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Extended Abstract

Abstract

With increasing environmental sustainability awareness significant attention on ecological traffic management (eco-TM) has come into the focus of researchers and practitioners. While different approaches have been applied to reach minimal pollutant production, the classic user equilibrium calculation with the pollutant production as travel costs instead of using travel times remains in the center of attention. However, the validity of such a direct transformation to find a user equilibrium is questionable. In this paper, a simplified analytical approach to examine the above aforementioned validity has been carried out, followed by a simulation approach to verify the results of the analytical approach. The initial result shows that the pollutant production function violates the usual assumption of a monotonous function (typically, emission has a minimum at travel speeds around 60 km/h). This means that substantial modifications to the algorithms that compute the user equilibrium have to be discussed since they do not work as intended when pollutant production is used as travel costs, especially in a transportation system with mixed speeds that cover a range around the minimum emission speed.

Introduction

With increasing environmental sustainability awareness significant attention on ecological traffic management (eco-TM) has been paid since last decades. Usually, eco-TM is performed by computing several scenarios and then selecting the one with minimal pollutant production. In fact, this does not try to minimize an objective function directly that describes the emission production as function of the traffic pattern in a given area. Another applied approach is to add a toll that takes pollutant production into account and to find a user equilibrium based on measured pollutant production. This approach can in principle embedded in the usual formulation of the user equilibrium (UE), i. e. emissions generated by vehicles will be used as travel costs instead of using travel times. However, the validity of such a direct transformation to find a user equilibrium is questionable, since the vehicular energy consumption does not monotonously increase with descending or ascending traveling speed [1]. For each vehicle type and even for each vehicular brand, there is usually an ideal traveling speed for the optimal energy consumption.

In this paper, an analytical approach is used to examine the validity of the classic user-equilibrium approach based on pollutant emission. A simulation will be used subsequently to verify the results from the analytical approach. Conclusions will be offered and at the end.

Analytical approach

Monotone validity

A classic simple example with one OD-pair and two routes is chosen here [2]. Assume that the two routes have exactly the same length with $L_1 = L_2 = 30\,\text{km}$, they have a linear travel-time function as function of demand q:

$$t_i(q_i) = T_i \left(1 + k\frac{q_i}{q_\infty}\right) \tag{1}$$

where q_∞ is a proxy of the link capacity, k is a factor that determines, how slow the travel time will be when capacity is reached, i. e. $(k+1)T_i$, and T_i is the travel time at free-flow speed ($q_i = 0$). The factor k can be link-dependent, but only one factor is used for all links here.

Pollutant, e. g. CO_2, typically has a more complicated function. A simplified form as a function of speed is adapted here with regard of analysis simplicity and shown below.

$$\tilde{e}(v) = c + dv^3 \tag{2}$$

The equation (2) is the production per unit of time. To compute the production along a link of length L_i, it has to be multiplied with the time needed to traverse the link, where this time is given by equation (1). Therefore the pollutant produced along a certain link turns out to be

$$\begin{aligned} e_i(q_i) = t_i(q_i)\left(c + dv^3\right) &= t_i(q_i)\left(c + d\frac{L_i^3}{(t_i(q_i))^3}\right) \\ &= T_i\left(c\left(1 + k\frac{q_i}{q_\infty}\right) + dV_i^3\frac{1}{\left(1 + k\frac{q_i}{q_\infty}\right)^2}\right) \end{aligned} \tag{3}$$

where V_i is the travel speed on link i at free-flow speed ($q_i = 0$). An alternative form of this equation is $e(v) = cL/v + dLv^2$. From this from, the constants can be made a bit more self-explaining. c is clearly the pollutant production when idling, while d is a complicated constant taking into account air drag, which depends on the vehicle form, front area and so on. However, by assuming an ideal speed v_0 with minimal pollutant production, the constant d can be written as $d = c/\left(2v_0^3\right)$ which results in:

$$e_i(q_i) = cT_i\left(1 + k\frac{q_i}{q_\infty} + \frac{1}{2}\left(\frac{V_i}{v_0}\right)^3\frac{1}{\left(1 + k\frac{q_i}{q_\infty}\right)^2}\right) \tag{4}$$

Since pollutant production is usually proportional to energy consumption, fuel consumption can be used as a general indicator of pollutant production. In most cases, v_0 has been set to $15\,\text{m/s}$ ($54\,\text{km/h}$), while $c = 1\,\text{l/h}$ is a good estimate for the fuel consumption of a vehicle when idling. According to the aforementioned assumptions, the relationship between travel time, fuel consumption and the number of vehicles can be illustrated in Figure 1 on the facing page. It is obvious, that the pollutant production function violates the usual assumption of a monotonous function, which also indicates that the algorithms to compute the user equilibrium can not work correctly with use of pollutant production as travel costs.

