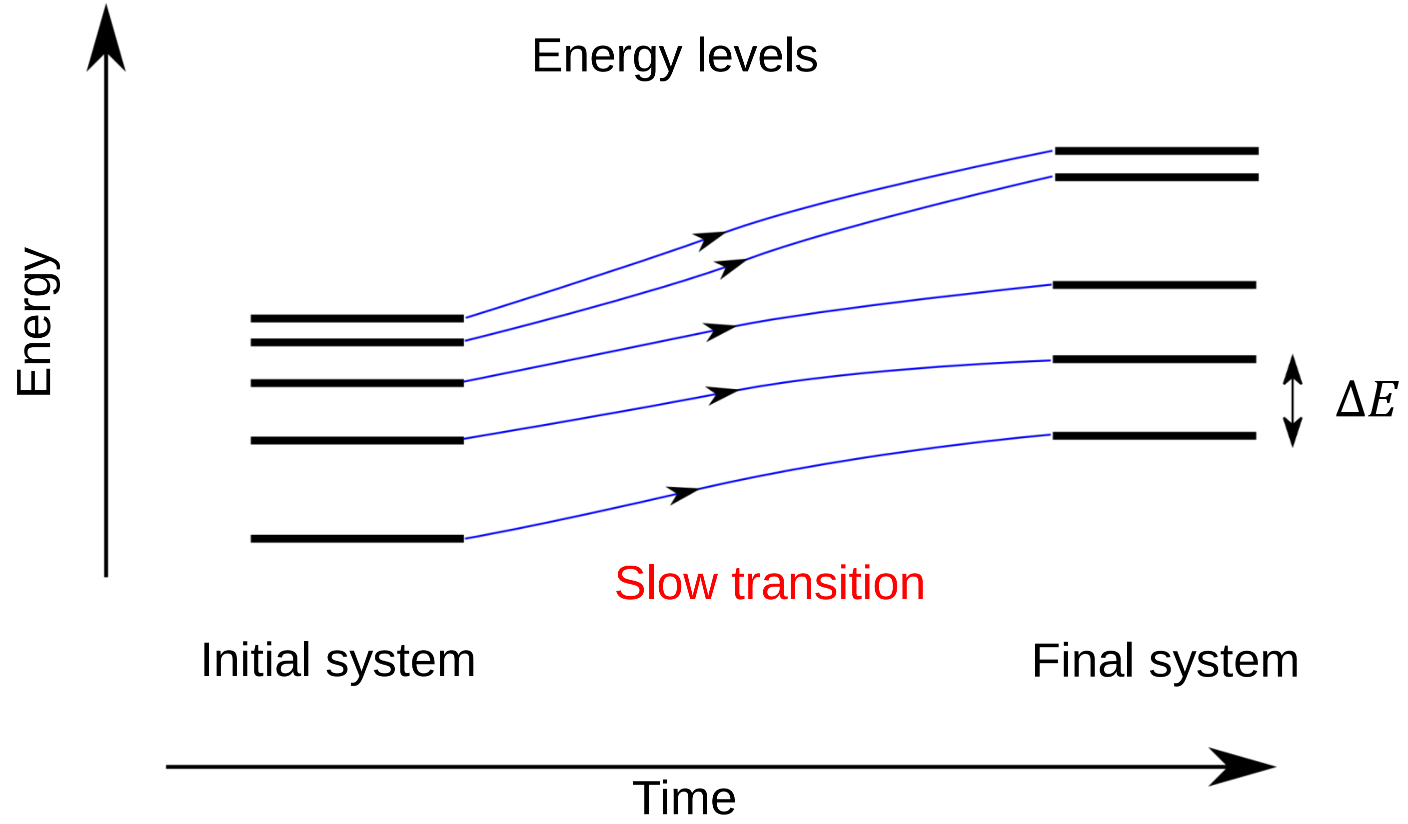
Adiabatic Quantum Computer – How does it work?

