Determination of Spectral Focusing Features of a Metamaterial Slab

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Abstract: The realization of flat superlenses is a major application area of metamaterials. A slab of double negative (DNG) material is capable of imaging with a resolution beyond the diffraction limit. The focusing quality depends primarily on the amount by which the original wavenumber spectrum of the source is restored behind the lens. Even a small deviation from the ideal case of $\epsilon_r = \mu_r =$ -1 limits the wavenumber spectrum of the transmitted field, which may result in a significant degradation of the focusing quality. In this work we determine the width of the transmitted wavenumber spectrum as a function of the configuration parameters and establish a relation between the spectrum width and the imaging quality. Restrictions imposed on the focusing characteristics and difficulties arising in full wave simulations will be pointed out.

Keywords: Metamaterial Slab, Focusing Properties, Spectral Analysis

1. Introduction

As a consequence of negative refraction, a slab with finite thickness d and material parameters $\epsilon_r = \mu_r =$ -1 (refractive index $n = \sqrt{\epsilon_r \mu_r} =$ -1) focuses waves emitted from a point source located at a distance l in front of the slab to a point at a distance of $d-l$ behind the slab [1, 2, 3]. However this process is highly sensitive to the slab parameters and even small deviations from their ideal values may result in a significant loss of the focusing quality. The effect of the deviation from the ideal material parameters (finite lens size, impedance mismatching) onto the imaging quality has been investigated theoretically and numerically in [4, 5]. In this work we follow the approach from [4] and apply a slight perturbation σ to ε_r and μ_r whilst retaining the refractive index of $n = -1$. The field of a point source is expanded into a Fourier integral over transversal plane waves (perpendicular to the optical axis of the slab), and the influence of the slab transmission coefficient onto each mode (plane wave component) is investigated numerically. An approximate method for determining the highest transmitted mode is formulated and the influence of the wavenumber spectrum width onto the focusing characteristics of the slab is studied. We show that the electromagnetic field behavior at the image plane and thereby the focusing quality of a metamaterial slab are mainly determined by the highest mode contained in the wavenumber spectrum of the transmitted field.

2. Problem Definition

A. Analytical Formulation

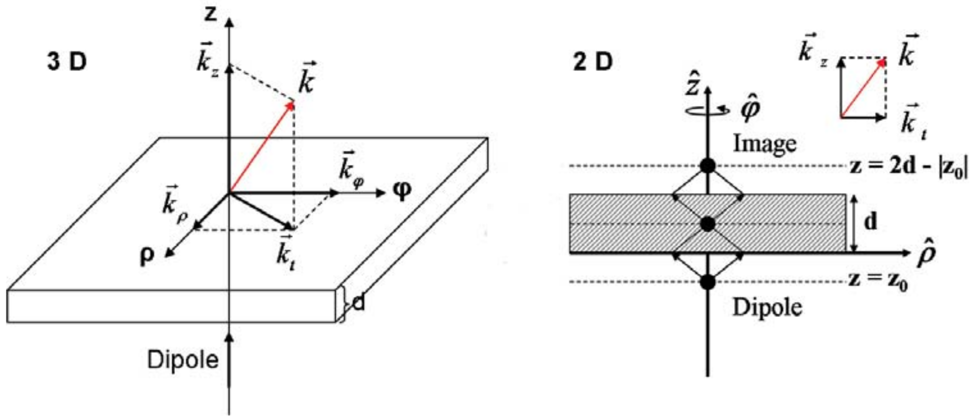


Fig. 1. A point source in front of an infinite metamaterial slab.

We consider the setup given in Fig. 1. Since the configuration is unbounded in the transversal direction the problem can be formulated in 2D by using circular-cylindrical coordinates (ρ, z). A Hertzian dipole with the impressed current density $J(\rho, z) = I_0\delta(\rho)\delta(z-z')\hat{\mathbf{z}}$ is placed on the z-axis at $z' = z_0 < 0$ in front of a DNG slab with material parameters $\varepsilon_r = -(1+\sigma)$ and $\mu_r = -\frac{1}{1+\sigma}$ and thickness d. The wave impedances Z and longitudinal wavenumbers k_z along the optical axis (**z**-axis) for TM-modes are

$$k_z^{air} = \sqrt{k_0^2 - k_t^2} \quad \text{and} \quad Z_0 = \frac{k_z^{air}}{\omega\epsilon_0}, \tag{1}$$

in free space and

$$k_z^{slab} = \sqrt{n^2 k_0^2 - k_t^2} \quad \text{and} \quad Z = \frac{k_z^{slab}}{\omega\epsilon_r} \tag{2}$$

inside the slab, where $k_t = \sqrt{k_\phi^2 + k_\rho^2}$ is the transvere wavenumber parallel to the plane of the slab interface. For a time-harmonic field of angular frequency ω the electromagnetic field is given behind the slab $z > d$ by [6, 7]

$$\boldsymbol{E}(\rho, z) = \frac{-I_0}{4\pi\omega\epsilon_0}\nabla\times\nabla\times\int_0^\infty \frac{k_t}{\sqrt{k_0^2-k_t^2}} J_0(k_t\rho) e^{i\sqrt{k_0^2-k_t^2}(z-d+|z_0|)} T(k_t) dk_t \tag{3}$$

