Several metrology applications require distribution of clock or frequency signals over long free-space links, while preserving their accuracy and stability.

Frequency dissemination through the atmosphere is generally degraded by stochastic delay effects of the transmission medium \([1,2]\). A two-way link configuration is generally used in order to measure and compensate for the instabilities introduced by this channel.

When even mobile terminals (such as aircraft, satellite, or space probes) are part of the transmission chain, their movement adds to the dynamic effects of the medium. Two-way satellite time and frequency transfer (TWTT) is generally used in order to cancel out the varying delays along the signal path. This configuration assumes path symmetry for both ways to eliminate the delays introduced by the transmitter, the receiver, the satellite, and the atmosphere \([3]\).

In optical fiber communications, bidirectional frequency transfers are also performed using optical carriers. The fiber disturbances are precompensated using a return link on the same fiber \([4,5]\).

In free-space optical links, the atmospheric index-of-refraction turbulence (IRT) distorts the wavefront, which causes intensity scintillation and phase distortions at the receiver side \([6]\). The remaining phase piston error will substantially decrease the stability of the received frequency reference. This piston error value will vary over time due to the optical path length variations caused by IRT \([7,8]\). This effect generates signal-phase fluctuations with bandwidth of up to kilohertz order \([9]\). From the receiver point-of-view, variations of the phase are detected as changes of the reference frequency. A two-way link concept could also be used with mobile optical free-space links to alleviate this problem, but they require high technological effort and also have very limited applicability in space-ground links, where the point-ahead angle prevents them from crossing the same volume of atmosphere for up- and downlink.

The scope of this Letter is to propose a new method for transmitting a reference frequency. We aim to keep the short-time stability and therefore compensate the instabilities introduced by the atmospheric IRT in a one-way configuration. The main target is scenarios with short contact times that do not allow long integration periods, such as links from/to low-Earth-orbit (LEO) satellites \([10,11]\). The main advantage of the proposed configuration is the hardware simplicity of the receiver, which only requires a high-bandwidth optical direct detection receiver. The technique should be robust to variations of the optical path length between satellite and ground station as through atmospheric piston or satellite motion instabilities. First Lab tests are promising for the robustness of the proposed system. But further investigations, such as long-range free-space tests, are planned for a conclusive characterization.

The concept is based on the assumption that two carriers transmitted through the atmosphere will experience the same delay effects if they are close enough to each other \([12]\). After crossing the atmosphere, both carriers, at the optical frequencies \(\omega_o\) and \(\omega_o + \omega_{RF}\), will have a stochastic phase delay \(\Delta \phi\), which will change over time due to changes in the atmospheric refractive index.

This phase delay can be assumed to be the same for both carriers in sufficient approximation (the remaining error will be evaluated later). The receiver squares the received field and thus mixes both carriers like in a heterodyne receiver configuration, where the local oscillator is transmitted together with the signal.

\[
\cos(\omega_o t + \Delta \phi) + \cos((\omega_o + \omega_{RF}) \cdot t + \Delta \phi)^2. \tag{1}
\]

After applying the square, only the cosine of the difference remains in the electrical signal. The difference between both carriers is the RF frequency, \(\omega_{RF}\), and the phase error introduced by the atmosphere cancels out.

For a first theoretical approach, we can relate the Allan deviation to the spectrum of the phase fluctuations. The Allan deviation \([13]\) is usually used for evaluating the frequency stability of a signal.

The Allan deviation is related to the structure function of the time fluctuations \(D_s(\tau)\), when assuming the mean value of the fluctuations to be constant \([6,14]\), where \(\tau\) is the integration time.

\[
\sigma_y^2(\tau) = \frac{1}{2\tau^2} E[4D_s(\tau) - D_s(2\tau)]. \tag{2}
\]
This last assumption is only true for relative short time periods, in which the atmospheric conditions do not change.

Time fluctuations can be expressed in terms of phase fluctuations $\varphi$ by using the carrier frequency $\omega_0$.

$$D_{\varphi}(\tau) = \langle [x(t+\tau) - x(t)]^2 \rangle = \frac{D_{\varphi}(\tau)}{\omega_0^2}. \quad (3)$$

The phase structure function $D_{\varphi}(\tau)$ is derived in [6] from the wave structure function and, in turn, from the first and second perturbation orders using the Rytov method.

The turbulence strength is characterized by the structure parameter of the refractive index $C_n^2$, which is used in the spectrum of the refractive index fluctuations. For the evaluation of the remaining fluctuations due to the separation between the two carriers ($\lambda_1, \lambda_2$), a structure parameter of the difference $C_{\Delta n}^2$ is introduced and used instead of $C_n^2$. $C_{\Delta n}^2$ is calculated writing the dependency of the refractive index difference with the wavelength, and it is related to the temperature structure parameter $C_T^2$ by following [6]:

$$C_{\Delta n}^2(z) = \left[ 5.8 \cdot 10^{-21} \left( \frac{\lambda_2^2 - \lambda_1^2}{\lambda_2^2 \lambda_1^2} \right) \frac{P(z)}{T^2(z)} \right]^2 C_T^2(z). \quad (4)$$

The wavelength is expressed in meter, the temperature in Kelvin, and the pressure in Pascal. The temperature structure parameter can be calculated using the Hufnagel–Valley model, which provides the height distribution of the structure parameter through the atmosphere. The pressure $P(z)$ can be represented by an exponential model [15] and the temperature $T(z)$ by a linear approximation [16].

