IMPACT OF TEC GRADIENTS AND HIGHER-ORDER IONOSPHERIC DISTURBANCES ON SPACEBORNE SINGLE-PASS SAR INTERFEROMETRY

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ABSTRACT
This paper analyzes the impact of a spatially inhomogeneous ionosphere on spaceborne single-pass SAR interferometry. For this, linear TEC gradients and higher-order irregularities are considered. It is shown that TEC gradients as low as 0.01 TECU/km may already noticeably affect the accuracy of an L-band cross-track interferometer, causing, e.g., horizontal and vertical offsets in the order of 1-2 m. Higher-order perturbations of the electron plasma lead to additional errors that vary nonlinearly with the length of the interferometric baseline. To predict these errors, we model the ionospheric irregularities as the product of a vertical profile and a second-order stationary stochastic process with a 3-D power-law spectrum. The interferometric errors are then derived via a set of projection integrals that express the expected phase error variance as a function of the baseline length and the angular extent of the synthetic aperture. With this model, we show that the phase errors of an L-band single-pass SAR interferometer may reach several tens of degrees under medium turbulence conditions. Since these phase errors are highly correlated among neighboring resolution cells, they cannot be reduced by multi-looking, thereby posing a possible challenge for multiple interferometer and SAR tomography.

Index Terms — SAR interferometry, ionosphere, TEC gradients, scintillation, irregularity spectrum, turbulence

1. INTRODUCTION
It has often been claimed that single-pass SAR interferometry is, up to a common range shift of both interferometric channels, not affected by the atmosphere. In [1], it has, however, been shown that the slightly different ray paths through a vertically stratified and horizontally homogeneous troposphere can cause notable differential phase and range offsets. Small differential tropospheric delays explain, for example, the systematic height errors of up to three meters that have been observed in the TanDEM-X mission [2]. Single-pass SAR interferometers operating at lower frequencies, like the planned Tandem-L and SAOCOM-CS missions, may furthermore be affected by differential ionospheric propagation effects. Since the ionosphere is a dispersive medium, one must here distinguish between differential range delay and interferometric phase advance, which cause opposite DEM offsets for SAR interferometry and radargrammetry. Such offsets have, together with additional intricacies from spectral shift, been analyzed in [1] for the case of a spatially homogeneous ionosphere, where it has been shown that height errors of several tens of meters must be expected at times of moderate to strong solar activity. In this paper, we extend these analyses to a space-variant ionosphere, where we consider in Section 2 linear slopes of the total electron content (TEC), and in Section 3 higher-order electron density perturbations. Due to space limitations, we restrict our discussion to differential effects that arise for the phase of a single-pass SAR interferometer. The impact of TEC variations on the focusing of single SAR images has already been treated in detail in the literature [3], [4], [5], [6].

2. LINEAR TEC GRADIENTS
We consider first a linear TEC gradient in the cross-track direction. Figure 1 illustrates how the rays from the two antennas intersect the ionosphere which is in this section modelled as a thin spherical shell layer. The distance \( \Delta x \) separating the two intersection points between the rays and the ionospheric shell can be approximated as

\[
\Delta x \approx \frac{h_{\text{iono}}}{h_{\text{sat}} \cdot \cos \theta_i} \cdot B \perp \quad (1)
\]

If the gradient of the (vertical) TEC in range direction \( x \) is given by \( \frac{dT E C}{dx} \), we obtain for the slant TEC difference \( \Delta \text{STEC} \) between the two acquisitions

\[
\Delta \text{STEC} \approx \frac{h_{\text{iono}}}{h_{\text{sat}} \cdot \cos^2 \theta_i} \frac{dT E C}{dx} \cdot B \perp \quad (2)
\]
The corresponding horizontal and vertical DEM offsets can then be derived in analogy to the derivations in [1] as

$$\Delta x_{\text{horizontal}} = \pm \frac{K}{f^2} \frac{\partial \text{TEC}}{\partial x} \frac{h_{\text{iono}}}{h_{\text{sat}}} r_{\text{slant}} \frac{1}{\cos \theta_i^2} \tag{3}$$

$$\Delta x_{\text{vertical}} = \pm \frac{K}{f^2} \frac{\partial \text{TEC}}{\partial x} \frac{h_{\text{iono}}}{h_{\text{sat}}} r_{\text{slant}} \tan \theta_i^2 \cos \theta_i^2$$

where $f$ denotes the radar carrier frequency, $r_{\text{slant}}$ the slant range, and $K = 40.28 \text{ m}^2/\text{s}^2$. The positive signs apply to radargrammetry and the negative signs to interferometry.

