Bayesian Train Localization Method Extended By 3D Geometric Railway Track Observations From Inertial Sensors

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Train Localization

Where exactly is the train?

Key process in safety critical railway systems:

- train control
- (semi) automated train driving
- collision avoidance

- **Requirements**
  - robustness, availability
  - high-precision (track selective localization)
  - onboard solution, no additional railway infrastructure

- **Our train localization approach**:
  - map (track network) based
  - uses multiple onboard sensors (GNSS, IMU)
  - probabilistic approach (estimation, uncertainty)
  - based on a sequential Bayesian filter: particle filter
  - includes positioning method by track geometric effects

Train localization definitions

- **Goal**: estimation of the topological train pose:

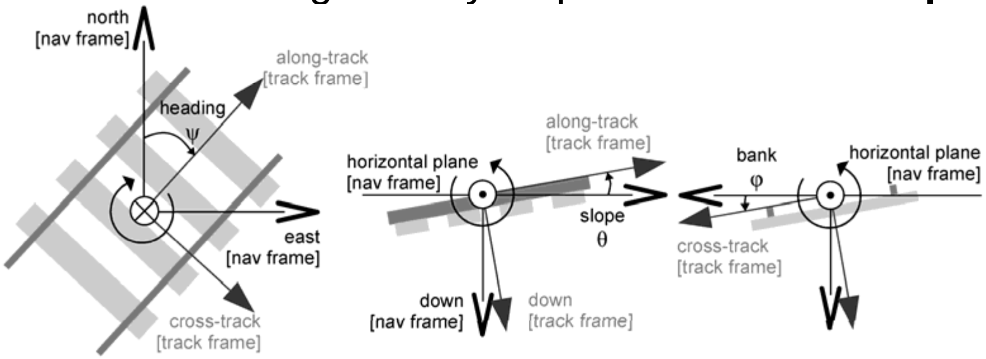
$$\underbrace{\mathsf{P}^{\mathsf{topo}}}_{\text{topological pose}} = \{ \underbrace{R}_{\text{track ID}}, \underbrace{s}_{\text{track position}}, \underbrace{dir}_{\text{direction}} \}$$

- **Map**-based approach
  - necessary for onboard-sensors-only approach
  - map contains information of the track network
    - **topological:** track IDs, connections (switch, crossing, track end)
    - **geometric:** position, attitude, curvatures
  - data access by the topological train pose:

$$\left.\begin{array}{l}\text{topological connections}\\ \text{geographic track position}\\ \text{track geometry}\end{array}\right\} = f_{map}(\mathsf{P}^{\mathsf{topo}})$$

Track Geometry I

- track geometry as parameter of **track position s:**



track attitude:

$\phi(s)$ bank
$\theta(s)$ slope
$\psi(s)$ heading

track attitude changes:

$\frac{d\phi}{ds}(s)$ bank change
$\frac{d\theta}{ds}(s)$ slope change
$\frac{d\psi}{ds}(s)$ curvature

- Track effects: geometry and train motion causes **turn rates** and accelerations

Turn rates (navigation frame)

$$\underbrace{\dot{\phi}}_{\text{roll rate}} = \frac{d\phi}{dt} = \frac{d\phi}{ds}\frac{ds}{dt} = \underbrace{\frac{d\phi}{ds}}_{\text{bank change}} \underbrace{\dot{s}}_{\text{speed}}$$

$$\underbrace{\dot{\theta}}_{\text{pitch rate}} = \underbrace{\frac{d\theta}{ds}}_{\text{slope change}} \dot{s}$$

$$\underbrace{\dot{\psi}}_{\text{yaw rate}} = \underbrace{\frac{d\psi}{ds}}_{\text{curvature}} \dot{s}$$

frame conversion →

Turn rates (train frame)