Changes in the mean temperature and wind velocity are neglected, which is not applicable for longer integration times. Therefore, a more accurate analysis is required for a full characterization.

In Fig. 1 with solid lines, the Allan deviation of the optical carrier is shown for a link to an LEO satellite with circular orbit at 500 km and 30° elevation angle, and using the Hufnagel–Valley $C_n^2$ profile with $C_n^2(0) = 10^{-12}$ m$^{-2/3}$ and the modified spectrum [6]. The outer scale $L_o$ is set to 100 m and the inner scale $l_i$ to 1 mm. The influence of the outer scale is clearly seen by the change of the slope for time integration times higher than 100 ms. The atmospheric wind is calculated using the Button model [17].

The dashed lines correspond to a 1 km horizontal link (HOR). In this case, the inner-scale effect is evident for integration times below 10 ms. The outer-scale effect is also present but at higher integration times than in the LEO link scenario.

Because of the fast satellite movement, the LEO-downlink slew rate makes the beam cross the atmosphere faster than in the HOR case. That accelerates the conversion to a statistically white process, meaning that the Allan deviation slope becomes $\tau^{-1}$ earlier than in the HOR case. This slope may then correspond to white frequency noise.

The diamonds and asterisk markers are the residual error of our system due to the separation of both carriers (100 MHz and also 10 GHz for comparison). The larger the separation, the higher the residual error the system will suffer. In both cases the modified spectrum is used. The residual error is higher at 10 GHz than at 100 MHz, but it is still approximately six orders of magnitude below the atmospheric influence on a single optical carrier.

First measurements in a laboratory test-bed were carried out with $f_{RF} = 100$ MHz to evaluate the proposed method. The system was built using a Brimrose acousto-optic modulator as frequency shifter. It is integrated with the Y-couplers, forming an interferometer with polarizing maintaining (PM) single-mode fiber. As shown in Fig. 2, the output is connected to the piston generator. The piston generator is a fiber wound around a cylinder that allows varying its diameter through a piezo-actuator, and it is used to generate dynamic path length fluctuations.

A phase comparator KV52Z CH7-48 was used to downconvert the 100 MHz signal into 1 Hz pulses. The time variations of the pulses were measured by a time-interval counter SR620 (TIC) and the Allan deviation calculated.

The noise floor of the measurement system was characterized by connecting the 100 MHz source to the phase comparator and measuring against itself. This is our reference curve (see Fig. 3); it is the best measurement

![Figure 1](image1.png)

**Fig. 1.** (Color online) Theoretically calculated Allan deviation for LEO downlink (500 km) and horizontal link (1 km), limited by IRT.

![Figure 2](image2.png)

**Fig. 2.** (Color online) One-way IRT-robust frequency dissemination system.
possible, limited only by the white noise of the measurement instruments.

The dashed line in Fig. 3 is the measurement of the piston introduced by the room noise plus the piezo-actuator. It was measured connecting the piston generator in one arm of the interferometer. The strength of the piezo-actuator disturbance is greater than in the simulated atmospheric scenarios in Fig. 1, in order to make it higher than the unavoidable acoustic noise in the laboratory and the noise floor. No calibration of the piston generator was carried out and no direct comparison with the above theory is expected. Our scope here is to get a first validation of the concept by comparison with an explicitly strong uncompensated scenario.

The solid lines are the performance of our configuration, with and without piston perturbations of the piezo-actuator. Because of the fiber interferometer, our system configuration is highly sensitive to acoustic noise and vibrations. Despite acoustic and temperature isolation, some instability at longer integration times is observed: slightly for $\tau > 10$ s and higher for $\tau > 1000$ s. An even better isolation of the system should improve the performance. Furthermore, a free-space implementation of the interferometer may be more immune to the vibrations than the actual system based on single-mode fibers.

In conclusion, we developed a concept for minimizing the atmospheric IRT influence on the optical transmission of an RF frequency reference signal, by using two optical carriers. Analytical calculations were carried out, showing that the remaining frequency instability introduced by the atmospheric dispersion depends on the separation between the carriers. However, the remaining error is several orders of magnitude smaller than the direct atmospheric influence on the carrier. The system was tested in the laboratory, and the first measurements show an improvement of at least four orders of magnitude in the short-term frequency stability, by using two optical carriers.

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References

Fig. 3. (Color online) Measured overlapping Allan deviation according to [18].