Deriving an estimate for the Fried parameter in mobile optical transmission scenarios

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Measuring the Fried parameter $r_0$ (atmospheric optical coherence length) in optical link scenarios is crucial to estimate a receiver’s telescope performance or to dimension atmospheric mitigation techniques, such as in adaptive optics. The task of measuring $r_0$ is aggravated in mobile scenarios, when the receiver itself is prone to mechanical vibrations (e.g., when mounted on a moving platform) or when the receiver telescope has to track a fast-moving signal source, such as, in our case, a laser transmitter on board a satellite or aircraft. We have derived a method for estimating $r_0$ that avoids the influence of angle-of-arrival errors by only using short-term tilt-removed focal intensity speckle patterns. © 2011 Optical Society of America

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1. Introduction
Atmospheric index-of-refraction turbulence (IRT) distorts optical waves and leads to reduced resolution of ground-based telescopes or reduced received signal stability in optical free-space communication systems. The strength of these wavefront distortions is classically quantified in astronomy by the long-term Fried parameter $r_0$, which is defined according to [1]. Simplified, the IRT of the atmosphere produces optical wavefront distortions with structure size $r_0$, which limit the resolving power of a telescope or the signal quality of an optical receiver. While $r_0$ is a long-term parameter of an unbounded plane optical wave inside the atmospheric propagation medium, it can practically only be measured with limited-size telescopes. This implies that the statistics of the remaining overall tilt of the optical wavefront over the telescope aperture needs to be regarded, especially when the ratio between telescope aperture diameter $D$ and $r_0$ is small.

When application scenarios are expanded from celestial observations toward mobile laser links like satellite downlinks to optical ground stations (OGSs) or optical aircraft links, the classical principle of a long-term (several seconds) observation of the received wavefront quality is no longer applicable due to the following reasons:

- Because of the fast lateral movement of the mobile link partner(s), the atmospheric volume traversed by the optical signal changes during the required long-term observation time. As this atmospheric volume cannot be regarded as isotropic, no valid long-term statistical parameter can be derived from such a dynamic measurement. Instead, one needs to reduce to intermediate-term statistical evaluations.

- The relative angular movements of the link partners requires some kind of agile tracking of the partner’s signal. This tracking could have either a nonnegligible remaining tracking error that might be much larger than the angle-of-arrival caused by the atmosphere (aAoA), or it could also be of such high quality that it perfectly tracks any incoming angle-of-arrival (AoA), thereby also removing the aAoA. But this aAoA needs to be taken into account for the $r_0$ determination. In both situations, no distinction between mechanical and aAoA effects can be made, again changing the statistics of an $r_0$ measurement.
These two facts require an adapted method for deriving an estimate for $r_0$ in mobile transmission scenarios.

2. Basic $r_0$ Estimation from a Focal Speckle Pattern

For the estimation of the Fried parameter $r_0$, different methods are available, e.g., the differential image motion monitor [2] or measurements with fast wavefront sensors. Another very simple and practical way is to observe the long-term focal speckle pattern produced from a point source at infinite distance onto a focal camera. This intensity distribution represents all atmospheric wavefront aberrations seen by the aperture, including the overall tilt (which causes lateral movements of the focal speckle patterns) and resembles a Gaussian distribution. The full width at half-maximum (FWHM) of this focal seeing disk (FSD) $\text{FWHM}_{\text{FSD}}$ can be used to calculate $r_0$ according to [3] (with $\text{FWHM} = 1.67 \times \sigma$ for the Gaussian FSD approximation), where $D > r_0$ must be observed ($D$: receive aperture diameter) to account for the error by the diffraction limited focal spot size:

$$r_0 \approx 0.98 \cdot \frac{\lambda \cdot f}{\text{FWHM}_{\text{FSD}}} = 0.59 \cdot \frac{\lambda \cdot f}{\sigma_{\text{FSD}}},$$

where $f$ is the effective focal length, $\lambda$ is the wavelength, and $\sigma_{\text{FSD}}$ is the sigma radius of the FSD. In fact, this method cannot measure $r_0$ larger than 0.96 $\cdot D$ as the minimum (diffraction limited) focal spot size of a circular aperture already has a FWHM of 1.024 $\cdot \lambda \cdot f / D$. In other words, this method, rather, estimates the seeing resolution of a specific telescope, where this resolution is either limited by the telescope aperture diameter $D$ or the atmospheric seeing given by $r_0$, whichever is smaller (see [4] for an explanation of the effective resolution of a telescope under atmospheric turbulence).

The drawback of this simple method is the fact that a perfect geometric axial telescope alignment toward the point source must be ensured during the long-term exposure of the FSD so as to allow only atmospherically induced wavefront tilts to enlarge the FSD. The effects of nonperfect tracking of the point source, e.g., by mechanical inaccuracies of the telescope mount or through the fast angular movement of the point source (which might be emitted from a fast moving satellite or an aircraft instead of a fixed star) will lead to extra broadening of the long-term FSD and thus spoil the measurement of $r_0$, as already stated above.

