Development of the Adjoint Approach for Aeroelastic Wing Optimization

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Abstract In the context of gradient-based optimization techniques for coupled aero-structure problems an efficient approach was sought to evaluate the gradient of the cost function with respect to the design variables; also called the sensitivities. The traditional approach to calculate the sensitivities, the finite differences, can become prohibitively expensive in high-fidelity optimizations context. For this reason an existing flow adjoint approach was further developed in order to suit coupled aero-structural systems. Then the developed approach was evaluated and tested. The results showed that the approach can provide accurate sensitivities in a very efficient way.

1 Introduction

Computing the sensitivities for gradient-based multidisciplinary optimizations with the traditional finite differences approach can be extremely expensive. For a coupled aero-structural system the sensitivities require two converged coupled computations for each design parameter with the central finite differences approach. This means that such technique can become prohibitively expensive for high-fidelity problems with large number of design parameters because the computational power needed is linearly dependent on the number of design parameters.
To get rid of this expensive dependency, the discrete adjoint approach is investigated here. In this approach, a Lagrange formulation is defined and used in such a way to eliminate this dependency, where only one converged coupled computation and one adjoint computation are needed to compute the sensitivities. The flow adjoint technique has been successfully developed in the DLR CFD code TAU [2], then

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tested for NS equations in 3D configurations [3], and has proven to be very efficient and cheap in comparison to finite differences. For this reason, this approach was suggested to be further developed for coupled aero-structural systems in the frame of the internal DLR-funded project MDOrmec.

2 Definition and Formulation of the Problem

The system to be solved can be defined as follows. The state variables of the coupled system is the vector W=[w u]; where (w) represents the flow variables and (u) represents the structure displacements. The design variables of this system are defined by D =[A T] ; where (A) and (T) are the shape design variables and the thickness of the structure elements, respectively. The cost function to be optimized (I) can be defined as a function of both the state and the design variables; I=I(W,D). Similarly the aerodynamic residual (Ra) and the structure residual (Rs) are defined; Ra=Ra(W,A) and Rs=Rs(W,T). Here the aerodynamic residuals are described by the flow equations (Euler or Navier-Stokes) and the structure equation is described by

$$Rs = Ku - f = 0\,, \tag{1}$$

where (K) is the stiffness matrix, (u) is the structural displacement and (f) represents the forces on the structure nodes.
The gradient of the cost function with respect to the design variables can be written in the vector format as:

$$\frac{dI}{dD} = \frac{\partial I}{\partial D} + \left[\, \frac{\partial I}{\partial w} \;\; \frac{\partial I}{\partial u} \,\right] \begin{bmatrix} \frac{dw}{dD} \\ \frac{du}{dD} \end{bmatrix}\,. \tag{2}$$

On the other hand the gradients of the aerodynamic and the structure residuals with respect to the vector of design variables have the same format as that of the cost function. To formulate the coupled adjoint equation, a Lagrange (L) is defined as:

$$L = I + \Psi R\,, \tag{3}$$

where the Lagrange multiplier

$$\Psi = \left[\, \psi \;\; \phi \,\right] \tag{4}$$

contains both the aerodynamic ψ and structure ϕ multipliers respectively, and the vector R=[Ra Rs] represents the residuals vector that contains both aerodynamic and structure residuals, respectively. After defining the Lagrange, its gradient with respect to the design variables is sought. Having the residual vector $R = 0$ means that the gradient of the Lagrange is equal to that of the cost function. The gradient of the Lagrange is:

$$\frac{dL}{dD} = \frac{dI}{dD} = \left(\left[\frac{\partial I}{\partial A} \ \frac{\partial I}{\partial T} \right] + \left[\psi \ \phi \right] \begin{bmatrix} \frac{\partial Ra}{\partial A} & \frac{\partial Ra}{\partial T} \\ \frac{\partial Rs}{\partial A} & \frac{\partial Rs}{\partial T} \end{bmatrix} \right) \left(\begin{bmatrix} \frac{dA}{dD} \\ \frac{dT}{dD} \end{bmatrix} \right) + \left(\left[\frac{\partial I}{\partial w} \ \frac{\partial I}{\partial u} \right] + \left[\psi \ \phi \right] \begin{bmatrix} \frac{\partial Ra}{\partial w} & \frac{\partial Ra}{\partial u} \\ \frac{\partial Rs}{\partial w} & \frac{\partial Rs}{\partial u} \end{bmatrix} \right) \left(\begin{bmatrix} \frac{dw}{dD} \\ \frac{du}{dD} \end{bmatrix} \right) . \tag{5}$$

