



ECTRI – FEHRL – FERSI Young Researchers Seminar 2015

TRANSPORT USER BENEFITS MEASURE FOR TRAVEL DEMAND MODELS WITH CONSTRAINTS

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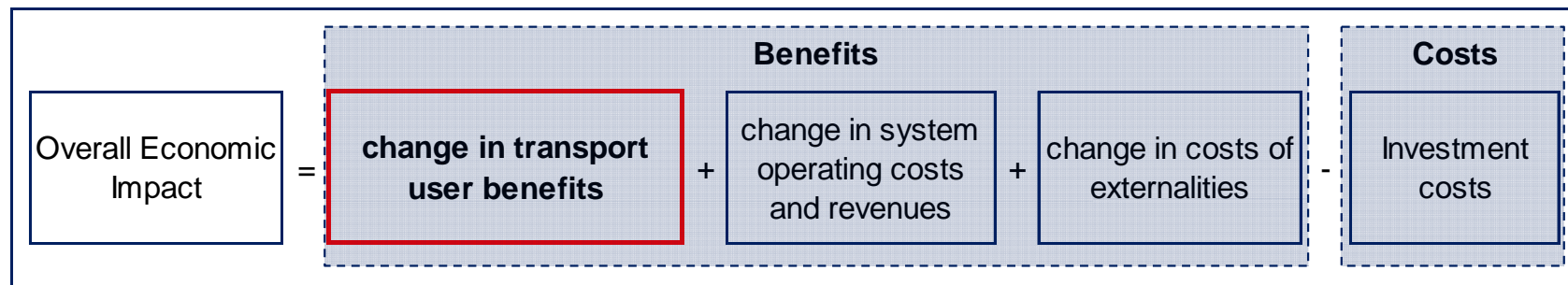
Institute of Transport Research

