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A BI-LEVEL MODEL OF DAILY TRIP CHAINS

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1. INTRODUCTION

The development of proper methodologies for modeling urban transport processes is of paramount importance once they are utilized as planning instruments that support decision-making. Models are applied within planning processes; they serve as instruments for the prediction of transport demand and help to monitor the performance of transportation systems in major cities. To this purpose, models have to deal with complex urban environments and processes on different spatial and/or temporal scales. In this sense, one would expect a urban transport model able to handle different challenges: to be responsive to changes that influence transport demand on a long-term, like new job opportunities or changes in land-use, and to be sensitive for processes with effects on the short-term like daily activity-scheduling or new shopping facilities.

Some of the well-known strategic urban transport models (such as ESTRAUS and EMME-2), simplify the treatment of complex urban systems in order to represent reality in a single scale system for each of the most relevant dimensions: time, space and population. Thus, most of these models lack of certain characteristics that approaches based on the concept of activity-based analysis address by understanding travel demand as derived from interdependent activities and trips throughout a day. At the same time, typical 4-Step-Models are trip-based and consider a small travel purposes; it distinguishes not more than 3 to 4 time periods and divides the demand side in limited user classes. Today these models are primarily applied to support strategic decision-making concerning environmental and fare policies as well as projects of road and public transport infrastructure.

In this paper, we develop a methodology that aims at introducing a more detailed analysis of processes and elements on different levels concerning users, space, time, activities (trip purposes) and travel modes. Thus, we make a first attempt to design a multi-scale model of the urban system, concentrating our research on a bi-level model, with micro and macro scales, formulated to describe the activities and trips performed in a single day by a heterogeneous population.

In formulating the model, we apply classical concepts drawn from the literature of dynamic systems, such as hierarchal and memory based processes, originally observed in ecological systems but subsequently extended to economic and social systems (see Gunderson and Holling, 2002). Although the approach closely follows ideas of systems simulation, we somehow depart from these methods when, following micro economic principles, we introduce equilibrium concepts that impose resources constraints in the consumers' behavior.

The model is applied to the case of Santiago City combining the existing macro scale models of land use and transport (MUSSA and ESTRAUS) with detail information of consumers' choices obtained from Santiago's Travel and Household Survey (EOD).

2. THE URBAN DYNAMIC SYSTEM

In this section we discuss some basic concepts of dynamic systems and how they are applied to the urban context.

The hierarchical search

One basic issue of the methodology is that, choice processes for activities and later on for related destinations and travel modes are modeled as they were made hierarchically. This means for example that an activity-plan (refers to a given number and order of activities, as defined in detail in the next section) is assigned to an individual whereas the activities of a plan are differentiated by those of primary and secondary importance.

The search process for an optimal activity plan, considering a specific optimal path, is extremely complex for human beings due to the huge amount of options and the information needed to analyze them. Such complexity means also high requirements for advanced computers. Additionally, the effects of constraints on resources and details of specific processes can only be properly described at the adequate scale.

In order to introduce a rational strategy to reduce complexity, the information can be organized into a hierarchical structure of space and time; this structure is common to several complex systems both in nature and in social organizations. Regarding spatial hierarchy the individual's decisions are first made on the upper – the macro level – and then further disaggregated to a micro level. The macro level is spatially characterized by the cities, comprising around 1,000 Traffic Analysis Zones (TAZ) in big cities such as Santiago. The micro level by their further distribution could comprise approximately 50,000 city blocks.

This structure is also adequate to model the dynamics of the subsystems, which are different at each level: at a micro level the speed of change of variables is higher (small time windows) than those at a macro scale, while changes at the macro scale have dominant impacts on the micro scale. For example, a secondary activity, such as daily shopping, takes place in a time window of the order of one hour and choices of their location usually occur in the vicinity of another primary activity; conversely, work is a primary activity consuming a large part of the time budget and changes in jobs locations usually happen in a time window of years in the context of the whole city.

The decision for destinations and modes of each activity is made simultaneously in reference to the transport system supply quality (times, costs). It goes along with an adjustment process to adjust modeled expected time expenditure to observed time-use of travelers to make sure that the complete activity-trajectory (refers to an activity-plan complemented by destinations and modes) is practicable. Second, this methodology is based on the definition of probabilities about the realization of specific activity-trajectories. This is based on the computation of conditional probabilities calculated directly from matrices containing trip distributions available at the TAZ-level of the city. The third methodological aspect addresses the transition processes of mode and destination choices between the macro- and micro level.