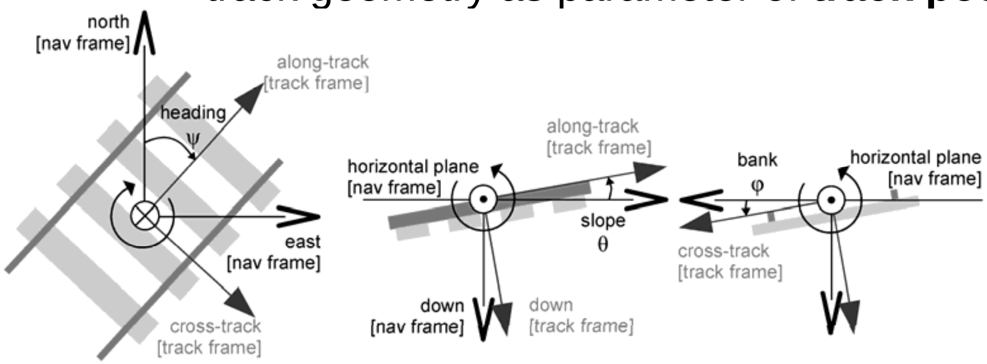
$$\omega^x = \dot{\phi} - \dot{\psi}\sin\theta$$

$$\omega^y = \dot{\theta}\cos\phi + \dot{\psi}\sin\phi\cos\theta$$

$$\omega^z = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$$

Track Geometry II

- track geometry as parameter of **track position s**:



track attitude:

$\phi(s)$ bank

$\theta(s)$ slope

$\psi(s)$ heading

track attitude changes:

$\frac{d\phi}{ds}(s)$ bank change

$\frac{d\theta}{ds}(s)$ slope change

$\frac{d\psi}{ds}(s)$ curvature

- Track effects: geometry and train motion causes turn rates and **accelerations**

Accelerations (navigation frame)

$$a^{\text{cross}} = \underbrace{\frac{d\psi}{ds}}_{\text{curvature}} \cdot \dot{s}^2$$

$$a^{\text{vertical}} = \underbrace{\frac{d\theta}{ds}}_{\text{slope change}} \cdot \dot{s}^2$$

frame $\longrightarrow$ conversion

Accelerations (train frame)

	train motion	gravity	cross track geometry (curvature)	vertical track geometry (slope change)
$a^x =$	$\ddot{s}$	$+g\sin\theta$		
$a^y =$		$-g\sin\phi\cos\theta$	$+\dot{\psi}\dot{s}\cos\phi$	$-\dot{\theta}\dot{s}\sin\phi$
$a^z =$		$-g\cos\phi\cos\theta$	$-\dot{\psi}\dot{s}\sin\phi$	$-\dot{\theta}\dot{s}\cos\phi$

Bayesian Approach

- Train state definitions

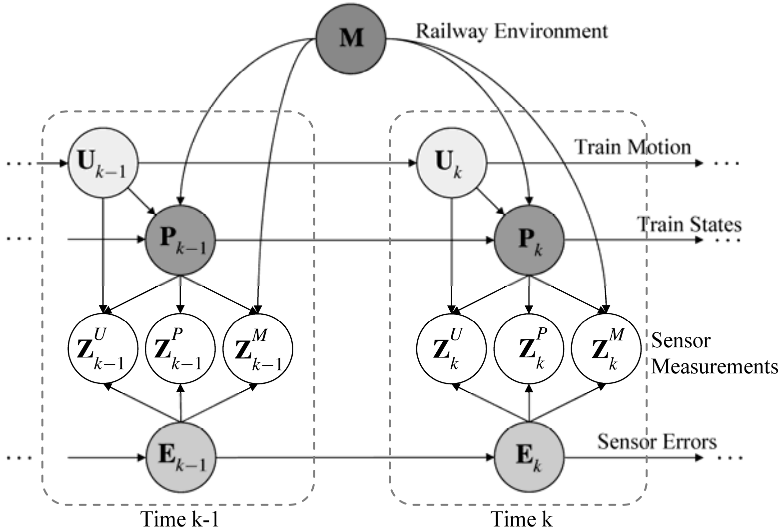
Train motion

$$\mathsf{U}_k = \{\Delta s_k, \dot{s}_k, \ddot{s}_k\}$$

Train pose

$$\mathsf{P}_k = \{\underbrace{R_k, s_k, dir_k}_{\mathsf{P}_k^{\mathsf{topo}}}, \underbrace{lat_k, long_k, alt_k}_{\mathsf{P}_k^{\mathsf{geo}}}, \underbrace{\phi_k, \theta_k, \psi_k}_{\mathsf{P}_k^{\mathsf{att}}}, \underbrace{\dot{\phi}_k, \dot{\theta}_k, \dot{\psi}_k}_{\mathsf{P}_k^{\mathsf{turn}}}\}$$