German Aerospace Center

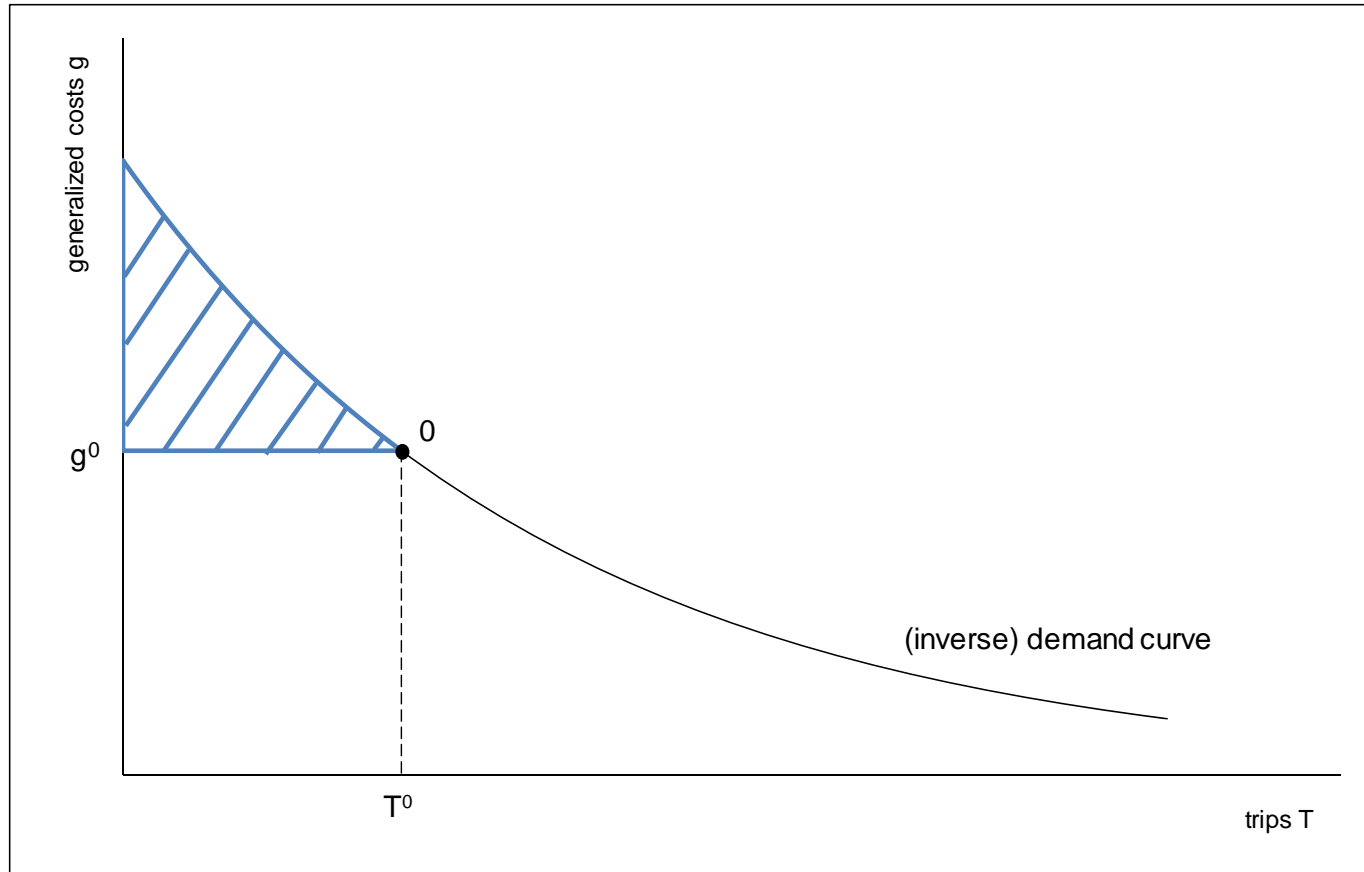


Overview

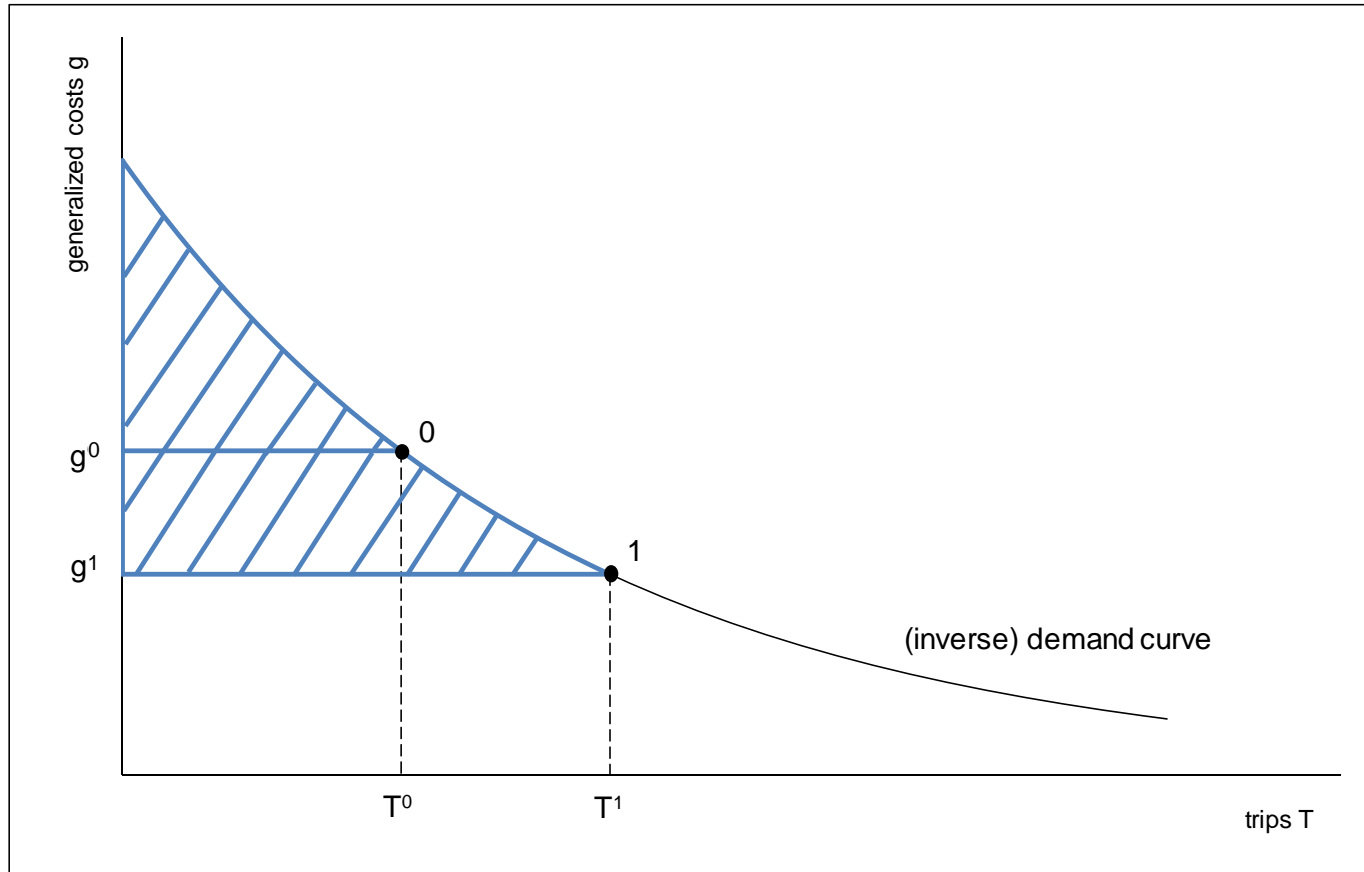
- Transport investments are often associated with high investment costs for society and cost changes for transport users
- Anticipation of welfare impacts is essential
- Decision makers need robust and easy decision advice
- Specific guidelines and rules are implemented in most countries
- Cost benefit analysis is the standard approach
- The basic calculation is:



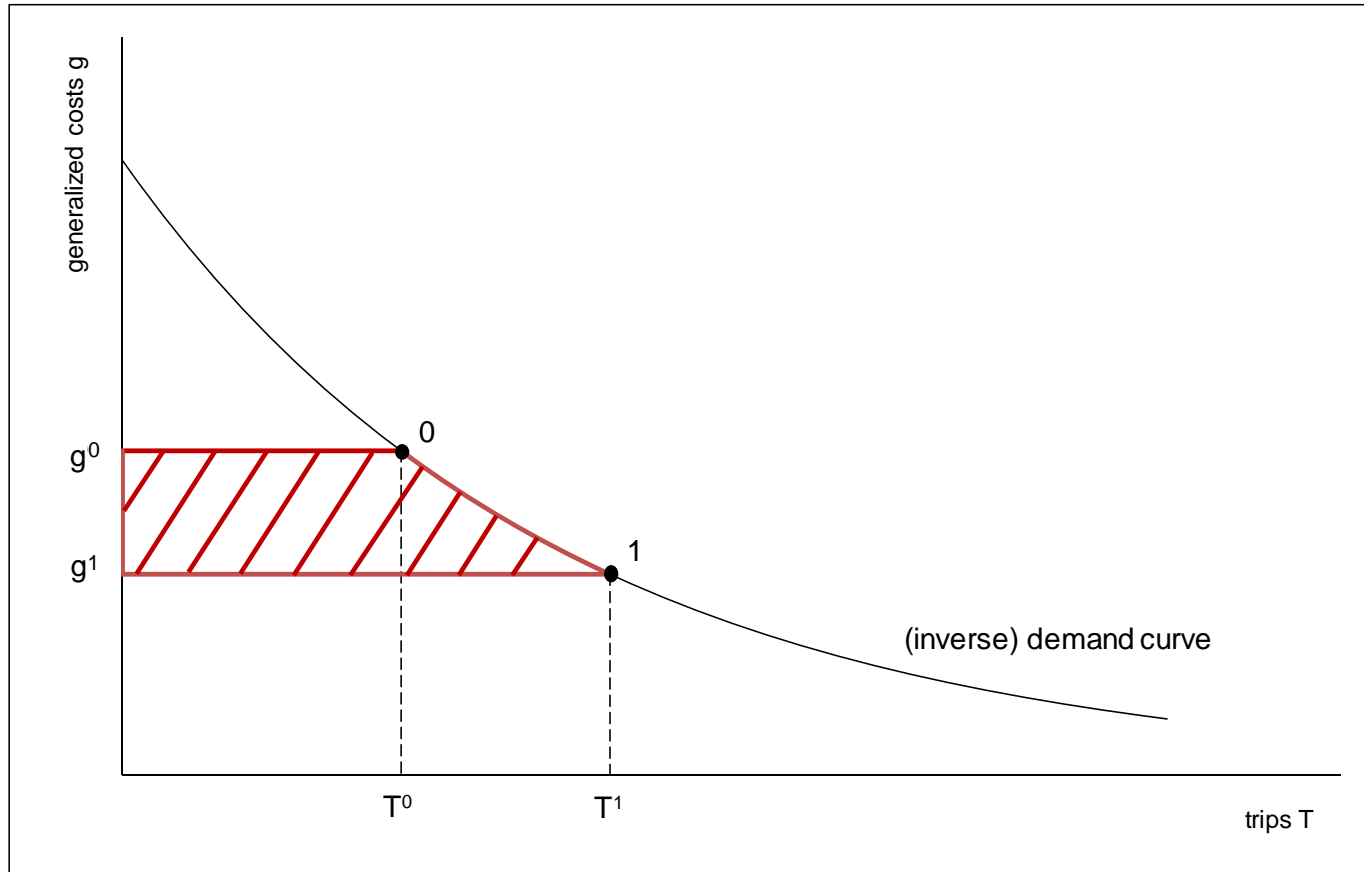
Consumer surplus – initial state



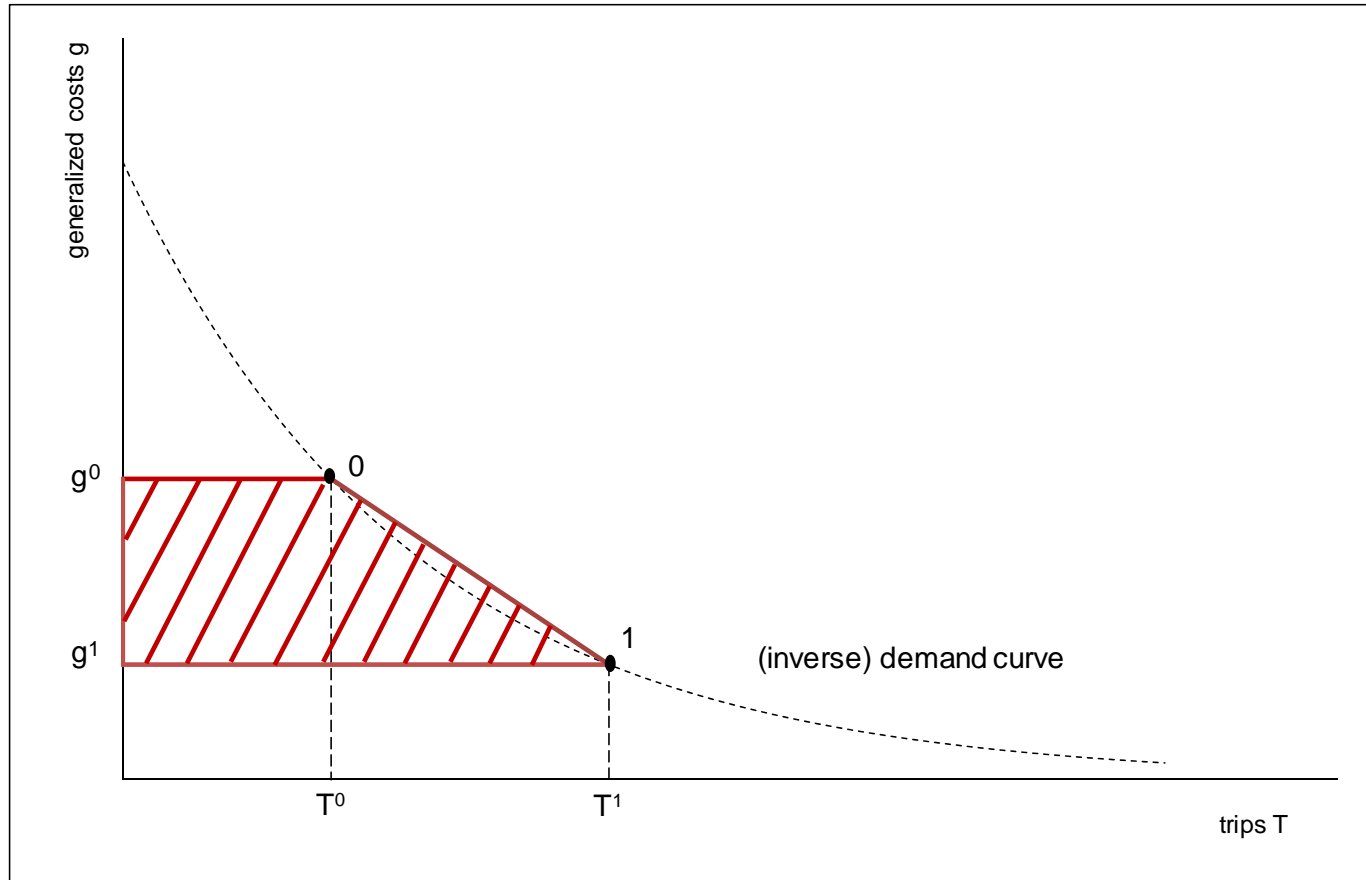
Consumer surplus – final state



Change in Consumer surplus



The Rule of the Half



Aim of the Study

- combine travel demand and benefits calculation for constrained travel demand models
- On the base of a trebly constrained model (EVA model)
- Change in consumer surplus as a benefit measure
- Compute a mathematical exact integration of the travel demand function



EVA logit model

EVA model

$$\begin{array}{l}
 T_{ijk} = f(g_{ijk}) \cdot a_i \cdot b_j \cdot c_k \\
 \left. \begin{array}{l}
 \sum_j \sum_k T_{ijk} = O_i \\
 \sum_i \sum_k T_{ijk} = D_j \\
 \sum_i \sum_j T_{ijk} = M_k
 \end{array} \right\} \text{constraints}
 \end{array}$$

- Consideration of different sets of constraints
- Utility maximization?

Logit model

$$\begin{array}{l}
 T_{ijk} = P_{ijk} \cdot T \\
 = \frac{e^{V_{ijk}}}{\sum_{i'} \sum_{j'} \sum_{k'} e^{V_{i'j'k'}}} \cdot T
 \end{array}$$

- Consideration of only one set of constraints
- Random utility maximization



EVA logit model

- Definition of a specific utility function
- Consideration of shadow prices

$$V_{ijk} = -g_{ijk} + \theta_i + \tau_j + \rho_k \quad \forall i, j, k$$

- The EVA logit model is then:

$$T_{ijk} = P_{ijk} \cdot T = \frac{e^{(-g_{ijk} + \theta_i + \tau_j + \rho_k)}}{\sum_{i'} \sum_{j'} \sum_{k'} e^{(-g_{ij'k'} + \theta_{i'} + \tau_{j'} + \rho_{k'})}} \cdot T$$