- Encode objective function in ground state of quantum system
- Initial system groundstate simple and implementable



Adiabatic Quantum Computer – How does it work?

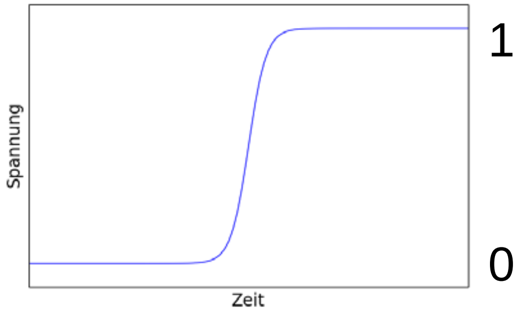
- Sufficiently *slow (adiabatic)* transition to final system
- Lowest energy gap ΔE determines runtime



Quantum Bits – Two Level Quantum Systems

Classical bits

- Voltage



Quantumbits (Qubits)

- State is Superposition of „0“ und „1“

$$|\varphi\rangle = a|0\rangle + b|1\rangle$$

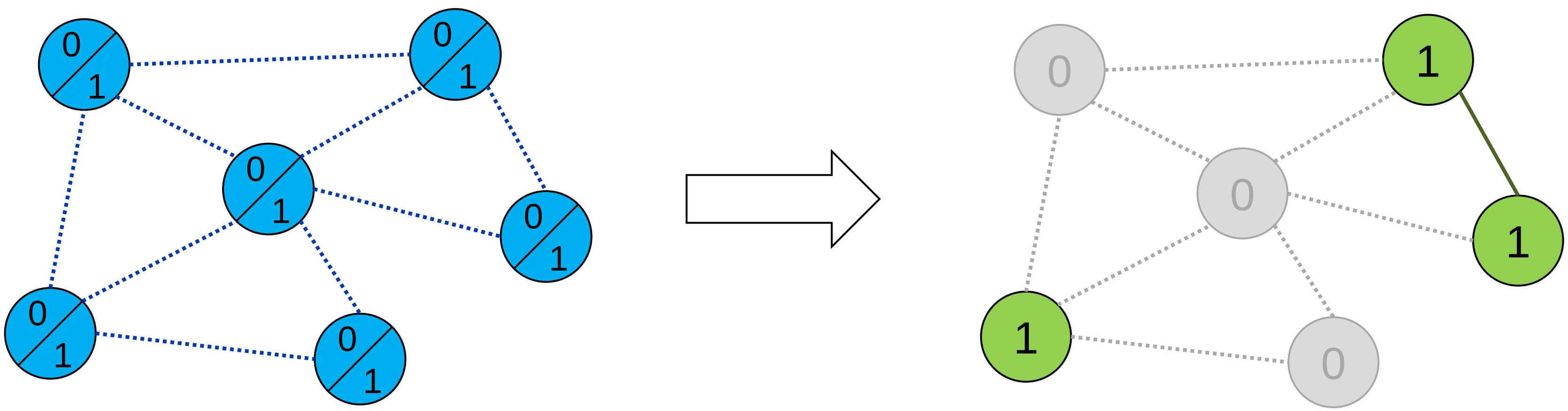
- Measurement changes the state

  $\rightarrow$ Observe „0“ with probability $|a|^2$
  $\Rightarrow |\varphi\rangle = |0\rangle$
  $\rightarrow$ Observe „1“ with probability $|b|^2$
  $\Rightarrow |\varphi\rangle = |1\rangle$

- Multiple Qubits $|\chi\rangle_{12} = |\psi\rangle_1 \otimes |\varphi\rangle_2$

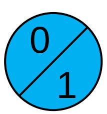
e.g. $|\chi\rangle_{12} = |0\rangle_1 \otimes |1\rangle_2$

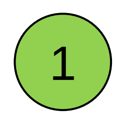
Adiabatic Optimisation Procedure



All qubits in „mixed state“

Solution

 $= \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right)$

 $= |1\rangle$

Farhi et. al., Quantum Computation by Adiabatic Evolution (2001)

Why use an Adiabatic Quantum Computer?