$$\boldsymbol{H}(\rho, z) = \frac{iI_0}{4\pi}\nabla\times\int_0^\infty \frac{k_t}{\sqrt{k_0^2-k_t^2}} J_0(k_t\rho) e^{i\sqrt{k_0^2-k_t^2}(z-d+|z_0|)} T(k_t) dk_t, \tag{4}$$

where $T(k_t)$ is the transmission coefficient of the slab,

$$T(k_t) = \frac{1}{\cos(\sqrt{n^2k_0^2 - k_t^2}\,d) + \frac{i}{2}\sin(\sqrt{n^2k_0^2 - k_t^2}\,d)\left[\frac{\sigma^2+2\sigma+2}{1+\sigma}\right]}. \tag{5}$$

The integration is performed over the entire wavenumber spectrum containing both propagating ($k_t < k_0$) and evanescent ($k_t > k_0$) modes. For the perfect imaging condition with $\epsilon_r = \mu_r =$ -1 ($\sigma = 0$ and $k_z^{slab} = k_z^{air}$) the transmission coefficient simplifys to

$$T(k_t) = e^{-i\sqrt{k_0^2-k_t^2}\,d} \tag{6}$$

so that each mode ($0 < k_t < \infty$) is transmitted uniformly trough the slab, creating a perfect reconstruction of the point source at the back focal plane. The transmission coefficient exhibits a singularity at the critical wavenumber

$$k_c = \pm k_0\sqrt{1 + \left[\frac{1}{k_0d}\tanh^{-1}\left(\frac{2+2\sigma}{2+2\sigma+\sigma^2}\right)\right]^2}. \tag{7}$$

The wavenumber k_c is associated with the excitation of surface waves and has a strong effect on the minimum feature resolvable by the slab. With $k_t > k_c$, the amplitude of the transmission coefficient decays rapidly (Fig. 2a) so that a cutoff wavenumber can be introduced. The cutoff wavenumber k_t^{max} defines the width of the transmitted spectrum and thus the focusing quality of the slab. In the next section we are going to analyze this numerically and formulate an approximate method for the determination of k_t^{max}.

B. Determination of the Cutoff Wavenumber

The intensity distribution along the image plane $I(\rho) = |\boldsymbol{E}(\rho, 2d - |z_0|)|^2$ for $-\infty < \rho < \infty$ can be evaluated numerically by integrating (3). The maximum intensity $I_{max}(0)$ is attained at the focal point $\rho = 0$ and $z = 2d - |z_0|$. Let us assume that the integration is truncated at a certain cutoff wavenumber $k_c < k_t^{max} < \infty$ so that we have

$$\int_0^\infty \frac{k_t}{\sqrt{k_0^2 - k_t^2}} J_0(k_t\rho)e^{i\sqrt{k_0^2-k_t^2}(z-d+|z_0|)}T(k_t)dk_t \cong$$

$$\int_0^{k_t^{max}} \frac{k_t}{\sqrt{k_0^2 - k_t^2}} J_0(k_t\rho)e^{i\sqrt{k_0^2-k_t^2}(z-d+|z_0|)}T(k_t)dk_t. \tag{8}$$

The calculated intensity at the focal point will then be given by $I_{app}(0)$ and we can measure the deviation from the exact value by using a relative error (Fig. 2b)

$$\delta_{rel} = \frac{|I_{max}(0) - I_{app}(0)|}{I_{max}(0)}. \tag{9}$$

The relative error vs. the cutoff wavenumber for different physical configurations is plotted in Fig. 2c (indicated by circles). For a relative error δ_{rel} the cutoff wavenumber can be estimated as

$$k_t^{max} = k_c - \frac{\ln|2\delta_{rel}|}{2d} \tag{10}$$

which is shown by the solid lines. The agreement between the obtained results and the approximation (10) is very good. Furthermore Fig. 2d shows the relation between the cutoff wavenumber and the thickness of the slab. In this figure σ is plotted against the cutoff wavenumber for an assumed relative error of $\delta_{rel} = 10^{-5}$. It can be concluded that the cutoff wavenumber degrades with increasing slab thickness.