$$\sum_j \sum_k T_{ijk} = O_i$$

$$\sum_i \sum_k T_{ijk} = D_j$$

$$\sum_i \sum_j T_{ijk} = M_k$$



Change in consumer surplus

- The change in consumer surplus is mandatory for defining welfare impacts
- The mathematical integration of the demand function is necessary
- Use either the single integration of each alternative or the antiderivative
- The antiderivative of the logit model is known (“logsum term”)
- The change in consumer surplus is then (“logsum difference”):

$$E(\Delta CS) = \left[\ln \left(\sum_i \sum_j \sum_k e^{V_{ijk}^1} \right) - \ln \left(\sum_i \sum_j \sum_k e^{V_{ijk}^0} \right) \right] \cdot T$$



Change in consumer surplus

- However: logsum difference fails for constrained models
- Reason: shadow prices for satisfying constraints
- Only the real benefit caused by changes in generalized costs is targeted
- A detailed analysis of the integral of the demand function is necessary
- Segmentation of the integral according to single variables



Change in consumer surplus

- Logsum difference approach with shadow prices can be partitioned into:

$$E(\Delta CS_r^*) = - \int_{g^0}^{g^1} \sum_i \sum_j \sum_k P_{ijk} dg_{ijk} + \int_{\theta^0}^{\theta^1} \sum_i \sum_j \sum_k P_{ijk} d\theta_i + \int_{\tau^0}^{\tau^1} \sum_i \sum_j \sum_k P_{ijk} d\tau_j + \int_{\rho^0}^{\rho^1} \sum_i \sum_j \sum_k P_{ijk} d\rho_k$$

