Exact G-functions for various leaf normal distribution functions applied to analytical two-stream radiative transport in turbid vegetation media

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Leaf normal distribution functions (LNDFs)

- Turbid media approach (Marshak and Davis, 2005): $g_{L}(\mathbf{x}, \mathbf{y}_{L}, t) \geq 0$ is the probability that a planar leaf at point $\mathbf{x}$ and at time t, its normal vector has the direction $\mathbf{y}_{L}$ is confined to the upper hemisphere S_1^+
- Parameterisation of S_1^+ as a function of the azimuth angle φ and the cosine μ of the zenith angle ϑ:

$$L_o \begin{pmatrix} \cos\varphi\,\sqrt{1-\mu^2} \\ \sin\varphi\,\sqrt{1-\mu^2} \\ \mu \end{pmatrix} := \hat{\boldsymbol{\omega}}(\mu, \varphi)$$

with the unith length L_o and the parameter $(\mu, \varphi) \in [0,1] \times [0, 2\pi]$
- Homogeneous and time independent LNDF with separated angular dependence $g_{L}(\mathbf{y}_{L} = \hat{\boldsymbol{\omega}}(\mu_{L}, \varphi_{L})) = g_\mu(\mu_{L})\, g_\varphi(\varphi_{L})$ with $g_\varphi(\varphi_{L}) := 1$
- Normalisation condition

$$\int_{[0,1]} g_\mu(\mu_{L})\, d\mu_{L} = 1$$

- Standard LNDFs which fulfil the above normalisation condition: purely horizontal, erectophile, extremophile, uniform, plagiophile, planophile, rather vertical and purely vertical leaves normals (Otto and Trautmann, 2008a)

Explicit analytical G-functions (GFs)

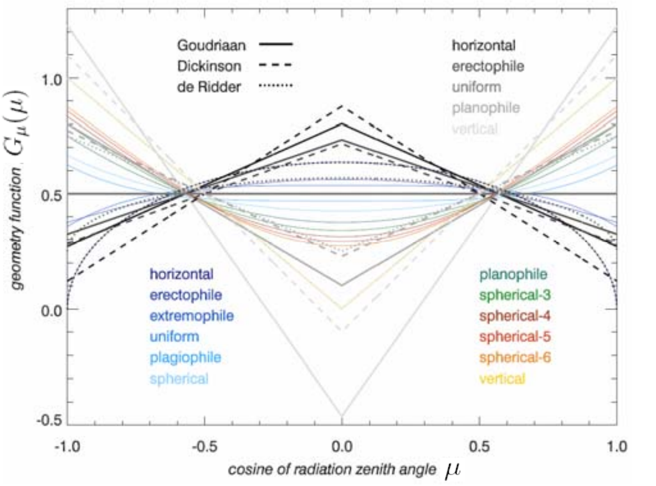
- Ross-Nilson intersection function is a geometry function which is calculated for turbid media as follows (Ross, 1981)

$$G(\mathbf{y}) := \frac{1}{A(S_1^+)} \int_{S_1^+} |\,\mathbf{e}_{\mathbf{y}} \cdot \mathbf{e}_{\mathbf{y}_{L}}|\, g_{L}(\mathbf{y}_{L})\, do(\mathbf{y}_{L}) \quad \text{with} \quad \mathbf{e}_{\mathbf{z}} := \frac{\mathbf{z}}{\|\mathbf{z}\|},\ \mathbf{z} \in \mathbb{R}^3$$

and is azimuthally independent and axially symmetric for standard LNDFs

$$G_\mu(\mu) := G(\mathbf{y} = \hat{\boldsymbol{\omega}}(\mu, \varphi)) \quad \text{and} \quad G_\mu(\mu) \equiv G_\mu(-\mu)$$

- Explicit analytical expressions for standard LNDFs (Otto and Trautmann, 2008a) in comparison to approximations (Goudriaan, 1977; Dickinson et al., 1990; de Ridder, 1997):