- QUBO / Ising is NP-hard

- Quantum speed-up assumed but subject to discussion

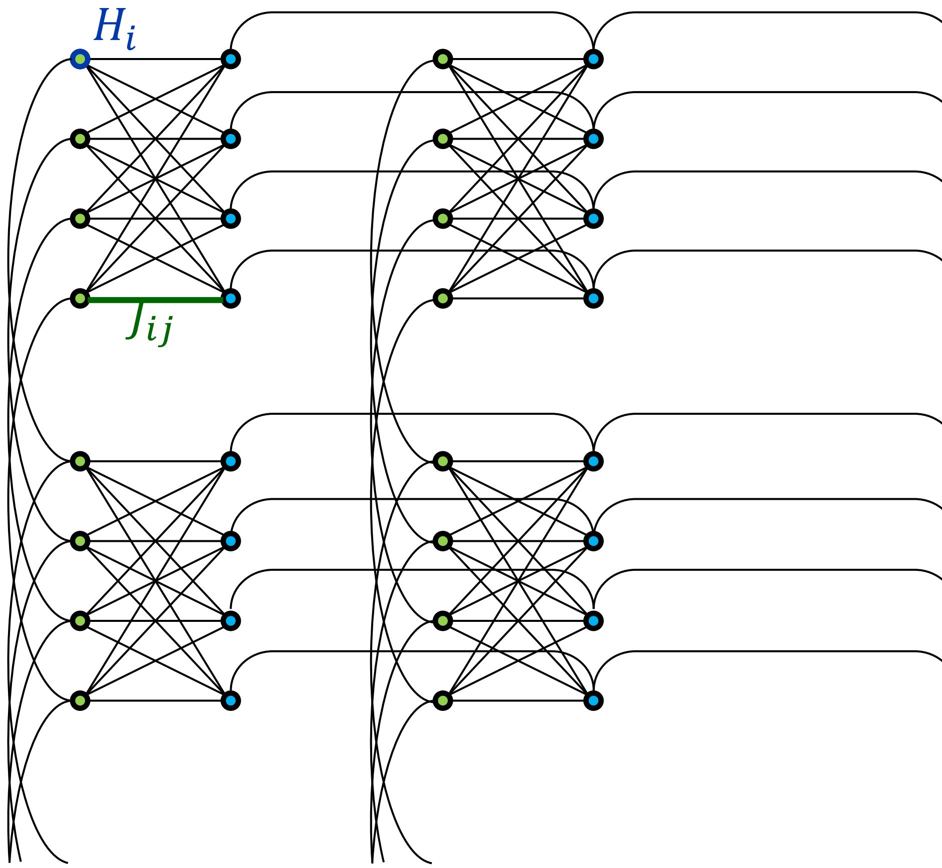
  (Defining and detecting quantum speedup, Rønnow et. al., Science 345, 1695, (2014))

- Map other NP-hard problems to QUBO/Ising

  (Ising Formulations of many NP problems, A. Lucas, Frontiers in Physics (2014))