- Possible simplifications:

$$E(\Delta CS_r) = E(\Delta CS_r^*) - \int_{\theta^0}^{\theta^1} \sum_i P_i d\theta_i - \int_{\tau^0}^{\tau^1} \sum_j P_j d\tau_j$$

- In the case of inelastic constraints it is:

$$E(\Delta CS_r^*) = 0, \quad P_i = \frac{O_i}{T} = \text{constant}, \quad P_j = \frac{D_j}{T} = \text{constant}$$



Change in consumer surplus

- It follows:

$$E(\Delta CS_r) = \sum_i \frac{O_i}{T} \cdot (\theta_i^0 - \theta_i^1) + \sum_i \frac{D_i}{T} \cdot (\tau_i^0 - \tau_i^1)$$

- Overall change in consumer surplus:

$$E(\Delta CS) = \sum_i O_i \cdot (\theta_i^0 - \theta_i^1) + \sum_i D_i \cdot (\tau_i^0 - \tau_i^1)$$

- The result is valid for inelastic constraints and constant shadow prices for modes
- Solutions for all different kinds of constraints have been derived



Example

- Synthetic example:
 - five travel zones
 - inelastic origin and destination constraints
 - one mode
 - travel time and travel cost
 - value of time of 10 Euro/h (0.1667 Euro/min)

initial state

generalized costs [Euro]

gij	1	2	3	4	5
1	4,08	3,25	6,25	5,50	3,25
2	4,08	4,08	4,78	4,58	5,00
3	6,25	4,37	4,08	4,00	4,00
4	5,50	4,58	4,42	4,08	4,08
5	4,50	6,33	4,00	3,25	3,67

final state

generalized costs [Euro]

gij	1	2	3	4	5
1	4,08	3,25	6,25	3,25	3,25
2	4,08	4,08	4,78	4,58	5,00
3	6,25	4,37	4,08	4,00	4,00
4	3,25	4,58	4,42	4,08	4,08
5	4,50	6,33	4,00	3,25	3,67



Example – EVA logit model

- Calculation of trips by the EVA logit model – **initial state**

Tij	1	2	3	4	5		Oi-resulting	Oi-given	θ_i
1	3,03	32,35	1,93	1,38	11,31		50,00	50,00	2,611
2	9,81	45,52	27,14	11,17	6,36		100,00	100,00	3,785
3	0,44	13,46	21,46	7,86	6,79		50,00	50,00	2,850
4	2,30	26,68	37,85	17,80	15,37		100,00	100,00	3,751
5	9,42	6,99	86,62	61,79	35,18		200,00	200,00	4,162
						500,00	500,00	500,00	
Dj-resulting	25,00	125,00	175,00	100,00	75,00	500,00			
Dj-given	25,00	125,00	175,00	100,00	75,00	500,00			
τ_j	2,581	4,116	4,299	3,211	3,065				



Example – EVA logit model

- Calculation of trips by the EVA logit model – **initial state**

Tij	1	2	3	4	5		Oi-resulting	Oi-given	θ_i
1	3,03	32,35	1,93	1,38	11,31		50,00	50,00	2,611
2	9,81	45,52	27,14	11,17	6,36		100,00	100,00	3,785
3	0,44	13,46	21,46	7,86	6,79		50,00	50,00	2,850
4	2,30	26,68	37,85	17,80	15,37		100,00	100,00	3,751
5	9,42	6,99	86,62	61,79	35,18		200,00	200,00	4,162
						500,00	500,00	500,00	
Dj-resulting	25,00	125,00	175,00	100,00	75,00	500,00			
Dj-given	25,00	125,00	175,00	100,00	75,00	500,00			
τ_j	2,581	4,116	4,299	3,211	3,065				

$$\begin{aligned}
 T_{11} &= P_{11} \cdot T \\
 &= \frac{e^{(-4,08 \text{ Euro} + 2,611 \text{ Euro} + 2,581 \text{ Euro})_{11}}}{\sum_{i'} \sum_{j'} e^{(-g_{ij'} + \theta_i + \tau_{j'})}} \cdot 500 \text{ trips} \\
 &= 3,03 \text{ trips}
 \end{aligned}$$