Thus, we develop a bi-level model with two levels in the spatial scale: macro (zones) and micro (blocks), with the city being the third and highest level.[1] For each scale we define a corresponding category for population, time windows, activities and land use. To decide appropriate spatial scales for each choice process we consider the following rules:

1. *Every process/constraint that exceeds the dimensions of the micro scale has to take place at the macro level. Conversely, every process/constraint that is fully embedded in a micro scale has to be described at micro scale.*
2. *The dynamic of a micro scale system is constrained by the slow moving variables at the macro level. Conversely, micro level fast moving variables influence macro variables.*
3. *The scale of each process/choice is directly associated with the amount of resources required.*

These rules have the following practical implications in building hierarchies:

- The amount of information required to make informed choices at each level is bounded by the human effort, so the computer power required may be considered a constant in defining an appropriate hierarchical structure.
- The micro level processes are faster than macro levels, respectively their variables and choices adjust quicker to equilibrium. Thus equilibrium iterations at the macro level are less frequent than at a micro level.
- Macro level processes consume a larger amount of scarce resources than micro level ones.

[1] We use capital indices for variables at macro level and lower case indices for variables at micro level.

- Macro levels resources accumulate physical and cultural stocks decided at lower levels.

The time-hierarchy of processes in a city may have the structure shown in Table 1.

Time windows	**Time units**	**Consumers**	**Suppliers**	**Regulators**
Very long term	**Centuries**	Cultural factors	City's buildings and network structure New cities	
Long term	**Decades-Years**	Family structure Education Car ownership Residence Job	Infrastructure: Buildings Roads/tracks Bridges Technology	Plans: Regulations Incentive policies Infrastructure plans
Short-very short	**Year-months**	Time and location of leisure activities Transport modes	Operations: level of services	
Very short	**Day-hours**	Route choice Local transport modes Walking destinations	Adjustment of operations: stops and delays	

Table 1. Time hierarchy for activities

Memory

In contrast with most systems, human behavior benefits from memory of past experiences and accumulation of physical and human capital. This implies that past failures and achievements are considered by the agents' search for future optimal choices.

The implication of this condition is that the specific process that implements the utilitarian criteria at every hierarchical level is naturally dynamic, as new choices depend on previous ones. How long back in time memory matters for a current choice is likely to be a number of periods back in time, were the period refers to the time-window as shown in table 1 and hence, is associated with the hierarchy where the choice takes place.

Memory has also the effect of slowing down the speed of the dynamic of choices, since building up the memory information is not instantaneous. This argument has implications in the choice modeling specification which should evolve with parsimonious changes.

Stochastic shocks

At every level of the hierarchy, unexpected shocks may occur. We consider two types of shocks: internal and external shocks. Internal shocks represent unpredictable responses of agents to internal stimulus, which may temporarily change the equilibrium of the system to an unstable situation but it does not, per se, change the long term equilibrium. In contrast, external shocks change the context of the system and the equilibrium permanently.

An internal shock at a macro level causes external shocks at a micro level, because it implies a change on available resources to perform the micro processes along the whole micro level time-window. For example, a new job may modify both the income available for all other activities and the available time budget if the job location or working hours changed. Therefore, all secondary activities will be confined to the new set of basic resources left (e.g. income and time) after primary activities have been defined.

Proposition 1: *Inter-scales shocks independence*

Stochastic shocks may be correlated if their respective sources are defined within the same time scale, but are naturally independent if these sources are defined in different scales. Additionally, micro scales shocks are conditional on macro temporal scale shocks.

Proof (pending)

General approach of a bi-level model

The simplest multi-scale model is the bi-level model, where we postulate that much of the features of a multi-scale case, or most of the important ones, can be analyzed by the following simple micro-macro model.

To consider the processes at the micro level, the decisions made at the macro level for an activity-trajectory are spatially disaggregated to the block level. Depending on the type of activity, traveler characteristics, the chosen mode and the specific characteristics of the blocks within one TAZ, trips “arriving” at a zone are further distributed. Therefore, detailed information about land-use and transport supply characteristics are processed to provide the necessary information for the micro level decisions. Next, we analytically show the transition from the macro to the micro level for a finite amount of disaggregated TAZ.

Be α and β the parameters controlling the choice process at the macro and micro level respectively. Be Z and z the variables of these processes respectively, and denote by $\overline{x}$ the expected value of any variable x, such that $\overline{x} = E(x+\varepsilon)$ with ε the variable random term that describes internal potential shocks. Let Y and y be the set of resources available for all processes at the macro and micro level respectively, where elementary resources include time, income, and any other resource required to perform processes at each hierarchical level.

Hence we define the choice in any given process k at the macro level, by the following probability:

$$P^{k,T}_{I \in C^{k,T}} = P(U^{k,T}_I, U^{k,T}) \quad \text{with} \quad U^{k,T}_I = U^k((Z_I(\bar{z}); Y, \alpha)^T) \qquad (1)$$

where the super-indices define a specific process *k* in the macro-period *T*. $U^{k,T}_I$ and $U^{k,T}$ represent respectively: the utility of option *I* and the expected highest utility across all options in the set $C^{k,T}$. Z_I is the set of variables characterizing option *I*. *Y* is the set of resources available for process *k* at period *T*. $\bar{z}$ is the expected status of the micro variables that define process *k*. α is the set of parameters associated with the choice process.