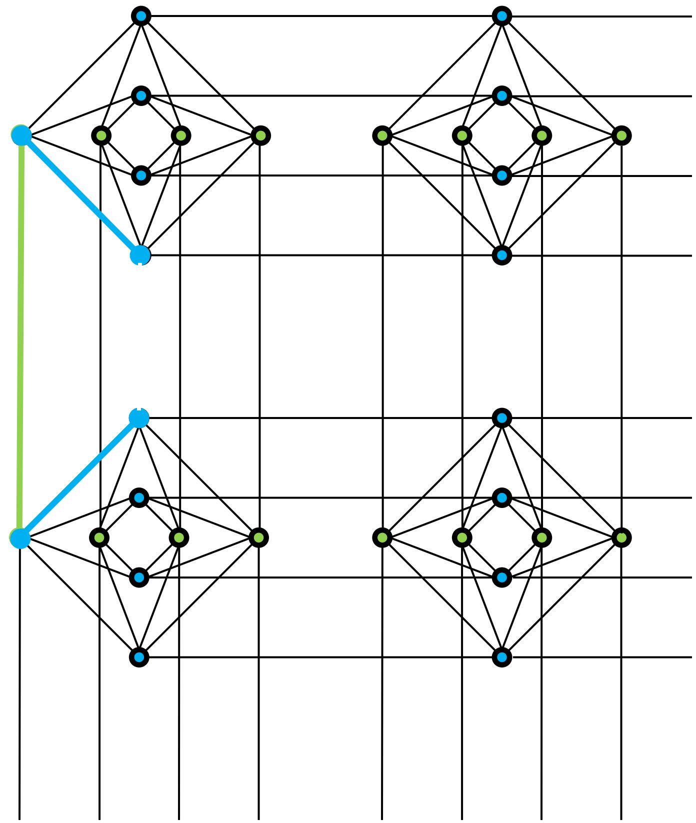
Hardware limitations on a D-Wave device

- *Unit cell* with 8 Qubits
  an 2 partitions
- 8x8 Unit cells on D-Wave On chip
  $\Rightarrow$ 512 Qubits
- Range of strenghts H_i and couplings J_{ij} limited and subject to uncertainties



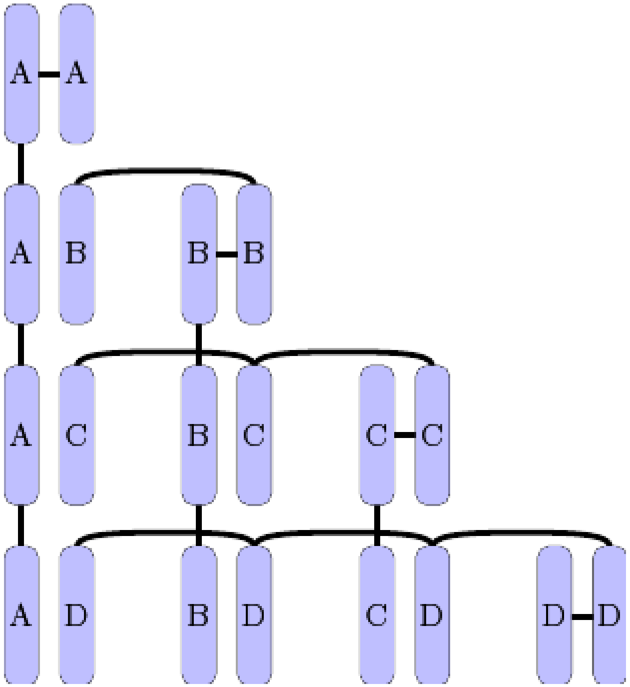
Hardware limitations on a D-Wave device

- Not all Qubits are connected
- How to realise complete graphs?
- Solution:
  - Connection via other qubits
  - Representation of a *logical* qubit with several physical qubits