Example – EVA logit model

- Calculation of trips by the EVA logit model – **final state**

Tij	1	2	3	4	5		Oi-resulting	Oi-given	θ_i
1	1,45	27,93	1,57	9,62	9,43		50,00	50,00	2,386
2	5,90	49,43	27,67	10,33	6,67		100,00	100,00	3,790
3	0,26	14,29	21,39	7,10	6,96		50,00	50,00	2,833
4	11,59	25,58	34,07	14,53	14,23		100,00	100,00	3,632
5	5,80	7,77	90,30	58,41	37,72		200,00	200,00	4,190
						500,00	500,00	500,00	
Dj-resulting	25,00	125,00	175,00	100,00	75,00	500,00			
Dj-given	25,00	125,00	175,00	100,00	75,00	500,00			
τ_j	2,068	4,194	4,313	3,128	3,107				



Example – change in consumer surplus

- Change in consumer surplus calculated by the adjusted logsum difference

total number of trips originating at zone i	
O ₁	50
O ₂	100
O ₃	50
O ₄	100
O ₅	200

difference of origin shadow prices	
$\theta_{1}^O - \theta_{1}^M$	0,224
$\theta_{2}^O - \theta_{2}^M$	-0,005
$\theta_{3}^O - \theta_{3}^M$	0,018
$\theta_{4}^O - \theta_{4}^M$	0,120
$\theta_{5}^O - \theta_{5}^M$	-0,027

ΔCS_i [Euro]
11,22
-0,49
0,88
11,96
-5,45
$\Sigma(i)$
18,11

total number of trips attracted to zone j	
D ₁	25
D ₂	125
D ₃	175
D ₄	100
D ₅	75

difference of destination shadow prices	
$\tau_{1}^O - \tau_{1}^M$	0,513
$\tau_{2}^O - \tau_{2}^M$	-0,078
$\tau_{3}^O - \tau_{3}^M$	-0,014
$\tau_{4}^O - \tau_{4}^M$	0,083
$\tau_{5}^O - \tau_{5}^M$	-0,042

ΔCS_j [Euro]
12,82
-9,69
-2,52
8,34
-3,19
$\Sigma(j)$
5,76

$\Sigma(i,j)$ **23,87** Euro

$$E(\Delta CS) = \sum_i O_i \cdot (\theta_i^1 - \theta_i^0) + \sum_j D_j \cdot (\tau_j^1 - \tau_j^0)$$



Example – change in consumer surplus

- Change in consumer surplus calculated by the Rule of a Half

Δg_{ij}	1	2	3	4	5
1	0,00	0,00	0,00	2,25	0,00
2	0,00	0,00	0,00	0,00	0,00
3	0,00	0,00	0,00	0,00	0,00
4	2,25	0,00	0,00	0,00	0,00
5	0,00	0,00	0,00	0,00	0,00

$\Sigma T_{ij}/2$	1	2	3	4	5
1	2,24	30,14	1,75	5,50	10,37
2	7,85	47,47	27,41	10,75	6,51
3	0,35	13,87	21,42	7,48	6,87
4	6,94	26,13	35,96	16,17	14,80
5	7,61	7,38	88,46	60,10	36,45

ΔCS	1	2	3	4	5
1	0,00	0,00	0,00	12,38	0,00
2	0,00	0,00	0,00	0,00	0,00
3	0,00	0,00	0,00	0,00	0,00
4	15,62	0,00	0,00	0,00	0,00
5	0,00	0,00	0,00	0,00	0,00

$$E(\Delta CS) = \sum_i \sum_j \frac{1}{2} \cdot (g_{ij}^0 - g_{ij}^1) \cdot (T_{ij}^0 + T_{ij}^1)$$

28,00 Euro

- Benefit is overestimated by 17%



Conclusion

- Multiple-constrained travel demand models are expressible in terms of a logit model
- Derivation of the “adjusted logsum difference” as a mathematical exact integral of the demand function of constrained models
- Applicable for all constrained models
- Allows measuring benefits caused by changing costs and changing attractions
- The approach is easy to use and to implement in existing models





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