Dynamic Bayesian Network



- Sequential Bayesian Filter: Factorized solution of the localization posterior

$$\underbrace{p(\{\mathbf{PUE}\}_{0:k}|\mathbf{Z}_{1:k}, \mathbf{M})}_{\text{localization posterior}} \propto \underbrace{p(\mathbf{Z}_k^{\mathsf{M}}|\{\mathbf{PE}\}_k, \mathbf{M}) \cdot p(\mathbf{Z}_k^{\mathsf{P}}|\{\mathbf{PE}\}_k) \cdot p(\mathbf{Z}_k^{\mathsf{U}}|\{\mathbf{PUE}\}_k)}_{\text{sensor likelihoods}} \cdot \underbrace{p(\mathbf{U}_k|\mathbf{U}_{k-1})}_{\text{1D motion model}}$$

$$\cdot \underbrace{p(\mathbf{P}_k|\mathbf{P}_{k-1}, \mathbf{U}_k, \mathbf{M})}_{\text{train state transition}} \cdot \underbrace{p(\mathbf{E}_k|\mathbf{E}_{k-1})}_{\text{sensor errors transition}} \cdot \underbrace{p(\{\mathbf{PUE}\}_{0:k-1}|\mathbf{Z}_{1:k-1}, \mathbf{M})}_{\text{recursion}}$$

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Particle Filter Implementation

- Particle representation:

$$\underbrace{p(\{\mathbf{PUE}\}_{0:k}|\mathbf{Z}_{1:k},\mathbf{M})}_{\text{localization posterior}} \approx \{\ \underbrace{x^i_{0:k}}_{\text{particle}}\ ,\ \underbrace{w^i_{0:k}}_{\text{weight}}\ \}_{i=1}^{N_p}$$

1. Proposal function:

$$\underbrace{q(\{\mathbf{PUE}\}_{0:k}|\mathbf{Z}_{1:k},\mathrm{M})}_{\text{proposed posterior}} \mathrel{\hat=} \underbrace{p(\mathbf{U}_k^{\ddot{s}}|\mathbf{Z}_k^{a_x},\mathbf{P}_k^{\mathrm{att}})}_{\text{sampled acc.}} \cdot \underbrace{p(\mathbf{U}_k|\mathbf{U}_{k-1}) \cdot p(\mathbf{P}_k|\mathbf{P}_{k-1},\mathbf{U}_k,\mathrm{M})}_{\text{train transition}} \cdot \underbrace{p(\mathbf{E}_k|\mathbf{E}_{k-1})}_{\text{error transition}} \cdot \underbrace{q(\{\mathbf{PUE}\}_{0:k-1}|\mathbf{Z}_{1:k-1},\mathrm{M})}_{\text{recursion}}$$

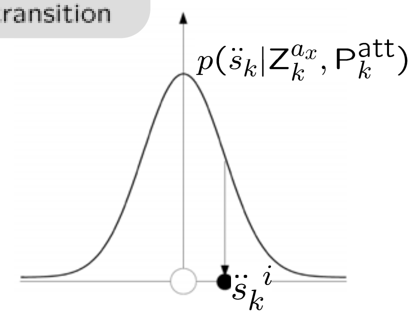
- particles exist only on tracks

2. Weight function:

$$w_k^i \propto \underbrace{w_{k-1}^i}_{\text{recursion}} \cdot \underbrace{p(\mathbf{Z}_k^{\mathrm{GNSS}}|\{\mathrm{PE}\}_k^i)}_{\text{GNSS likelihood}} \cdot \underbrace{p(\mathbf{Z}_k^{\omega_x}|\{\mathrm{PUE}\}_k^i) \cdot p(\mathbf{Z}_k^{\omega_y}|\{\mathrm{PUE}\}_k^i) \cdot p(\mathbf{Z}_k^{\omega_z}|\{\mathrm{PUE}\}_k^i)}_{\text{gyroscope likelihoods}} \cdot \underbrace{p(\mathbf{Z}_k^{a_y}|\{\mathrm{PUE}\}_k^i) \cdot p(\mathbf{Z}_k^{a_z}|\{\mathrm{PUE}\}_k^i)}_{\text{accelerometer likelihoods}}$$