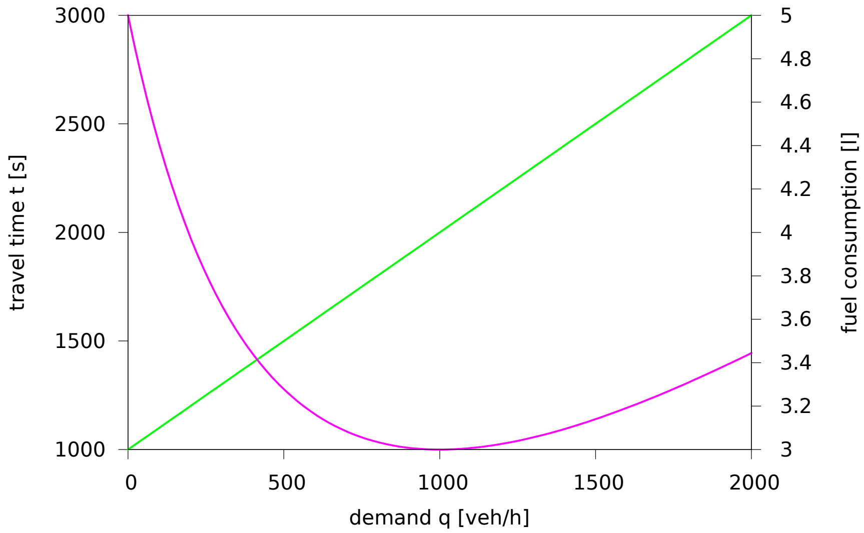

Figure 1: Relationship between travel time, fuel consumption and traffic demand. The travel-time eq. (1) and the pollutant function eq. (4) are as functions of demand. The parameters have been set so, that the minimum in the pollutant versus speed curve occurs at $v = 15\,\mathrm{m/s}$. The parameter settings here and for Figure 2 are: $L_1 = L_2 = 30\,\mathrm{km}$, $T_1 = T_2 = 1000\,\mathrm{s}$, c and v_0 are described in the text, the capacity on each link has been set to $q_\infty = 2000\,\mathrm{veh/h}$ and $k = 2$ has been used.

User equilibrium validity

For the travel times, the user equilibrium can be computed as usual [3], by minimizing the objective function:

$$Z(q_1, q_2) = \sum_{i=1}^{2} \int_0^{q_i} d\omega t_i(\omega), \tag{5}$$

with $t_1(q_1) = t_2(q_2)$ and $q_1 + q_2 = Q$, where Q is the total demand for travel. The same formulation can then be used with pollutant production for reaching a eco-based user equilibrium. In the two routes example, two constraints will now be

$$e_1(q_1) = e_2(q_2), \tag{6}$$

$$q_1 + q_2 = Q, \tag{7}$$

To get the solution we can either solve $e_1(q_1) = e_2(Q - q_1)$ directly or construct the complete objective function which leads to:

$$E(q) = cTq \left(1 + \frac{1}{2} k \frac{q}{q_\infty} + \frac{1}{2} \left(\frac{V}{v_0}\right)^3 \frac{1}{1 + k\frac{q}{q_\infty}}\right), \tag{8}$$

$$T(q) = Tq \left(1 + \frac{k}{2} \frac{q}{q_\infty}\right), \tag{9}$$

$$Z^{(e)}(q_1) = E(q_1) + E(Q - q_1), \tag{10}$$

$$Z^{(t)}(q_1) = T(q_1) + T(Q - q_1), \tag{11}$$

which is a one-dimensional curve, parametrized by the demand Q. Note, that the two additional solutions cannot be directly inferred from the condition $e_1(q_1) = e_2(Q - q_1)$ or $\partial E(q)/\partial q = 0$, since they stick to the boundary of the valid UE's solution region.