At the micro level we define the following choice probability:

$$P^{k,t}_{i \in C^k(I)} = P(U^{k,t}_i, U^{k,t}) \quad \text{with} \quad U^{k,t}_i = U^k((z_i, y(I), \beta(I))^t) \qquad (2)$$

The dependence of a micro choice process at *t* on the state of the macro level at period *T*, with $t \subset T$, is represented in two ways: by the available resources after macro choices have been decided and by the behavior parameters associated with this micro level. $C^k(I)$ denotes the choice set associated with process *k* restricted to option macro-level option *I*.

Intra-hierarchy level interactions (*cross systems*): Resources available for any process *k* are assumed shared by all other processes at the same period *T*. This implies that choices of process *k* are dependent on all other choice processes competing for the same resources in the resources set. Hence, a process *k* defines a subsystem interacting with other subsystems of the same hierarchy level and time period.

Inter-hierarchy levels interaction (*cross hierarchies*): There are two directions of interactions, from macro to micro, or from slow to fast variables (downwards interaction), and from micro to macro (upwards interaction). The downwards interaction is defined as twofold in (2), through available resources and by the state of behavior parameters. This type of interaction is critical for the micro and dependent level because it may affect drastically and suddenly the equilibria conditions, thus inducing severe instability on the micro system. For the upwards interaction we define the expected value of $\bar{z}$ as an index of expected value at equilibrium in the micro-level across time period *T*, which is in turn defined as a function of the expected values associated with the micro-level time windows *t*; that is, $\bar{z}^T = f((\bar{z}^t)_{t \in T})$. These interactions can only be observed along a period of time of several *T* periods. The upwards interaction may also have drastic consequences on the macro level equilibria if the cumulative variable overpass perceived or natural boundaries, such as the cumulative increase in energy consumption or in congestion.

Memory and hysteresis. The effect of memory on any choice process *k* can be introduced in the inter-temporal definitions of variables and parameters. Processes holding memory have inertia, which affects the speed of the dynamics, since the forces for change are undermined by the forces of retaining the observed past equilibrium. Some choice process may be hysteretic, which means that the process is not fully irreversible. This feature is particularly relevant in long term forecasting and policy making, since it implies that reversing policies that induced undesired equilibrium implies an effort much larger than the proportional effort used for attaining such wrong equilibrium.

Resources variability. Some resources are naturally fixed, e.g. time for daily activities, other are variable on time. Variable resources, like income, depend on individuals choices made in the past, such as stock of durables, and shocks affecting the individuals' wealth, thus we can say $Y^T = Y(P^{-t}, \varepsilon^T); y^t = y(Y^T, P^T)$. Thus, engagement on activities that affect accumulation of resources for the future is crucial defining the size of the individuals' future resources available and affects also crucially the way he/she expends resources currently available.

3. THE DAILY TRIPS MODEL FORMULATION

The general framework for the bi-level model is applied in this section to the problem of modeling daily trips. We consider an isolated day, where each individual in a city population has to choose the activities to be performed during the day and the sequence of such activities, called the *activity plan*. In addition, the individuals decide the best way to perform such a plan: the location and travel mode for each activity given the location of other activities in the plan, called the *plan path*. With the generation of complete activity-trajectories (plans and paths) we create a highly disaggregate travel demand being in line with approaches of activity-based demand-generation (see Bowman and Ben-Akiva, 2000; Bhat et al., 2008; Balmer et al., 2005).

A model is built upon the following general normative hypothesis of human behavior. First, individuals are rational beings able to make complex decisions by means of their preferences on performing activities. Secondly, the model recognizes that the process of activity scheduling and travel decisions across time and space requires a highly efficient strategic search to limit the cognitive burden of inspection of alternatives and choice making to a human capacity.

The strategic search is modeled assuming that individuals simplify the time and space scales by means of associating the appropriate scale level to each action. This is modeled by considering a setting where the scales of time and space are defined as a two levels (macro-micro) hierarchical system.

The rational assumption implies that all choices are made by individuals maximizing their utilities; both at micro and macro level, on activities and time, and that this maximization process must be consistent across scales.

The daily travel process

Although a day is a short term time window, a daily trip pattern represents the travel demand composed by the long term activities (macro choices) valid at the day analyzed, plus the short term activities (micro choices) optimized in that day. Therefore, it represents a demand model of a given point in time, but across the time scales, where the macro scale activities are fixed in the short run, but are variable in the long run.

Analytically, the activity plan s is defined as a sequence of activities to be performed during the day. Each activity plan can be associated with an individual n, residing at a home location i, which we will denote $s(n,i)$. The individual travels to each activity and then spends time on performing the activity, which is defined as a stage in the plan. Figure 1 shows a scheme of an activity plan with $\bar{e}$ stages, with travel times (tt) and activity times (ta) at each stage e.