Note that the DEM offsets are, as for the case of a constant TEC, independent of the baseline length. Figure 2 shows the predicted horizontal and vertical interferometric DEM offsets for an L-band interferometer operating at two different TEC gradients. It becomes clear that TEC gradients above 0.01 TECU/km can already lead to noticeable horizontal and vertical errors in the order of 1-2 meters that increase with increasing incident angles. Typical TEC gradients are characterized by a high degree of spatial and temporal variability, ranging from less than 0.01 TECU/km up to 0.08 TECU/km; under extreme conditions TEC gradients may reach values of 0.5 TECU/km [7], [8]. From this, it becomes clear that ionospheric effects should be taken into account when designing bistatic L-band SAR missions like Tandem-L or SAOCOM-CS.

A TEC gradient in azimuth will, to first order, add a linear phase ramp to each synthetic aperture. As a result, both SAR images, and hence also the DEM, will be shifted in azimuth. Assuming zero-Doppler geometry and no TEC variations in range, the two SAR image acquisitions will be affected by (almost) the same phase gradients, and therefore no additional height offsets beyond those of a constant ionosphere are expected. This applies even to the case of a bistatic acquisition with a non-vanishing along-track baseline, as long as the centroid of the common processed Doppler spectrum does not become too large. Joint azimuth and range gradients can, however, cause additional errors, e.g., due to mis-registration in azimuth.

### 3. Ionospheric Turbulence

The equatorial, auroral and polar ionosphere are often characterized by notable ionospheric irregularities [9], [10]. To study the impact of such irregularities on single-pass SAR interferometry, we assume a so-called frozen ionosphere which exhibits only spatial but no temporal fluctuations of the electron density within the observation interval. Referring to Figure 3, we model the ionosphere as a locally flat slab with a vertical electron density profile $g(z)$. The local electron density irregularities are moreover described by a 3-D homogeneous stochastic electron density distribution $N_e(x, y, z)$. The different incident angles at height $z_0$ are denoted by $\theta_1$ and $\theta_2$, while the angles $\xi_1$ and $\xi_2$ refer to the formation of the synthetic aperture.

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$$C_{N_e}(\Delta x, \Delta y, \Delta z) = E[N_e(x, y, z) \cdot N_e(x + \Delta x, y + \Delta y, z + \Delta z)]$$

where $E[\cdot]$ denotes the expectation operator. In the ionospheric literature it is common praxis to use instead of the autocorrelation function $C_{N_e}(\Delta x, \Delta y, \Delta z)$ the 3-D electron density power spectrum $S_{N_e}(f_x, f_y, f_z)$ which is...
related to $C_{N_e}$ by a 3-D inverse Fourier transform [9]

$$C_{N_e}(\Delta x, \Delta y, \Delta z) = \iiint \limits_{-\infty}^{\infty} S_{N_e}(f_x, f_y, f_z) \cdot \exp\left(j 2\pi (f_x \Delta x + f_y \Delta y + f_z \Delta z)\right) \cdot df_x df_y df_z$$

(5)

Based on this second-order stochastic model, we may now analyze the statistical dependencies of the phase fluctuations for two rays that penetrate the ionosphere as illustrated by the red arrows in Figure 3. To ease the analysis, we assume that the electron density variations, and therefore the local variations of the refractive index, are small within the size of the first Fresnel zone, which is in the order of 350 m for a typical L-band SAR. This allows us to use a simplified wave propagation model where we replace the wave equation and its forward-scattering solutions in terms of diffraction integrals by a geometric optics approximation which provides the range and phase offsets via a mere integration of the electron density along the ray paths [12]. We will later see that this approximation is well justified for the considered case of a moderately perturbed ionosphere where second-order correlation $C_\phi(\theta_1, \theta_2)$ of the phase fluctuations of the single-pass SAR interferometer can be derived in case of small phase errors as

$$C_\phi(\theta_1, \theta_2) = E[\varphi(\theta_1) \cdot \varphi(\theta_2)] = \iiint \limits_{-\infty}^{\infty} H_p(f_x, f_y, f_z; \theta_1, \theta_2) \cdot S_{N_e}(f_x, f_y, f_z) \cdot df_x df_y df_z$$