The expensive terms in this equation are the ones that relate the flow and the structure state variables to the design variables, namely (dw/dD) and (du/dD). They are expensive because they force the designer to perform one flow and structure computation for each design variable, which can be extremely expensive for high fidelity optimization. To get rid of these terms, the second part of the RHS in this equation is set to zero by finding the suitable vector Ψ that fulfils this condition. Since this vector was not defined so far, it can be chosen to satisfy any condition, and the condition here is to have the equation equal to zero:

$$\left[\frac{\partial I}{\partial w} \ \frac{\partial I}{\partial u} \right]^T + \left(\left[\psi \ \phi \right] \begin{bmatrix} \frac{\partial Ra}{\partial w} & \frac{\partial Ra}{\partial u} \\ \frac{\partial Rs}{\partial w} & \frac{\partial Rs}{\partial u} \end{bmatrix} \right)^T = 0 . \tag{6}$$

Equation (6) is called the coupled adjoint equation and is independent of the number of design variables. As soon as it is solved, the Lagrange multipliers vector Ψ is defined and used to solve equation (5) to get the gradients of the cost function w.r.t the design variables.

$$\frac{dI}{dD} = \left(\left[\frac{\partial I}{\partial A} \ \frac{\partial I}{\partial T} \right] + \left[\psi \ \phi \right] \begin{bmatrix} \frac{\partial Ra}{\partial A} & \frac{\partial Ra}{\partial T} \\ \frac{\partial Rs}{\partial A} & \frac{\partial Rs}{\partial T} \end{bmatrix} \right) \left(\begin{bmatrix} \frac{dA}{dD} \\ \frac{dT}{dD} \end{bmatrix} \right) . \tag{7}$$

This means that the solution of only one aero-structure coupling, one coupled adjoint system of equations and one gradient equation will provide the sensitivities.
To test the coupled adjoint approach, it was suggested, as a start, to consider a cost function that is exclusively related to aerodynamic terms, like drag, lift or a combination of both. The effect of the structure displacements will still be taken into account, and the resulting sensitivities will include information about the static aero elasticity of the problem. As a consequence, the terms that depend on the structure variables (T), will be pushed to zero, and the gradient equation (7) will shrink to:

$$\frac{dI}{dD} = \frac{\partial I}{\partial D} + \psi \frac{\partial Ra}{\partial D} + \phi \frac{\partial Rs}{\partial D} . \tag{8}$$

3 Implementation and Resolution of the Coupled Adjoint System

After defining the problem to solve, the next step was to identify the terms of the coupled adjoint system and to check which of these terms is already efficiently obtainable and which needs to be derived in order to avoid costly computations of these terms.

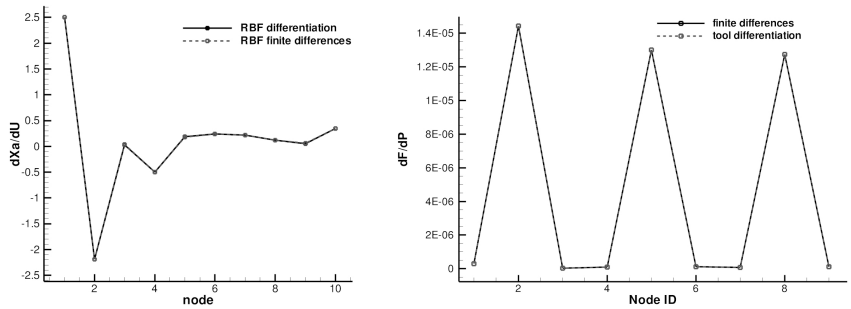
In the system of equations (6), the terms related to pure aerodynamic adjoint are already derived [2], namely ($\frac{\partial I}{\partial w}$) and ($\frac{\partial Ra}{\partial w}$), and the rest should be derived for the purpose of this work. The first term to be derived in this system of equations is the term ($\frac{\partial I}{\partial u}$) which represents the sensitivity of the cost function with respect to the displacement in the structural mesh. Another term that depends on this displacement is the term ($\frac{\partial Ra}{\partial u}$) which represents the sensitivity of the aerodynamic residuals with respect to the displacement in the structure mesh. If the displacement in the structural mesh changes, the aerodynamic mesh will also change and this affects both the aerodynamic cost function and the aerodynamic residual. Those two terms can be rewritten as:

$$\frac{\partial I}{\partial u} = \frac{\partial I}{\partial Xa}\frac{\partial Xa}{\partial u}\,,\ \frac{\partial Ra}{\partial u} = \frac{\partial Ra}{\partial Xa}\frac{\partial Xa}{\partial u}\,, \tag{9}$$