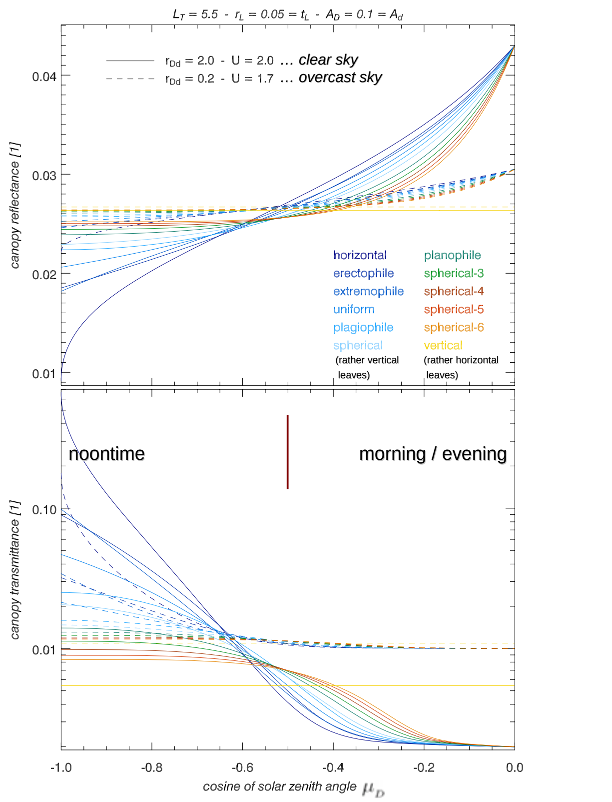


Two-stream approximation

- Time independent and horizontally homogeneous radiative transfer equation for turbid vegetation media assuming bi-Lambertian, elastic leaf scattering
- Two-stream approach for the upward and downward diffuse irradiances $E_\pm(L, \lambda)$ where the variable *L* is the leaf area index and λ the wavelength
- Analytical solution of the two-stream system of equations for standard LNDFs

$$\frac{d}{dL}\mathbf{E}(L,\lambda) = \begin{pmatrix} \alpha_1(\lambda) & -\alpha_2(\lambda) \\ \alpha_2(\lambda) & -\alpha_1(\lambda) \end{pmatrix} \cdot \mathbf{E}(L,\lambda) + D(L,\lambda) \begin{pmatrix} -\alpha_3(\lambda) \\ \alpha_4(\lambda) \end{pmatrix}$$

(Otto and Trautmann, 2008b) for a homogeneous layer using exact GFs
- Input parameters:
surface albedo of direct A_D and diffuse light A_d, total leaf area index L_T, cosine solar zenith angle $\mu_D < 0$, diffusivity factor U, hemispherical leaf reflectance and transmittance r_L and t_L, downwelling direct and diffuse irradiances at the top of the vegetation layer E_D and E_d
- Simulation of the canopy reflectance (CR) and transmittance (CT) as a function of the cosine of the solar zenith angle for two ratios of incident direct and diffuse light $r_{Dd} = E_D / E_d$ in the UV-VIS spectral range:



Radiation transport depends significantly on the orientation of the foliage (LNDF) as well as on the sky conditions above the vegetation:
+ CR for overcast skies is larger/lower at noon/morning-evening than for clear skies
+ CT of rather horizontal foliage (deciduous forests) is always larger under overcast sky conditions

- The one-layer solution can be extended to vertically inhomogeneous medium

References:
de Ridder, Radiative transfer in the IAGL land surface model, J. Appl. Meteorology, 36, 12-21, 1997.
Dickinson, Pinty, Verstraete, Relating surface albedos in GCM to remotely sensed data, Agric. For. Meteorology, 52, 109-131, 1998.
Goudriaan, Crop meteorology: a simulation study. Center for Agric. Publ. and Doc., Wageningen, The Netherlands.
Marshak and Davis, 3 D Radiative Transfer in Cloudy Atmospheres, Springer, 2005.
Otto and Trautmann, A note on the G-function with respect to radiative transfer in turbid vegetation media, J. Quant. Spectrosc. Radiat. Transfer, under review, 2008a.
Otto and Trautmann, Fast analytical two-stream radiative transfer methods for horizontally homogeneous vegetation media, Wissenschaftliche Mittelungen aus dem Institut für Meteorologie der Universität Leipzig, Band 42, 17-32 , 2008b. http://www.uni-leipzig.de/~meteo/ORGA/LIM_Bd_42.pdf
Ross, The radiation regime and architecture of plant stands, Dr. W. Junk Publishers, The Hague, 391 p., 1981.