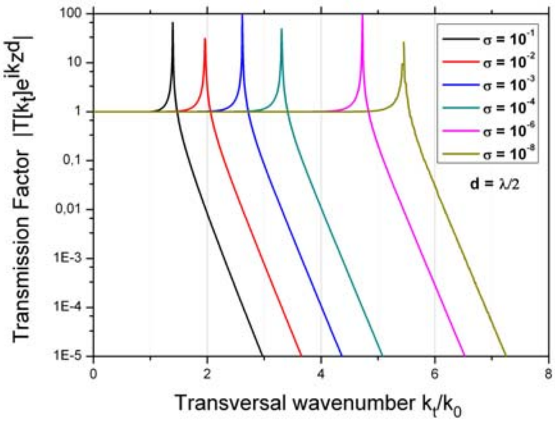
3. Discussion

The resolution enhancement R depends primarily on the value of k_t^{max} and can be defined as the ratio of $R = k_t^{max}/k_0$ [8]. Approaching the ideal case (as $\sigma \to 0$ and hence $k_t^{max} \to \infty$) means that each mode emitted from the point source will be transmitted uniformly through the slab and will be recovered perfectly at the focal point. On the other hand for a nonideal slab ($\sigma \neq 0$) the focusing quality will degrade according to the value of the cutoff wavenumber $k_t^{max} < \infty$. Therefore k_t^{max} limits the minimum feature resolvable by the slab. As k_t^{max} increases, the minimum resolvable feature $\lambda_{min} = 2\pi/k_t^{max}$ will decrease. In the ideal case with $k_t^{max} = \infty$ the complete wavenumber spectrum will be contained in the image resulting in an arbitrarily fine resolution. This can also be inferred from the width $\Delta\rho$ (Fig. 2b) of the main lobe of the intensity pattern.

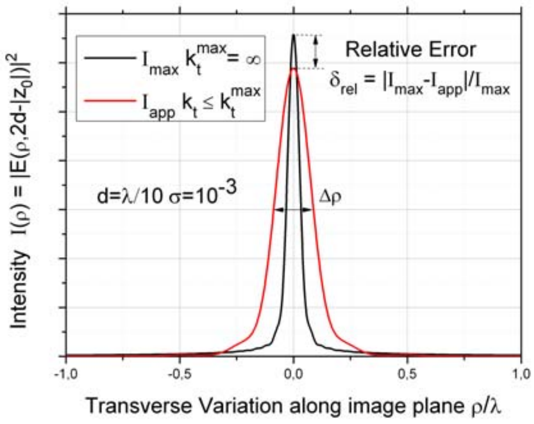
In our previous work [4] the focusing properties of a metamaterial lens were analyzed numerically by the transmission line matrix (TLM) method with negative refractive index materials modeled by matching an inter cell network to a standard 3D SCN node. Discretization of the structure plays an important role [9] and it is expected that the cell size Δl should be at least comparable with the minimum resolvable feature ($\Delta l \approx \lambda_{min}$), in order to achieve accurate results.

4. Conclusion

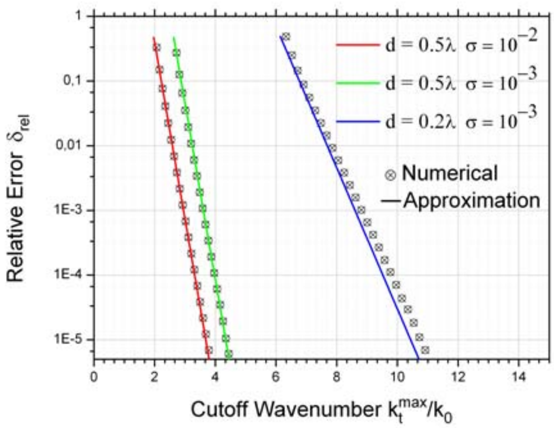
The imaging characteristics of a 2D metamaterial slab have been studied numerically for material parameters perturbed from their ideal values of $\varepsilon_r = \mu_r = -1$. It has been shown that the resolution enhancement depends primarily on the width of the wavenumber spectrum of the modes transmitted through the slab. An approximate analytical estimation for the cutoff wavenumber which limits the resolution has been given. For the nonideal case ($\sigma \neq 0$) the resolution enhancement is strongly dependent on the material parameters (ε_r, μ_r) and thickness of the slab and even a small deviation from ideal values may result in a significant degradation of the imaging quality. The derived estimation for the cutoff frequency can be used for a priori estimations of the required discretization step in TLM-based numerical simulations of metamaterial focusing structures.



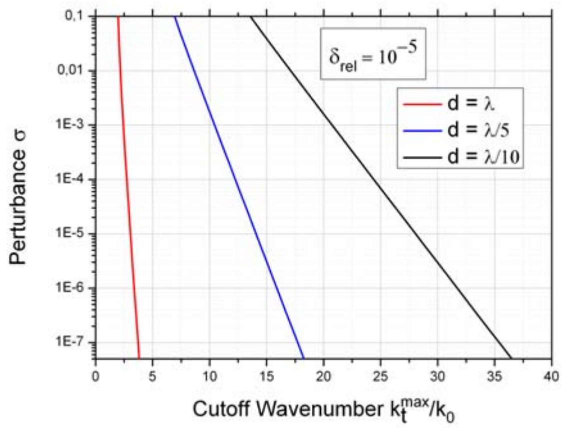
(a)



(b)



(c)



(d)

Fig. 2. (a) Dependence of the transmission factor on the transversal wavenumber for a slab of thickness $d = 0.5\lambda$ and several values of σ (b) Intensity distribution along image plane showing the difference between I_{max} and I_{app} for a slab of thickness $d = \lambda/10$ and $\sigma = 10^{-3}$ (c) Comparison of the relative error: solid lines - approximation (10), circles - numerical integration (3) (d) Relation between the cutoff wavenumber and the perturbation parameter for several values of the lens thickness.

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