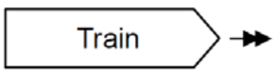
Transition Model

- train transition (proposal func.) $p(\mathbf{U}_k^{\ddot{s}}|\mathbf{Z}_k^{a_x},\mathbf{P}_k^{\text{att}}) \underbrace{p(\mathbf{U}_k|\mathbf{U}_{k-1})}_{\text{1D motion model}} \cdot \underbrace{p(\mathbf{P}_k|\mathbf{P}_{k-1},\mathbf{U}_k,\mathbf{M})}_{\text{train state transition}}$



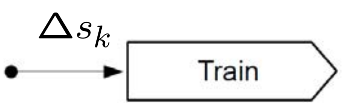
Algorithm:

1. **Sampling of the train acceleration measurement distribution:**



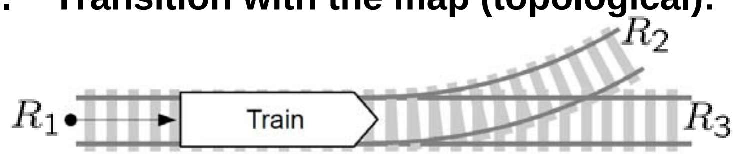
$$\ddot{s}_k{}^i \sim \mathcal{N}(\ddot{s}\,{}^i|Z_k^{a_x} + g\sin(\theta), \sigma_a^2)$$

2. **1D motion transition**:



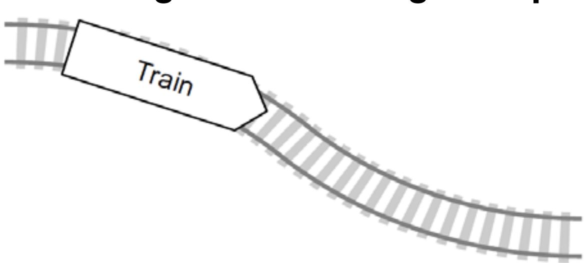
$$\underbrace{\mathsf{U}_k}_{\text{train motion state}} = f_{\text{1D motion}}(\mathsf{U}_{k-1},\ddot{s}_k) = \begin{pmatrix} \dot{s}_{k-1}\Delta t + \ddot{s}_k\frac{\Delta t^2}{2} \\ \dot{s}_{k-1} + \ddot{s}_k\Delta t \\ \ddot{s}_k \end{pmatrix}$$

3. **Transition with the map (topological):**



$$\mathsf{P}_k^{\text{topo}} = f_{\text{map(I)}}(\underbrace{\mathsf{P}_{k-1}^{\text{topo}}}_{\text{previous pose}}, \underbrace{\Delta s_k}_{\text{1D transition}})$$

4. **Retrieving the remaining train pose (geometric):**



$$\mathsf{P}_k = \{\mathsf{P}_k^{\text{topo}},\mathsf{P}_k^{\text{geo}},\mathsf{P}_k^{\text{att}},\mathsf{P}_k^{\text{turn}}\} = f_{\text{map(II)}}(\underbrace{\mathsf{P}_k^{\text{topo}}}_{\text{actual pose}}, \underbrace{\dot{s}_k}_{\text{speed}})$$

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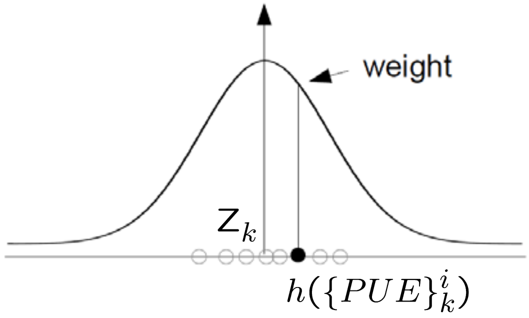
Measurement processing

- Likelihood: probabilistic model of sensor measurements by Gaussian distribution

$$\mathcal{N}\left(\underbrace{h(\{PUE\}_k^i)}_{\text{argument}} \,\middle|\, \underbrace{Z_k}_{\text{mean}}, \underbrace{\Sigma_Z}_{\text{covariance}}\right)$$