As shown in Figure 1, both low and high traveling speeds result in more pollutant production than a so-called ideal traveling speed with minimal pollutant production. If the demand is small, e. g. 1000 veh/h, and there is only high-speed traffic in the two routes example, the possible solutions with the aforementioned objective function can be calculated and illustrated in Figure 2. When the demand is only 1000 veh/h, the following situation arises: start with a share of 0.5, i. e. half of the vehicles drive on route 1, and the other half drive on route 2. The condition $e_1(q_1) = e_2(q_2) = e_2(Q - q_1)$ is then fulfilled, but this is not a stable set-up and even not the optimal solution, since the fuel consumption can be further reduced when one vehicle switches to the other route. Such a route switch increases the traffic flow on this route, and then reduces the respective traveling speed. The pollutant production will also accordingly be reduced. Therefore all drivers will immediately switch to the route with more traffic. This phenomena leads to the surprising situation that a stable eco-based UE solution in this case is given by either $p = (1, 0)$ or $p = (0, 1)$, where p is the vector of shares q_i/Q. This changes, of course, for large demand, or for links where the maximum speed is below the minimum of the pollutant curve (city traffic). Figure 2 indicates that the minimal fuel consumption occurs with a share of 0.5 when the demand is 3000 veh/h. Furthermore it also shows that ,in the situation with the demand of 2000 veh/h, there are still the both minima at the boundaries ($p_1 = 1$ or $p_2 = 1$).

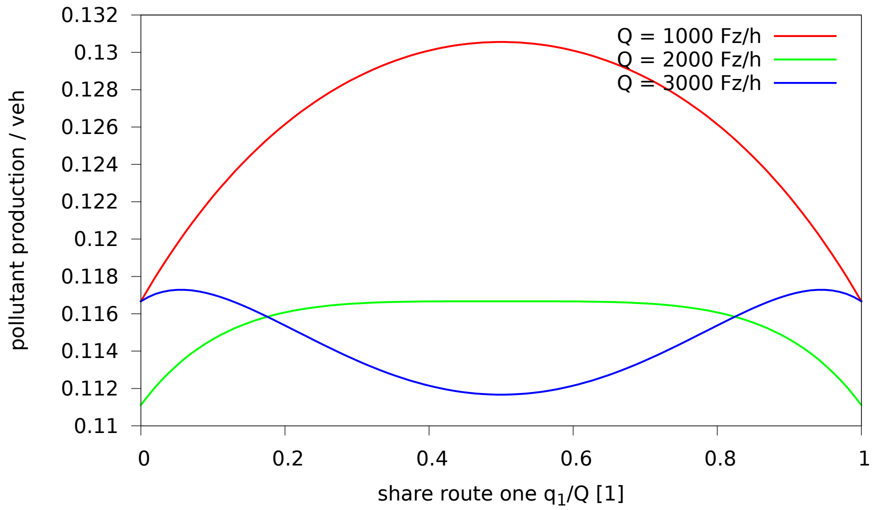

Figure 2: Pollutant production per vehicle, i. e. $Z^{(e)}(q_1)/Q$ as function of the share of vehicles using route 1.

Remarks and perspective

The initial result shows that the pollutant production function violates the usual assumption of a monotonous function, which also indicates that the respective algorithms to compute the user equilibrium must now deal with the fact, that the UE solution is not unique. This non-uniqueness will have consequences for all approaches trying to seek eco-optimal solutions in large transportation systems, and right now we are speculating that the convergence problems we faced with such a simulation are caused by this non-uniqueness, which of course gets clouded when a real heterogeneous transportation system is under consideration.

What is even more disturbing is that the solutions that came out of such an approach are completely counter-intuitive and that it is highly unlikely that they will ever be realized in reality. Squeezing all the demand on one link to force vehicles to drive slower to achieve an eco-optimal solution is a funny idea, but nothing that is realistic. We do not have a good answer to this question, therefore we rise it here.

Obviously, the UE approach can, in fact, still be used for a eco-TM in a traffic system where the speed limit is smaller than the ideal speed with minimal pollutant production. In this case, only the right branch of the pollutant curve in Figure 1 is used and everything is still working as intended.

Currently, a simulation study is undertaken, since true emission functions are more complicated than the simple approach used here. The microscopic traffic simulation software SUMO [4] and the HBEFA-based emisson model [5], already implemented in SUMO, are used with the aforementioned two routes example and with a real network. The respective results will be shown in the final contribution.

References

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