Overcome connectivity limitation

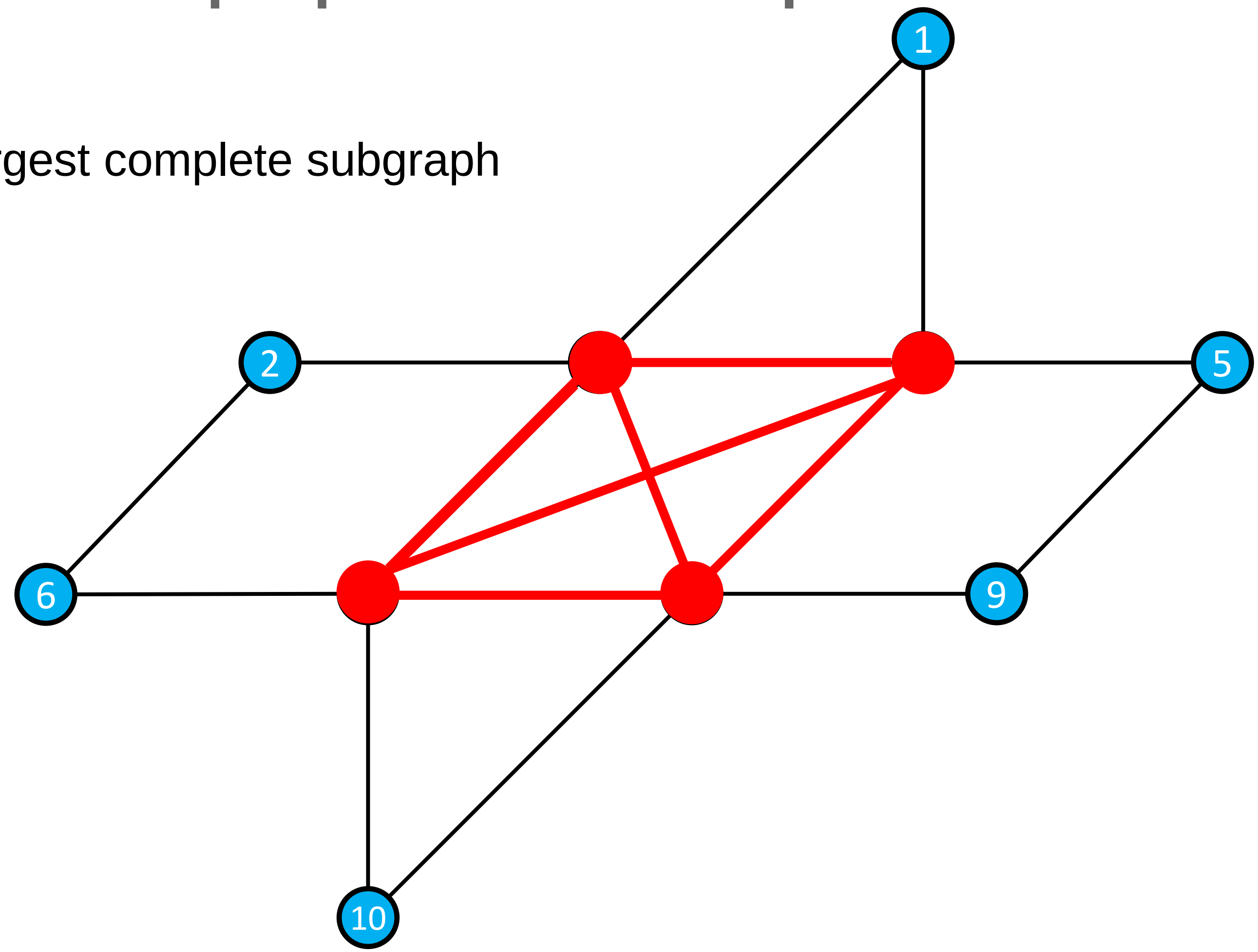
- Rule for implementing a complete graph onto the D-Wave hardware:



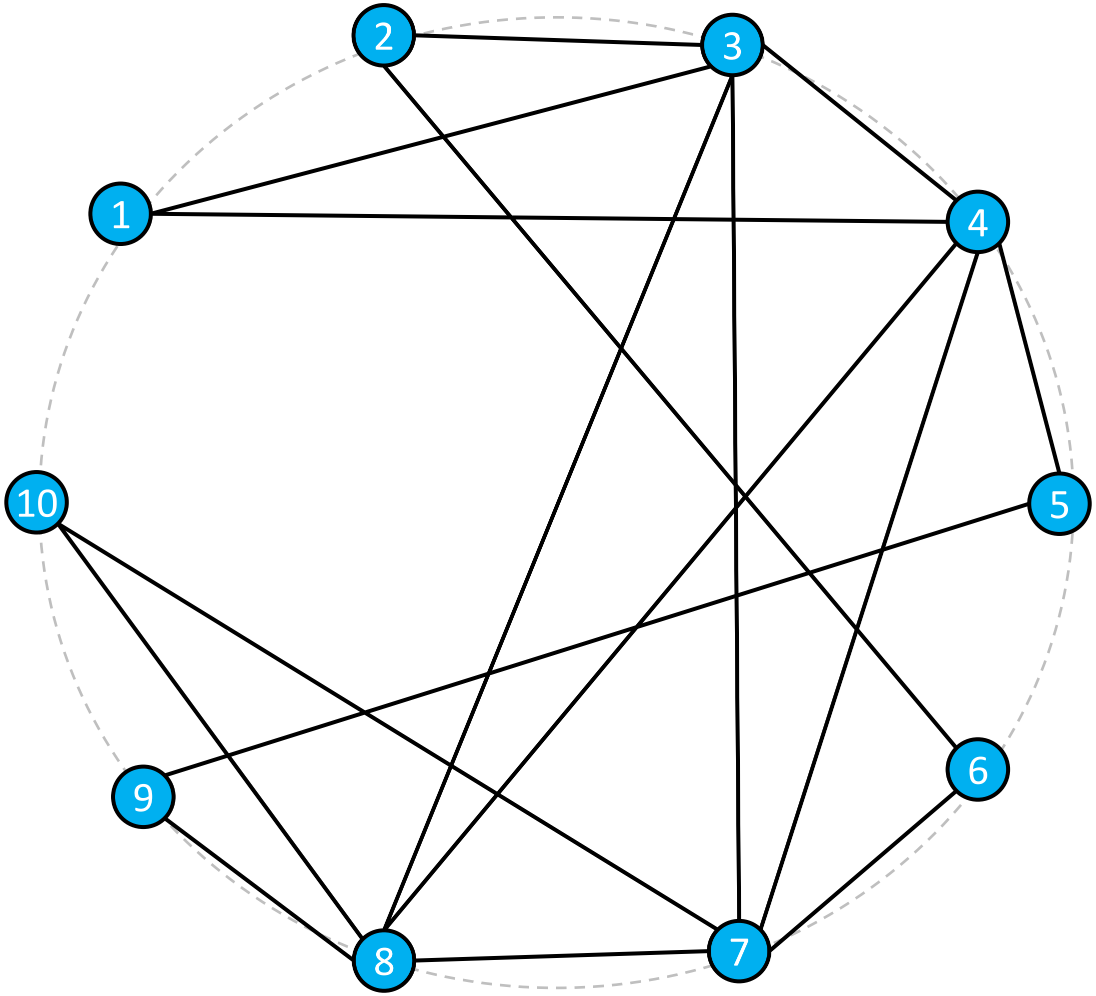
Source: D-Wave Sys. Simulator Software Documentation