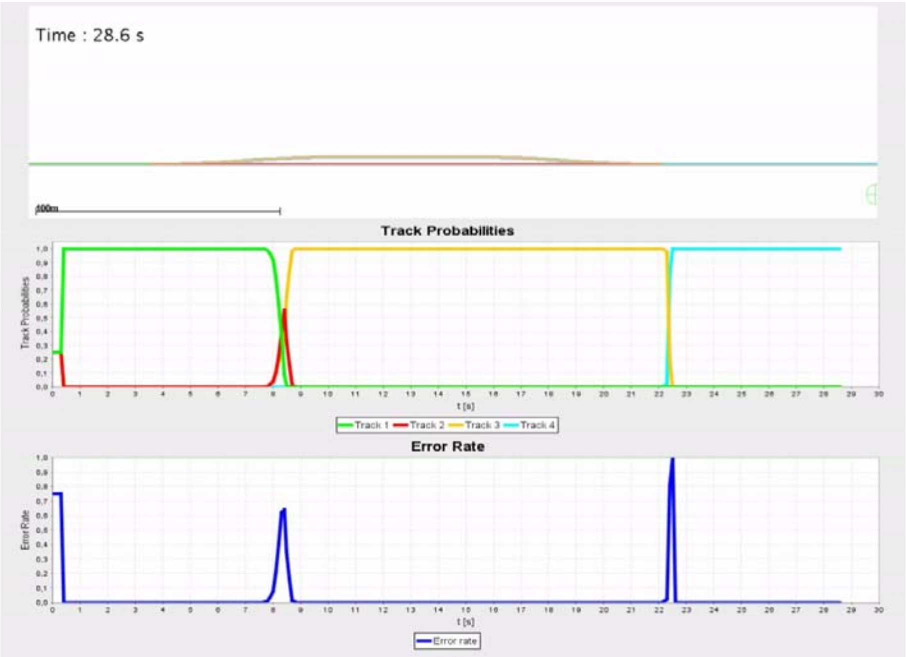
Sensor likelihoods (weight function)

- **GNSS** position likelihood $p(\mathbf{Z}_k^{\text{GNSS}}|\{\text{PE}\}_k^i) \hat{=} \mathcal{N}\left(h_{\text{GNSS}}(P_k^{i,\text{geo,att}}, \vec{d}) \,\middle|\, C_{\text{earth}}^{\text{nav}}(\mathbf{Z}_k^{\text{GNSS}}), \Sigma_{\text{GNSS}}\right)$
  - computed in NED coordinate frame

- **IMU** (train frame):
  - acceleration likelihoods [y,z]: $p(\mathbf{Z}_k^a|\{\text{PUE}\}_k^i) \hat{=} \mathcal{N}\left(h_a(P_k^{i,\text{turn,att}}, \dot{s}_k^{\,i}) \,\middle|\, Z_k^a, \sigma_a^2\right)$
  - turn rate likelihoods [x,y,z]: $p(\mathbf{Z}_k^\omega|\{\text{PUE}\}_k^i) \hat{=} \mathcal{N}\left(h_\omega(P_k^{i,\text{turn,att}}, \dot{s}_k^{\,i}) \,\middle|\, Z_k^\omega, \sigma_\omega^2)\right)$

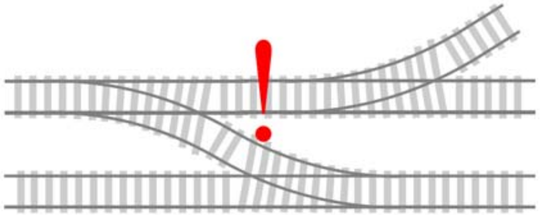
Proof of Concept Simulations 3

GNSS only



GNSS and IMU (geometric track features)

Summary and Conclusion



- train localization method
  - Bayesian approach with a **particle filter**
  - **onboard solution** with multiple sensors: **GNSS, IMU**
  - **map** in the transition model: particles exist only on tracks

- train accelerations and turn rates are depended on track geometry and speed
- **direct use of acceleration and turn rate measurements for localization**

  → **no integration** necessary compared to other GNSS/IMU localization methods

- **promising results in critical railway scenario** simulation
  - track precise accuracy with geometric track features
  - robust during GNSS outages

- **Future work:**
  - further robustness by feature detectors (switch way detector)
  - transition and sensor models extended by sensor errors (e.g. inertial drift)
  - map generation / verification by SLAM method

Thank you for your attention.

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