Classical thermal energy balance equation for thermoelectric devices

**Introduction**
- Performance of thermoelectric devices – in the framework of continuum theory
- TE effects - interference of two irreversible processes: Heat transport and charge carrier transport
- Onsager-de Groot-Callen theory: Thermoelectrics as a kind of „field theory “ in non-equilibrium thermodynamics
- Description via differential equations - thermal energy balance equation

**Thermal energy balance**
- Coupling of Fourier’s and Ohm’s law
- Transport coefficients - Onsager relations
- Local energy balance \( \frac{\partial \theta}{\partial x} = \mathbf{J} \cdot \mathbf{E} \)
- Divergence of the heat flux different terms \( \mathbf{V} \cdot \mathbf{q} = \mathbf{J} \cdot \mathbf{T} + \mathbf{J} \cdot \mathbf{E} - \frac{1}{\sigma} \mathbf{V} \cdot (\kappa \mathbf{E}) \)
- Representing Peltier, Thomson effects, Joule heating, Fourier heat conduction

**Performance of a TE element**
- Performance depends on:
  - Material properties
  - Working/boundary conditions like junction temperatures and heat fluxes, load resistance, electrical current
  - Contact quality (resistance)
  - Coupling to the surrounding (convection, radiation)
  - Geometry/shape of the TE elements

**Performance of thermoelectric elements**
- Constant properties model means no temperature (physical homogeneity) nor position dependence (chemical homogeneity) of the material properties
- Heat balance – analytic solution
- Exact Performance values of a TE element
- Commonly 1D case with constant cross-sectional area used, here TE generator
- Heat flow (hs) \( Q = K \Delta T + \iota A T \sim \iota^2 R \)
- Heat flow (cs) \( Q_c = K \Delta T + \iota A T \sim \iota^2 R \)
- B. power: \( P_d = Q_h - Q_c = (\Delta T - I R) I \)
- Efficiency: \( \eta = \frac{P_d}{\Delta S} \)
- Thermal conductance: \( K = \frac{\Delta S}{\iota T} \)
- Internal resistance: \( R = \frac{\iota}{\Delta S} Q \)

**Ioffe-CPM approximation**
- Representing Peltier, Thomson effects, Joule heating, Fourier heat conduction

**Optimum performance – Example**
- Maximum power output:
  \( I_{opt} = \frac{a \Delta T}{2 R_m} \)
- Maximum efficiency:
  \( \eta_{max} = \frac{t_a - t_h}{t_h - t_f} + \frac{1}{4 R_m} \)
- Example: \( T_h = 400K, T_c = 300K, \)
  \( L = 5mm, A_c = 1mm^2 \)

**Performance of elements with variable cross-section – Example: Truncated Cone**

**Quasi 1D approach**
- Generalized thermal energy balance
  \( \frac{d}{d z} \left[ K A_c(x) \right] = \frac{1}{l^2} \frac{\partial \theta}{\partial x} \)
- Truncated cone
  \( A_c(x) = A_{cm} + (x - L/2) s_A \)
- \( s_A = \frac{\Delta A_c}{L} \) ... shape parameter
- Definition of a generalized aspect ratio
  \( \Gamma = \int_0^L \frac{dz}{A_c(x)} = \frac{1}{s_A} \ln \left( \frac{A_{cm}}{A_{cm}} \right) \)

**FEM simulation of shaped elements**
- Comparison of the analytical quasi-1D model and 2D FEM simulation
- Distribution of heat flux - Integration

**FEM simulation for non-trivial geometry**
- Shaped elements – 2D or 3D calculation
- FEM software ANSYS workbench
- rotational symmetry 3D problem reduced to 2D

**FEM software ANSYS workbench**

**Variation of the shaped parameter**
- Differences between 1D analytical model and 2D simulation
- Efficiency independent of the shape
- Power output best for \( s_A = 0 \) (cylinder), contractions to [1]

**Performance calculation (1D, const. A_c) for an example material**

**Performance of thermoelectric elements**

**Performance of a TE element**

**Funding**

**Acknowledgments**

**References**

**Fig. 2 Scientists - Non-equilibrium Thermodynamics**

**Fig. 3 Thermocouple and thermoelectric leg**

**Fig. 4: Performance calculation (1D, const. A_c) for an example material**

**Fig. 5: 3D and 2D FEM calculation**

**Fig. 6: Shaped element - Truncated cone.**

**Fig. 7: Heat flux hot side (left) – cold side right.**

**Fig. 8: Comparison of the performance between 1D analytical model and 2D simulation.**