Maximum Clique problem - Example

- Find largest complete subgraph



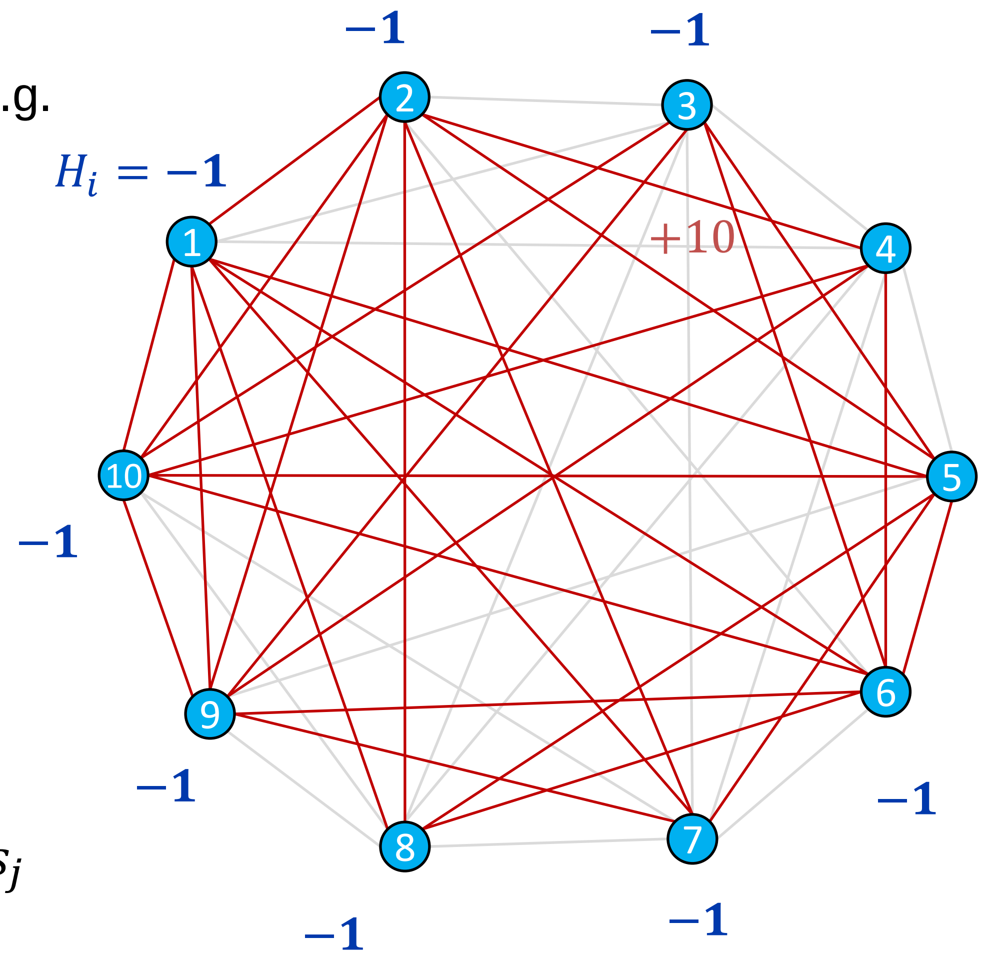
Maximum Clique problem - Example



Clique problem as QUBO

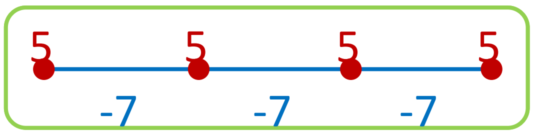
- Set negative strength at all nodes, e.g. $H_i = -1$
- Penalise all edges of graph complement with positive couplings, e.g. $J_{ij} = +10$
- Edges of the graph itself can be activated without penalty,

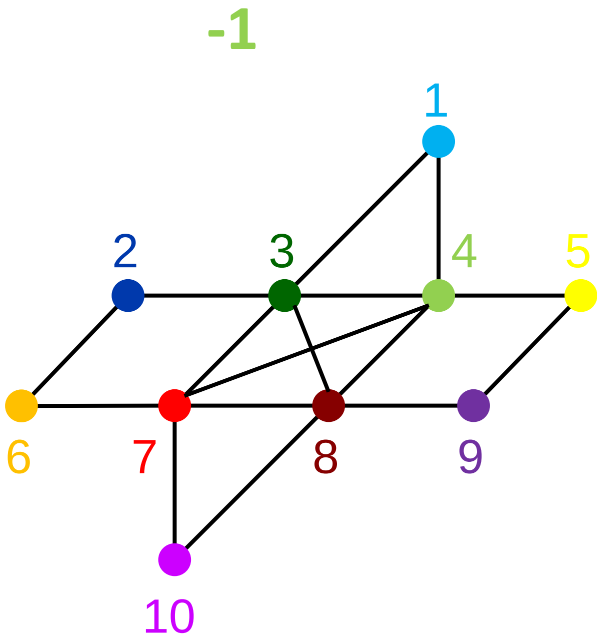
$$E(s_1, s_2, \ldots, s_n) = \sum_i H_i s_i + \sum_{ij} J_{ij} s_i s_j$$