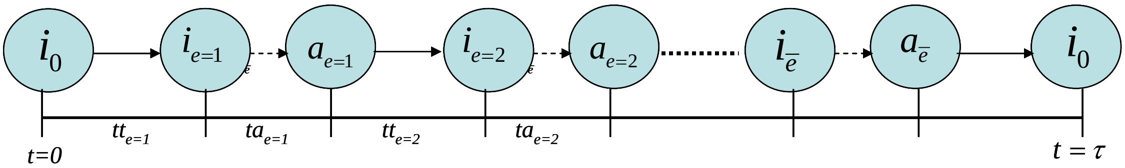

Figure 1: Timeline of an activity plan

Figure 2 describes a single stage, where the trip is represented in a bi-level space: the trip starts at a micro zone i, then travel by transport mode (denoted m^-) through a micro level network to either another location in the same zone (micro-space choice) or until it enters the macro level network at a macro level zone (the abstract centroid I), travels by mode M along the macro level network to the macro zone destination (abstract centroid J) where it enters the micro level network, to travel by mode m^+ to the micro zone j.

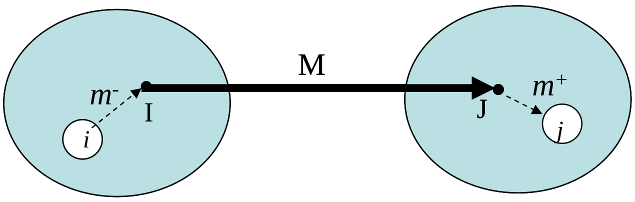

Figure 2: Trip legs of a stage

The bi-level model is a simplified way to represent how individuals simplify the spatial system and the transport options; at the same time provides the basis for our modeling approach, where transport modes, network and space are described in two different scales.

Notation

Notation of indices

N, n: indices for macro and micro scale population categories
I, J: indices for macro-zones scale of spatial representation
i, j : indices for micro-zones scale of spatial representation
s: index for a daily activity plan

e: index for a stage (activity) in the daily plan; stages are ordered according to the sequence of activities described in the plan s
A, a: indices for activities, also called trip purposes in the macro scale.
T, t: indices for time scales at macro and micro aggregation levels
M, m: indices for transport modes at macro and micro aggregation levels
m^-, m^+: indicates the mode used in the micro-zones, (-) the at the trip origin and (+) at the trip destination
$\#X$: the dimension of vector whose elements are indexed by X

Notation of processes and variables

$\xi_{s/n,i}$: Path of daily activity plan s, for an individual type n residing at zone i. It is composed by the sequence of stages of the plan where at each stage one activity is performed, then $\xi_{s/n,i} = \left\{ \xi_{e/s,n,i_{e-1}} ; \forall e \in C_s \right\}$, with C_s the set of stages in plan s and i_{e-1} represents the origin of the previous stage. This representation describes the location and transport mode for each stage of the plan as described next.

$\xi_{e/s,n,i_{e-1}}$: Expected path of the specific stage e in the activity plan s, from the origin of the previous stage i_{e-1} to the destination of the current stage, including the set of zones and modes involved at the micro and macro levels. Figure 1 shows that each stage involves a maximum of three trip-legs: micro, macro and micro legs, denoted as $\xi_{e/s,n,i_{e-1}} = \left\{ v_{k/e}, k \in (1,2,3) \right\}_{sni_{e-1}}$.

$v_{k/e}$: The trip-leg is a pair of destination and mode, which can be performed either in the macro or the micro spaces. If two consecutive activities are performed within the same macro zone then there is only one trip-leg that occurs within, which we denote by $\xi_{e/s,n,i_{e-1}} = (v_{1/e})_{sni_{e-1}} = \left\{ (j,m) \right\}_{seni_{e-1}}$, and where the single leg corresponds to the movement from i_{e-1} to j using mode m, at a micro spatial scale. However, if the secuential activities are performed in two different macro zones, it involves three trip-legs, then $\xi_{e/s,n,i_{e-1}} = (v_{1/e}, v_{2/e}, v_{3/e})_{sni_{e-1}} = \left\{ (I, m^-), (J,M), (j, m^+) \right\}_{seni_{e-1}}$, involving both the micro and macro spatial scales. Note that in a multi-level system with N levels, the maximum number of trip-legs for a path is $2N$-1; in our case N=2 so the path is composed by three trip-legs.

For example, in the stage described in Figure 2:

i) $v_{1/e}$=$((j,I),m^-)$, which means the decision if the activity of eth-stage is performed within the same zone than activity e-1, e.g. stay in the same zone and choose the micro destination zone j in a micro-spatial scale; or choose a destination elsewhere in the macro-spatial scale and then travel to the zone's centroid I; then choose also the transport mode to reach j or I in the micro-spatial scale.
ii) If the k-location is in a different zone, then decide (J,M), the location zone and transport mode in the macro-spatial scale.

iii) Then, choose a location at the micro-spatial scale j within zone J, and a mode m^{+} to travel to that point from the zone's centroid J.

$\tau(\xi_{s/n,i})$: total time required to complete the specific plan s, when it is performed by individual n residing at zone i (i.e. $s(n,i)$), including the time required for activities and travel.

tt_e: travel time for stage e.
ta_e: time spent in the activity at stage e.