respectively, where Xa represents the aerodynamic volume mesh.
By knowing that the terms ($\frac{\partial I}{\partial Xa}$) and ($\frac{\partial Ra}{\partial Xa}$) are already derived in the flow solver TAU [6], it becomes clear that only one term ($\frac{\partial Xa}{\partial u}$) should be derived here, instead of deriving both ($\frac{\partial I}{\partial u}$) and ($\frac{\partial Ra}{\partial u}$). This term represents the sensitivity of the movement in the aerodynamic mesh with respect to that in the structural mesh, which means that this term depends on the interpolation algorithm used between CFD and CSM meshes. In this case the mesh interpolation tool used is the Radial Basis Function (RBF) tool developed in TAU.
The term ($\frac{\partial Xa}{\partial u}$) was found by differentiating the RBF interpolation tool as explained in [5], and the results were evaluated against values computed by the costly finite differences for some selected nodes on a 3D LANN wing test case. Figure (1a) shows excellent matching in the results.
The second term to be derived is ($\frac{\partial Rs}{\partial w}$) which represents the sensitivity of the structural residuals with respect to the change in the flow state variables. While it is possible in the DLR-TAU solver to exchange between the conservative and the primitive variables, this term was derived with respect to the primitive variables, as this made the derivation much easier.
According to equation (1), only the force term (f) is of interest here,

$$\frac{\partial Rs}{\partial w} = \frac{-\partial f}{\partial w} = -\frac{\partial f}{\partial p}\,, \tag{10}$$

where p is the pressure; the only primitive variable that directly affects the force. This term is found by differentiating the tool that interpolates the pressure from the CFD mesh into forces on the CSM mesh. In this work, a linear interpolation tool is used.
To test and evaluate this term, it was compared to the results of finite differences. The test was done on some nodes of the LANN wing in different directions and the result of this comparison can be seen in Figure (1b).
The last term needed to solve the coupled adjoint system of equations is ($\frac{\partial Rs}{\partial u}$) which represents the sensitivity of the structural residual with respect to the structural displacement. This term is equal to the stiffness matrix (K) that is present in the struc-



(a) evaluation of the differentiation of the Mesh interpolation tool (term $\frac{\partial Xa}{\partial u}$) for LANN wing

(b) evaluation of the differentiation of the pressure interpolation tool (term $\frac{\partial Rs}{\partial w}$) for LANN wing

Fig. 1: Evaluation of the derived terms

tural residual equation (1).
After deriving all the required terms in the described system of equations, a way to solve the coupled adjoint system of equations is investigated. Such system of equations is usually solved through the lagged iterative method [1]. After arranging the system of equations it will look as the following:

$$\begin{aligned} \frac{\partial Ra^T}{\partial w}\psi^{Tn} &= -\frac{\partial I^T}{\partial w} - \frac{\partial Rs^T}{\partial w}\phi^{Tn-1} \\ \frac{\partial Rs^T}{\partial u}\phi^{Tn} &= -\frac{\partial I^T}{\partial u} - \frac{\partial Ra^T}{\partial u}\phi^{Tn} \end{aligned} \tag{11}$$

The lagged iterative method suggests initializing the process for $\phi^{n-1} = 0$, then solving the first equation for ψ^{Tn}, after that the second equation is solved for ϕ^n where this iterative process keeps going on until a convergence criteria is reached. The system can now be solved and the adjoint fields can be computed and inserted in the gradient equation.

4 Implementation of the Gradient Equation

Once the coupled adjoint field is computed, the last step is computing the gradient by solving the equation (8) which can be rewritten as:

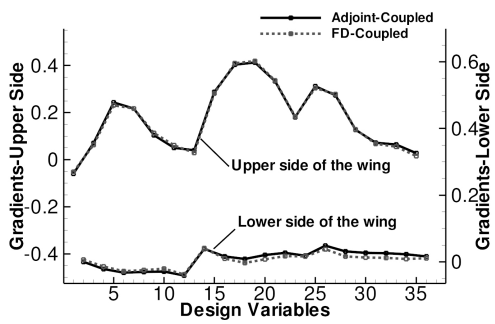
$$\frac{dI}{dD} = \frac{\partial I}{\partial Xa}\frac{\partial Xa}{\partial D} + \psi\frac{\partial Ra}{\partial Xa}\frac{\partial Xa}{\partial D} + \phi\frac{\partial Rs}{\partial D} \tag{12}$$

The terms in (12) not differentiated so far are ($\frac{\partial Xa}{\partial D}$) and ($\frac{\partial Rs}{\partial D}$). The first term describes how the aerodynamic mesh at the flight shape changes by varying the design

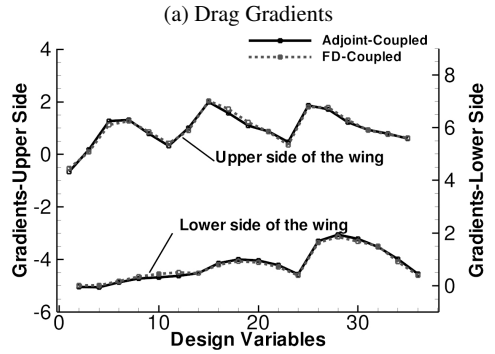
variables. The second term represents the sensitivity of the structure residuals with respect to the design variables. In this term, the change in the stiffness matrix due to the movement of the design variables is neglected, and only the change in the structural forces is taken into account. Both terms are cheaply computed through finite differences.