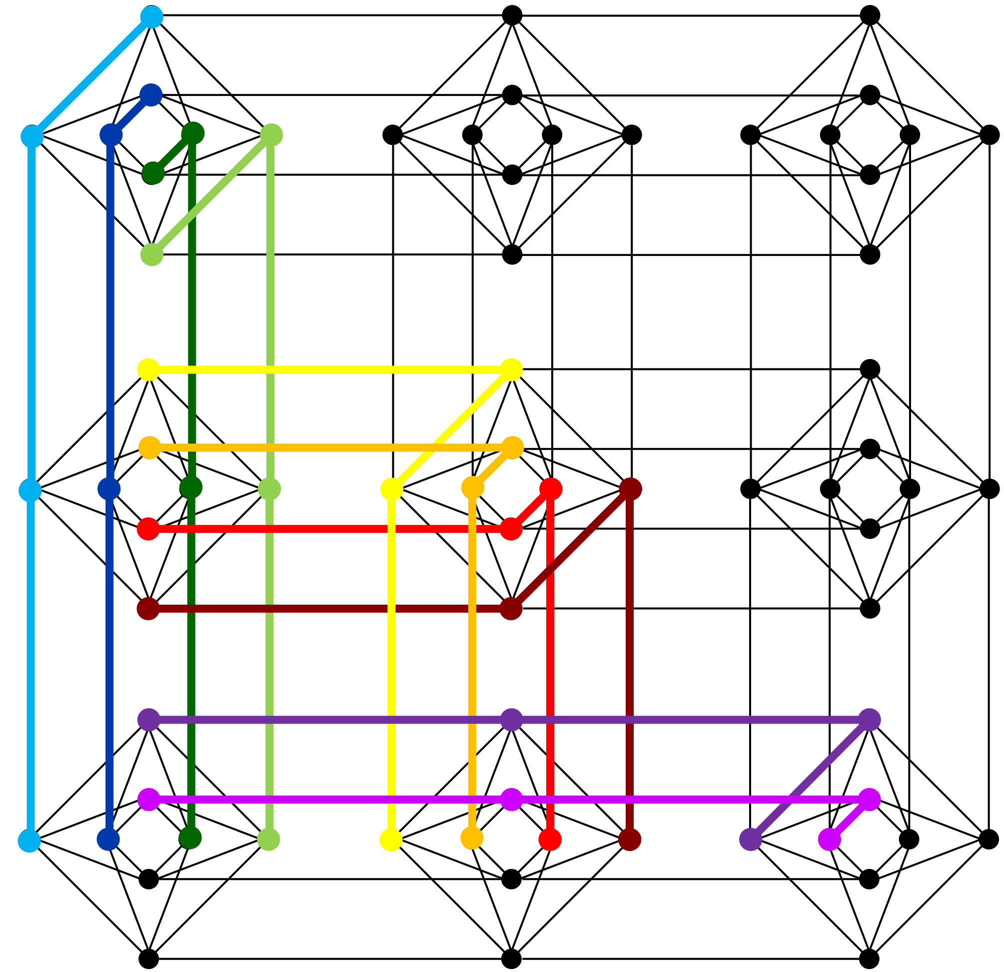


Cliquen Problem on D-Wave Device

- 1 logical qubit → 4 physical qubits







Outlook

- What kind of problems can be converted to QUBO?
- Combine conventional algorithms with AQC (hybrid approach)
- Investigate scaling behaviour