Therefore, a method has been tested using the size of the short-term tilt-removed focal intensity speckle (FIS) patterns and backcalculate the values toward a standard long-term $r_0$. A perfect tilt removal thereby is performed through simple image processing by centering the FIS image of the focal camera around its center of gravity.

The measurements used in this paper were taken during the Kirari Optical Downlinks to Oberpfaffenhofen (KIODO) 2009 optical satellite downlink trials using the Japan Aerospace Exploration Agency (JAXA) Optical Inter-Satellite Communications Engineering Test Satellite and the DLR OGS at Oberpfaffenhofen (OGS–OP) near Munich [5]. In this paper, we use data from the downlink that took place on 28 August 2009 from 04:05:39 a.m. to 04:09:01 a.m. local time. Of course, the presented principle can be applied to any optical free-space link scenario.

3. Introduction of Tilt-Removed and Short-Term Parameters

Only for illustration of the method do we introduce the parameters “short-term $r_0$” $r_{0,\text{st}}$, and “tilt-removed short-term $r_0$” $r_{0,\text{tr-st}}$ (they have no real physical meaning). Tilt removal from the impinging optical wavefront can be performed either by perfectly tracking the AoA (e.g., by an ideal tip–tilt mirror) or—more conveniently—by calculating the individual AoA from the measured (tilt-included) FIS and accordingly cutting out the centered focal speckle pattern by image processing.

By shortening the exposure time of the focal camera down to well below the atmospheric coherence time (typically 1 ms or shorter in extreme scenarios), the focal speckle pattern is frozen in time and does not move laterally nor change its shape. Figure 1(a) shows a typical FIS as seen during the optical satellite downlink.

Values for the tilt-removed instantaneous speckle size $\sigma_{\text{FIS-tr-st}}$ are derived from the sigma radius of the Gaussian fit to this FIS. The size of the tilt-removed FSD $\sigma_{\text{FSD-tr}}$ can either be found by a Gaussian fit over the long-term tilt-removed focal intensity distribution (which is found by superposing the N short-term tilt-removed FIS; see Fig. 2(b)) or by calculating the sliding mean of $N$ times the single focal FIS radius—both methods yield the same result as expected. The averaging time typically should be

![Fig. 1. Typical tilt-removed (a) short-term FIS and (b) long-term tilt-removed FSD (right, overlay of 48 samples, equivalent to 1 s exposure) measured during KIODO–2009. Both images show 990 $\mu$m $\times$ 990 $\mu$m of the focal intensity image, signal wavelength $\lambda = 847$ nm, receiver aperture diameter $D = 0.4$ m, and focal length $f = 8.3$ m. These values imply a minimum diffraction-limited focal spot sigma radius of $\rho_{\text{bl}} = 0.61 \times \lambda \cdot f / D = 10.8 \mu$m, which would only be seen under perfect seeing (without any atmospheric distortions). Note that all aAoA and all tracking AoA have been removed from these images.](image-url)
The variance of the aAoA moves around 1 s to have enough independent samples for a reliable calculation of statistical parameters (regarding a typical IRT-induced bandwidth of 100 Hz), while the atmospheric volume properties stay sufficiently constant during this observation time (which depends on the lateral velocity of the link partner). A shorter time should be applied for extremely fast movements.

In Fig. 3, all \( \rho_{\text{FIS-tr-st}} \) samples and their 1 s sliding mean (solid curve) are plotted against the link elevation.

4. Relation from Short-Term Tilt-Removed Parameters to \( r_0 \)

To derive a relation between the different effects of broadening the focal intensity pattern, we use the sum of the variances of the lateral focal deviations from a point image, assuming isotropic Gaussian statistics of the diverse uncorrelated deviations, which will produce, again, a Gaussian statistic of the broadening of the focal intensity spot:

\[
\sigma_{\text{FSD}}^2 = \langle \rho_{\text{FIS}}^2 \rangle = \langle \rho_{\text{FIS-tr-st}}^2 \rangle + \langle \rho_{\text{aAoA}}^2 \rangle, \tag{2}
\]

with \( \langle \rho_{\text{aAoA}}^2 \rangle \) representing the broadening of the FSD by aAoA.

According to formula 4.58 in [6], Hardy states the variance of the aAoA \( \beta_{\text{aAoA}} \) over an aperture \( D \) as

\[
\langle \rho_{\text{aAoA}}^2 \rangle = 0.182 \cdot \frac{\lambda^2}{D^{1/3} \cdot r_0^{5/3}}; \quad r_0 \leq D. \tag{3}
\]

From \( \langle \rho_{\text{aAoA}}^2 \rangle \) the variance of the focal aAoA movement \( \langle \rho_{\text{aAoA}}^2 \rangle \) is derived by multiplication with \( f^2 \).