Theoretical bases of the bi-level model

i) The utilitarian stochastic behavior

Each individual is assumed to know a set of alternative activity plans C_{ni} (exogenously defined) from where he/she chooses under the criteria of a rational being. Additionally, the individual chooses the set of locations and transport modes to perform the activities in the plan, called the plan path ($\xi_{s/n,i}$). The plan path has to be feasible with regard to time and space; therefore we impose two constraints on the individual's choices.

a) *Feasible total path time:* total time spent on activities and traveling has to match with the daily time budget, and
b) *Spatially consistent path sequence*: each stage must start at the location chosen at the previous stage in the path.

The rational behavior criterion implies that the individual chooses the option that maximizes his/her (indirect) utility. Following the literature on time assignment and activity based modeling (see Jara-Díaz and Martínez, 1999), the indirect utility of an activity plan is assumed to depend on the time assigned to each activity (ta), the opportunities and the quality of the activity defined by the land use in the neighborhood where the activities are performed (l) and the travel time (tt) required, then $U_{ni}(s) = U_{ni}((ta,l,tt)_{e\in s})$.

We define the utility for a specific path to perform the activity plan $U_{ni}(s,\xi_s)$ and develop the model assuming the following separable quasi-linear form:

$$U_{ni}(s,\xi_s) = \left[\sum_{e\in s}\left[U_{ni}(ta_e)\right] + \varepsilon_{sni}\right] + \sum_{e\in\xi_s}\left[U_{ni}(tt_e, l_e) + \varepsilon_{eni}\right] \tag{1}$$

The utility function $U_{ni}(s,\xi_s)$ is really an expression for the indirect utility, and is defined as $U_{ni}(s,\xi_s) = U_{ni}((ta,tt,l)_{e\in(s,\xi_s)})$, with each l_e representing the quality given by the land use. The assumption behind Expression (1) decouples utility depending on the path from the utility of the activity itself. The resulting optimization problem is

$Max\left[U_{ni}(s,\xi)\right]_{(s\in C_{ni},\xi\in C_{sni})}$ where the constraints (time budget and path consistency) are explicit in the definition of the sets C_{ni} containing all feasible plans given the constrains of ni, and C_{sni} containing all feasible paths for each plan.

The first set of terms on the left hand side of (1) defines the utility associated with the time spent on each activity of the plan *s*, with $\frac{\partial U}{\partial t_a}\in\Re$ (positive or negative depending on the type of activity). The second set of terms defines the disutility associated with travel choices on each stage of the path ξ_s which is normally assumed such that $\frac{\partial U}{\partial tt_e}\in\Re^-$ (negative if traveling is *per se* undesirable).

Under Proposition 1, random terms ε_{sni} and ε_{eni} represent independently distributed shocks, associated respectively, with the utilities attained at performing the activities and at traveling. The rationale comes from noting that ε_{sni} represents shocks on the decision on what activities engage and for how long, which involves long term choices like jobs and educations; on the other hand ε_{eni} represents shocks on short term choices associated with traveling.

Therefore, the above setting with quasi-linear and independently distributed random utilities, allows us to write (see Annex 1) the following joint probability that individual *n* residing at zone *i* performs a plan of activities (*s*) and path (ξ_s):

$$P_{ni}(s,\xi_s)_{s\in C_{ni},\xi_s\in C_{sni}}=P_{s/ni}(s\in C_{ni})\cdot P_{\xi/nis}(\xi_s\in C_{sni}) \tag{2}$$

which multiplies the marginal probability of choosing a given activity plan *s* from the set C_{ni}, denoted as $P_{s/ni}$, times the conditional probability of choosing a path $P_{\xi/nis}$ given the activity plan *s*.

We note, however, that the above developments imply the more specific assumption that shocks affecting the long term activity, like accepting a job, and the shocks associated with the destination choice of the trip to work are independent. This may not be totally realistic; an alternative assumption is that long term activities and their locations, both belong to the long term choice set (macro time), while the associated transport choices, as well as other short term activities belong to a micro time scale of the model. This leads to an different set of relations worth investigating and comparing.

ii) The activity plan choice model

The consumer is assumed to maximize utility from performing activities in a given day. He/she chooses the set of activities and the duration that maximizes utility. By equation (1), travel costs are considered homogeneous across options or negligible compared to the utility drawn from activities because traveling consumes/produces less time (and other) resources than activities do. This may be restated: micro decisions do not interfere in macro decisions, except when they build up an amount of resource consumption that affect the macro level.