5 Results

To validate the developed technique, the previously mentioned test case; 3D LANN wing, was used. The wing has around 10000 grid nodes on the structure side and around 30000 grid nodes on the CFD side. The solver used on the structure side is ANSYS Mechanical and DLR-TAU is used to solve the CFD Euler equations. The validation included the gradients of drag and lift, and was performed at a Mach number of 0.84 and an angle of attack of 0.6 degree. The wing is described using 60 design variables that are defined by the free form deformation technique [7]. Figure (2) presents the evaluation for drag and lift respectively for 36 design variables.



(a) Drag Gradients



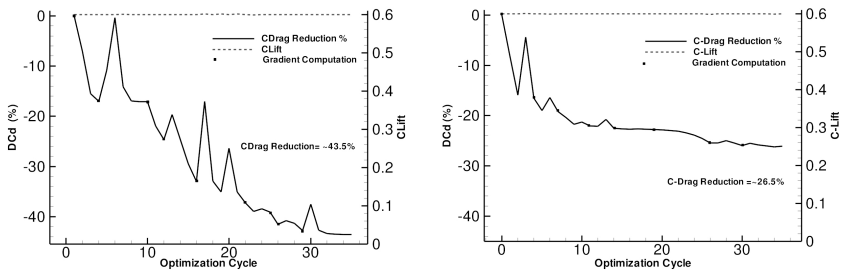
(b) Lift Gradients

Fig. 2: Gradients evaluation

Even though only aerodynamic design variables were employed, the effect of the static aero elasticity was taken into account while computing the gradients. The figure shows a very good matching between the finite differences and the coupled adjoint gradients. The time needed to get the gradients on a single processor with the central finite differences approach is around 72 hours, where only 5 hours were needed to get the gradients using the coupled adjoint formulation; 2.5 hours for each cost function.

To bring the coupled adjoint approach into application, drag reduction optimizations were performed, one under the constraint of constant lift solely, and a second at constnat lift and thickness. The previous test case was taken for the optimizations. A python-based optimization environment (Pyranha) was used, [4] and a conjugate gradient optimization algorithm was employed. Figure (3) presents the convergence of the cost function throughout the optimizations. The figure shows that the first optimization reduced the drag by 43.5% of its initial value, where the second optimization, with thickness constraints, reduced it by 26.5%.

Figure (4) shows the airfoil profiles and the pressure coefficients at $\eta = 0.5$ of the wing for both optimizations, where the dashed lines represent the initial states and the solid lines represent the optimized states.

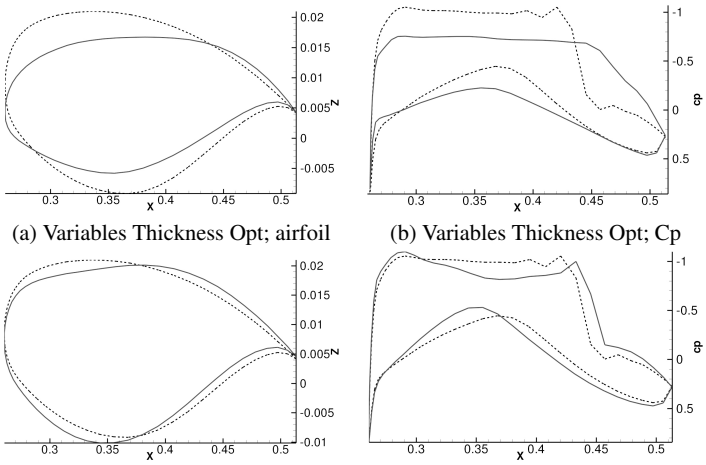


(a) Variables airfoil thickness (b) Constant airfoil thickness

Fig. 3: Optimizations convergence

6 Conclusions

In this study, the adjoint approach for a coupled aero-structural system was successfully derived and implemented. The coupled system was solved using TAU on the CFD side and ANSYS Mechanical on the CSM side. The advantages of such approach are to compute accurate gradients and to considerably reduce the time needed to compute them compared to the traditional approach. After evaluating the gradients obtained by the adjoint approach, two optimizations were performed, one reducing the drag at constant lift, and the other at constant lift and thickness. Both optimizations showed large drag decrease.



(a) Variables Thickness Opt; airfoil (b) Variables Thickness Opt; Cp

(c) Constant Thickness Opt; airfoil (d) Constant Thickness Opt; Cp

Fig. 4: Initial and Optimized states at $\eta = 0.5$

This work will be further developed in order to take structural cost functions into account, and to include the Navier-Stokes equations on the CFD side.

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