By measuring \( \rho_{\text{FIS-tr-st}} \) with the short-exposure focal camera picture, we can calculate the relation between an \( r_{0\text{-st}} \) and an \( r_{0\text{-tr-st}} = 0.59 \cdot \frac{\lambda \cdot f}{\rho_{\text{FIS-tr-st}}} \) by applying the relations stated in (1)–(3) to short-term parameters:

\[
\left( \frac{0.59 \frac{\lambda \cdot f}{r_{0\text{-st}}}}{r_{0\text{-tr-st}}} \right)^2 = \left( \frac{0.59 \frac{\lambda \cdot f}{r_{0\text{-tr-st}}}}{r_{0\text{-tr-st}}} \right)^2 + 0.182 \left( \frac{f \cdot \lambda}{D^{1/3} \cdot r_{0\text{-tr-st}}} \right)^2,
\]

where \( r_0 < D \) has to be regarded to account for the limitations of the approximation formula (3). We can then express the relation between \( r_{0\text{-st}} \) and its tilt-removed variant \( r_{0\text{-tr-st}} \) (measured via \( \rho_{\text{FIS-tr-st}} \)):

\[
r_{0\text{-tr-st}} = r_{0\text{-st}} \cdot \left[ 1 - 0.52 \cdot \left( \frac{r_{0\text{-st}}}{D} \right)^{1/3} \right]^{-\frac{1}{2}}. \tag{5}
\]
is somewhat contradictory to the 
\[ r_0 \approx \frac{8}{D + \rho_{\text{FIS}} \cdot \frac{10}{0.59 \cdot \lambda^2}}; \quad r_0 \leq D. \] 

Of course, a short-term \( r_{0,\text{st}} \) derived from a single FIS has no statistical meaning; however, any longer term \( r_0 \) values can be derived from vectors of \( r_{0,\text{st}} \) by averaging, where a sliding-mean algorithm is recommended for smoother data representation.

Obviously, the transition region from real short-term measurements “st” (single samples with exposure times below the atmospheric coherence time) to a real long-term estimation of \( r_0 \) needs to be account for. We suggest, therefore, to put the averaging time in seconds for the calculation of a modified (intermediate-term) Fried parameter into the subscript to clearly state the averaging effect, e.g., \( r_{0,0.1} \) or \( r_{0,10} \). The effect of two different averaging times is shown in Fig. 6.

It shall be clarified again that the introduction of the tilt-removed \( r_0 \) parameters in this paper is for derivation purposes only because they have no real physical meaning.

5. Summary

We have deduced a method to find an estimate for \( r_0 \) in a dynamic link scenario. A classical \( r_0 \) cannot be derived in unstable scenarios where the atmospheric volume traversed by the link changes very rapidly (in seconds or even shorter), such as in a satellite downlink. To account for the impracticality of measuring the real aAoA in such mobile (tracked) link scenarios, a way of determining \( r_0 \) from the size of the short-term tilt-removed FISs has been derived. The introduction of a short-term \( r_{0,\text{st}} \) is somewhat contradictory to the basic meaning of the Fried parameter, which is, by definition, a long-term parameter. By stretching the definition of a long-term \( r_0 \) to average values of only \( n \) seconds (\( \rightarrow \)”no more”), we are able to estimate this essential parameter also in dynamic or mobile scenarios.

Further work should involve a relation from these short-term parameters to other parameters for the atmospherically distorted optical wave, namely, the instantaneous heterodyning efficiency \( \eta_{\text{het}}(t) \), and so to determine the performance of a coherent or fiber-coupled receiver.

Appendix A: Abbreviations and Symbols

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<th>Abbreviation/ Symbol</th>
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<td>Atmospheric AoA</td>
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<td>( D )</td>
<td>Receiver aperture diameter</td>
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<td>( f )</td>
<td>Focal length of the telescope system</td>
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<td>FIS</td>
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</tr>
<tr>
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<td>FWHM</td>
<td>Full width at half-maximum</td>
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The author wishes to thank the Japanese National Institute of Information and Communications Technology, and JAXA for making possible the KIODO downlink campaigns. Special thanks go to the members of the DLR Optical Communications Group for performing the ground station measurements during KIODO-2009.

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<td>IRT</td>
<td>Index-of-refraction turbulence</td>
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<td>KIODO</td>
<td>Kirari Optical Downlinks to Oberpfaffenhofen</td>
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<tr>
<td>( r_0 )</td>
<td>Fried parameter, atmospheric coherence length</td>
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<td>Short-term ( r_0 ) (for exemplification only)</td>
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<td>Tilt-removed short-term ( r_0 ) (for exemplification only)</td>
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<tr>
<td>( \rho_{\text{dl}} )</td>
<td>Sigma radius of diffraction limited (ideal) focal spot size</td>
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<tr>
<td>( \rho_{\text{aAoA}} )</td>
<td>Focal angular deviation value due to atmospheric angle-of-arrival tilt</td>
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<tr>
<td>( \rho_{\text{FIS,\text{tr-st}}} )</td>
<td>Sigma radius of one FIS sample</td>
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<tr>
<td>( \sigma_{\text{FSD}} )</td>
<td>Sigma radius of long exposure FSD</td>
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<td>( \sqrt{\rho_{\text{aAoA}}^2} = \sigma_{\text{aAoA}} )</td>
<td>Sigma of aAoA over the Rx aperture in rad (assuming normal statistics)</td>
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<tr>
<td>( \sqrt{\rho_{\text{aAoA}}^2} = f \cdot \sigma_{\text{aAoA}} )</td>
<td>Sigma radius of focal spot broadening by aAoA</td>
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