Then, at the macro level the consumer problem is:

$$\underset{s \in S}{Max} \quad U_{ni}(s) = \left[\sum_{e \in s} \left[U_{ni}(ta_e) \right] + \varepsilon_{sni} \right] \tag{3}$$

or

$$P_{s/ni}(s \in C_{ni}) = P(U_{ni}(s) \geq U_{ni}(s'); \forall s' \in C_{ni}) \tag{4}$$

where C_{ni} is the set of feasible activities that consumer *n* can perform in a day if residing at location zone *i*. The feasible set is defined by the constrained imposed by the income budget and the time budget, as well as other –at least theoretically present-, like physical, psychological and sociological constraints. $U_{ni}(s)$ is the indirect stochastic utility that the consumer perceives from performing the activity plan *s;* defined as timely ordered along the day. For example, denote W for work, Sh for shopping and Sp for sports; the set W-Sh-Sp is defined as different than Sp-W-Sh in C_{ni}. Thus, the size of C_{ni} represents the combinatory of elemental activities, and elemental activities are defined by classifying activities into elemental classes.

The choices of different activities, however, belong to different time scales, as shown in Table 1. Therefore, the model considers a hierarchical choice process, where long term activities belong to the macro scale and define the core of the activities in the set (primary activities) meanwhile the micro scale activities are secondary and decisions for them conditional on the macro scale. This implies that the activity plan's choice probabilities are given by:

$$P_{s/ni}(s \in C_{ni}) = P_{A/ni}(A \in C_A) \cdot P_{a/niA}(a \in C_a) \tag{5}$$

where $C_A \subset C_{ni}$ is the set of long term (macro) activities feasible for individual *n* residing at *i*, $P_{A/ni}$ is the probability of choosing the set $A \in C_A$, $C_a \subset C_{ni}$ is the set of feasible short term (micro) activities and $P_{a/niA}$ is the probability of choosing the set $a \in C_a$ at the micro temporal level conditional on the set *A* at the macro level.

iii) The path choice model

Conditional on the ordered activity set performed daily, the individual decides the best locations to perform the activities and the associated transport modes. As these decisions are taken in mutual dependency, we calculate a combined probability for location and mode choices (see Jonnalagadda et al., 2001). However, location of macro activities belong to the macro time-spatial scale, while the location of micro activities belong to the micro time-spatial scale, and between macro and micro scale decisions there is independency.

The individual's problem is:

$$\underset{e \in E_{ni}}{Max} \quad U_{ni}(\xi_s) = \sum_{e \in \xi_s} \left[U_{ni}(tt_e, l_e) + \varepsilon_{eni} \right] \tag{6}$$

with E_{ni} the set of locations to visit and transport modes available to travel for individual *n* residing at *i*. This optimization problem is defined given the set of activities, their time and sequential order; hence the optimization is over trips destinations and mode choices.

This represents the classical travel demand model, but innovations here are twofold: by formulating a bi-level model and by strictly complying with the given trip sequence of the activity-plan.

An important point is that the activities sequence $e_s=(1,2,\ldots\#e)$ is exogenous in this sub-model, then for any stage e_s, the stage starting point must coincide with the previous trip end point j_{e-1}. This implies that choices at stage *e* are conditional on the choices of stage *e-1*. This condition is known as *trip chaining*, which embeds a difficult combinatorial optimization because the search space is large (see McGuckin et al., 2005).

The trip chaining condition is imposed at any stage by introducing a modification in the utility function such that the previous stage trip end defines the reference point of the next activity, denoted as j_{e-1}. Moreover, trip chains also impose a dependency between mode choices, since for example an individual can only have the auto option available if in the previous stage the auto had already been used. Therefore, accessibility indices of alternative locations/transport modes at the current stage depend on the choices at the previous stage. Then, we define following the individual's trip chain conditional utility for each stage: $U_{nie} = \left[U_{ni}(tt_e, l_e; \xi_{e-1}) + \varepsilon_{eni} \right]$. Under the assumption that dependency between utilities associated to different stages is explicitly modeled by the systematic term of the utility, then the random terms of problem (6) may be assumed independent. Therefore, problem (6) may be restated as:

$$\underset{e \in E_{ni}}{Max} \quad U_{ni}(\xi_s) = \sum_{e \in \xi_s} \left[U_{ni}(tt_e, l_e \xi_{e-1}) + \varepsilon_{eni} \right] = \sum_{e \in \xi_s} \underset{(i,m) \in E}{Max} \left[U_{ni}(tt_e, l_e, \xi_{e-1}) + \varepsilon_{eni} \right] \tag{7}$$

and the following choice probabilities hold:

$$P_{ni}(\xi_s) = \prod_{e \in s} P_{ni}(\xi_e; \xi_{e-1}) \tag{8}$$

Equation (7) and (8) states that, at each stage *e* in the activity plan *s*, the problem of an individual *n* residing at *i*, observed at the starting time of the stage, is to choose an optimal location and transport mode path.

Proposition 2 (bi-level spatial search)

The bi-level model has the benefit compared with the mono-level model of reducing the search space compared with the one level micro-scale model while keeping the search at a micro-spatial scale; the reduction scales down the search space by the same ratio between the dimensions of the macro and micro spaces.