(6)

where $H_p(f_x, f_y, f_z; \theta_1, \theta_2)$ is given by

$$H_p(f_x, f_y, f_z; \theta_1, \theta_2) = \frac{\lambda^2 r_e^2}{h_i^2 \cos \theta_1 \cos \theta_2} \int \limits_{-\infty}^{\infty} g(z') g(z'') \cdot \exp\left[j 2\pi f_z (z' \tan \xi' - z'' \tan \xi'')\right] \cdot \int \limits_{-\infty}^{\infty} h(\xi') h(\xi'') \cdot \exp\left[j 2\pi f_x (z' \tan \theta_1 - z'' \tan \theta_2) + f_y (z' - z'')\right] dz' dz''$$

Here, $\lambda$ denotes the wavelength, $r_e$ the classical electron radius, $\theta_1$ and $\theta_2$ the slightly different incident angles of the two SAR channels, $\xi$ the instantaneous squint/azimuth angle, $h(\xi)$ the combined azimuth weighting from the antenna pattern and the SAR processing filter, and $h_i$ a normalization constant that is given by the integral of $h(\xi)$ over all $\xi$. The phase correlation can therefore be derived from a weighted 3-D projection of the power spectral density $S_{N_e}(f_x, f_y, f_z)$. To get a first idea of how higher-order TEC variations may impact single-pass cross-track interferometry, we model the three-dimensional electron density spectrum by an isotropic power law model [13]

$$S_{N_e}(f_x, f_y, f_z) = \frac{A}{(f_x^2 + f_y^2 + f_z^2 + f_0^2)^{\frac{4}{3}}}$$

(7)

Figure 4 shows an example of the predicted standard deviation of the interferometric phase errors assuming the parameters provided in the figure caption. It becomes clear that already moderate levels of ionospheric disturbance can cause significant interferometric phase errors (the chosen parameters correspond to a vertically integrated strength of turbulence of $C_n L = 6.5 \cdot 10^{-3}$). It is important to note that these interferometric phase errors will not be reduced by multi-looking, since they are highly correlated among neighboring resolution cells. Irregularities in the ionosphere may therefore pose a potential challenge for both single-baseline DEM generation as well as multi-baseline SAR interferometry and tomography that combine the data from multiple single-pass acquisitions. The same applies to multistatic SAR systems that acquire multi-baseline data in a single pass of the satellite formation [15].
4. DISCUSSION

We have shown that ionospheric TEC gradients and higher-order plasma irregularities may have a notable impact on SAR images that are well suited to analyze higher-order perturbations in the ionosphere. Further opportunities arise from the combination of single and repeat-pass interferometric SAR data.

Besides these challenges, single-pass interferometry and tomography offer also new opportunities to investigate the ionosphere and its impact on wave propagation. One example is the examination of small-scale electron density perturbations by a satellite formation employing two or more platforms that are mutually displaced in the along-track direction [15], [16]. Figure 5 illustrates such a SAR train for the case of three fully active radar satellites that may be operated in alternating bistatic or a more advanced MIMO-SAR mode [17]. Since each satellite can transmit and receive, we obtain in total 3 monostatic and 6 bistatic SAR images (3 bistatic images are, however, expected to be, within the start-stop approximation, redundant due to reciprocity). If we further assume that the scene is stationary and all images are processed within the same Doppler band, we may, for small bistatic angles, adopt the monostatic-bistatic equivalence principle. We would, therefore, expect that all acquired mono- and bistatic SAR images are, up to mutual time shifts, equivalent. Deviations between the images could, however, occur due to the fact that the rays pass through different portions of the ionosphere. For this, one should note that spatial inhomogeneities in the ionosphere are often considered as temporally constant (or “frozen”), while the whole irregularity pattern may drift with velocities of several hundreds of meters per second, preferably in zonal directions [18].

REFERENCES