Proof:

Denote [#*i*] the number of micro zones in each macro zone and [#I] the number of macro zones. In a classical one-level micro scale model the dimension of the search space for any given stage is $\Omega_i = [\# i]^2 \cdot [\# I]^2$, and in a macro model it reduces to $\Omega_I = [\# I]^2$; in the bi-level model the dimension of the search space is $\Omega_{i-I} = [\# i] + [\# I - 1]^2 \cdot [\# i] \approx [\# I]^2 \cdot [\# i]$, then $\frac{\Omega_i}{[\# i]} = \Omega_{i-I} = [\# i] \cdot \Omega_I$.

Despite the reduction in the search space, we recognize that the bi-level search space is still large in real applications. A further reduction of the search space is obtained considering search rules in the macro space, based on the assumption that individuals do not search the complete space for every location choice but the sub-set of macro zones that offer better opportunities for the individual. This requires further criteria to define explicitly what means better opportunities.

iv) Equilibrium conditions

Assume travel times between micro and macro zones exogenous. Then the equilibrium condition imposes that the time spent in the activity plan should comply with the time budget, or that it is less. These means that the set of activities to perform in the activity plan and the path choice $(s, \xi_s, \xi_{e/s})$ are adjusted so that the time budget restriction is complied. The adjustment to time budget may imply the selection of a new plan or a new path, in both cases it will affect micro scale first and then macro scale choices if time deficit makes it necessary.

3. MODEL APPLICATION

In this paper we apply the bi-level model to the city of Santiago using information ready available. The objective of this application is to study a case where data at micro and macro scales are available and the model is used to integrate them consistently. Moreover, in this application we use a meso-scale model ready available and we improve its travel demand prediction to a more disaggregate micro-scale level.

For the micro level, we use the origin destination trips survey, EOD 2001, which contains interviews to households collecting information of the household and its members' characteristics and trips of each member during one day (see SECTRA, 2002 and Olguín, 2008). Besides trips the daily activities performed are described as well as their location, travel modes and time of every stage, with a large set of activity types.

Additionally we use the meso-scale land use and transport model MUSSA-ESTRAUS that represents the macro level. A travel origin destination matrix, per mode, activity (or trip purpose) and time period is available from this model for the same population of Santiago city, and year, than the EOD survey.

The approach considers that the MUSSA-ESTRAUS model is used to simulate policies associated to the land use or transport subsystems, which provides estimates of the travel demand and route assignment equilibrium at a macro zone level and for peak and of peak periods for the entire city. At this level activities are aggregated into one residential and three non-residential activities (work, education and other). At a micro level, studies are performed in a more ad-hoc way without imposing equilibrium rules but using detail information of local conditions and simulating choices. It is assumed that the micro level might no be necessarily studied for every macro zone in the city but only for those of special interest like those most directly affected by the policies or projects under study. In figure 3 the interaction of the model in Santiago city is described.

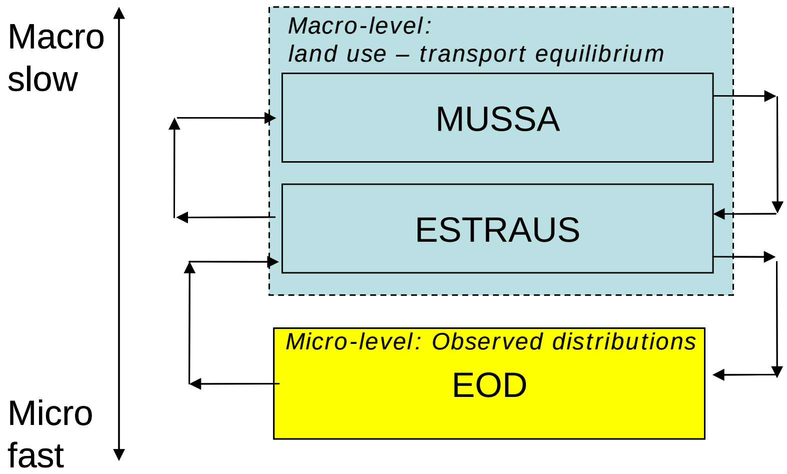

Figure 3: The bi-level model implementation to Santiago city

The methodology follows a top-down approach based on three steps: the activity plan choice, the macro scale land use-transport model and the micro scale travel demand step.

Choose an activity plan

In the first, all activities are classified into activity classes and then into macro and micro temporal activities, regarding to their duration. Hence, the chains derived from the data need to be classified e.g. with regard to the maximum duration of micro (secondary) activities if a macro (primary) activity is not reported or distributions of starting and durations times of the activities. Macro scales activities are those whose destination is adjusted in long term, say a year or more, like jobs or education location choices. Those whose location is adjustable within a day are defined in the micro scale. Then the set of activity plans observed in the EOD survey are analyzed according the activities engaged, first by the macro scale, secondly by the micro scale activities; the location of the micro level is thereby conditional on the location of the macro level activities.

For example, consider the individual n[th] plan being: $s_n(Home,\ Work,\ Shopping,\ Work,\ Home)$, with each element defined as X=(location, duration). *Work* and *Home* belong to the macro scale and *Shopping* to the micro scale, then we classify this activity hierarchically as: $s_n = (X_{0n}, X_{1n}, x_{2n}, X_{3n}, X_{0n})$, where the macro activity plan is *(Work, Work)*, and the micro is *Shopping*; the location of *Home* as the starting point for the plan is relevant for the location choice of *Work*.

The model calculates, for each individual in the EOD sample, the probability of choosing a given activity plan, as the following probability:

$$P_{ni}(s) = P_{ni}(X \in s) \cdot P_{ni/X}(x \in s) \tag{9}$$

where $P_{ni}(X)$ is the marginal probability of choosing the ordered set of macro activities X, and $P_{ni/X}(x)$ is the probability of choosing the set x of activities at the micro scale conditional on the choice X. Equation (9) holds under the inter-scales shocks independence (Proposition 1).

The model calculates these probabilities subject to the conditions that the observed distribution of plans in the population should be reproduced.

Macro model of land use and transport equilibrium

In the second step the macro model is run in a standard way to provide travel OD flow matrices at the macro scale, which yields macro scale estimates of the probabilities (P) of performing trips by period, aggregate activity and household cluster.

The macro travel demand generates travel demand probabilities P_{NIJMAT} that are calculated by the proportion of trips with the following indices for macro clusters out of the total trips with same indices *JMAT*: N for household type, I and J for zones, M for transport modes, A for trip purpose which aggregates activities, and T for time period.

Note that these macro probabilities are estimated by the macro model for each scenario of policies.

Micro model of travel choices

In the third step the micro scale model disaggregates the macro probabilities to obtain micro probabilities (p). In this process we introduce a non homogenous distribution of P estimates into micro estimates of travel demand by means of the transition macro-micro probabilities. For example,

$$p_{nIJMat/(a\in A, t\in T, n\in N)} = P_{NIJMAT} \cdot \pi_{ni/IJM} \tag{10}$$

is the micro level probability an individual n from cluster N makes the macro trip-leg defined by *IJM*, in a trip stage to visit activity a of class A, at time t in period T. It is calculated as the macro scale probability P_{NIJMAT} times the macro-micro transition probability $\pi_{ni/IJM}$; such that $\sum_{\substack{n\in N\\ i\in I}} \pi_{ni/IJM} = 1$. These transition probabilities differentiate the likelihood of choosing a destination-mode option J depending on the trip-leg origin and socioeconomic characteristics of the traveler.

For trip-legs with origin and destination within a macro zone the micro probabilities are directly estimated from the EOD distributions.

Building activity plan probabilities

The previous steps provide estimates of all the macro and micro probabilities which are required to calculate trip chains probabilities and the activity plans probabilities of equation (8). The calculations are:

i) Build stage probabilities: $P_{ni}(\xi_{e\in s}) = \prod_{k\in e} p_{v_{k\in e},nis}$ (11)

where $p_{v_{k\in e},nis}$ is obtained as defined in (10).

ii) Build paths probabilities: $P_{ni}(\xi_s) = \prod_{e} P_{ni}(\xi_{e\in s})$ (12)

iii) Calculate activity plan probabilities: $P_{ni}(s) = P_{ni}(X \in s) \cdot P_{ni/X}(x \in s) \cdot P_{ni}(\xi_s)$ (13)

Adjust probabilities to equilibrium

Macro probabilities are adjusted by running the macro model to each scenario. Then the time spent in the plan (τ_{nis}) is calculated and compared with the time budget τ_n ; then the time excess is calculated by $\Delta\tau_{nis} = \tau_n - \tau_{nis}$. The procedure adjusts probabilities in order to reduce the time excess.

In the adjustment procedure if the excess time is below a given threshold, then the time and location of macro scale activities are considered fixed, and micro scales travel choice probabilities are subject to an adjustment factor which is defined inversely proportional to the time spent in the micro stage. If time excess is larger then the threshold, than all probabilities are subject to an adjustment.

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Annex 1: The optimal stochastic behavior

From the random utility theory, the probability of choosing the set (s, ξ_s) from a feasible set of alternatives is:

$$P_{ni}(s, \xi_s)_{s \in C_{ni}, \xi \in C_{sni}} = \Pr ob\Big[U_{ni}(s, \xi_s) \geq Max\big\{U_{ni}(s, \xi_s)_{s \in C_{ni}, \xi \in C_{sni}}\big\}\Big] \qquad \text{(A1)}$$

Consider the assumptions of quasi-linear and independently distributed random utilities:

$$U_{ni}(s, \xi_s) = \left[\sum_{e \in s}\big[U_{ni}(ta_e)\big] + \varepsilon_{sni}\right] + \sum_{e \in \xi_s}\big[U_{ni}(tt_e, l_e) + \varepsilon_{eni}\big]$$

to write

$$Max\big\{U_{ni}(s, \xi_s)_{s \in C_{ni}, \xi \in C_{sni}}\big\} = Max\left[\sum_{e \in s}\big[U_{ni}(ta_e)\big] + \varepsilon_{sni}\right]_{s \in C_{ni}} + Max\left[\sum_{e \in \xi_s}\big[U_{ni}(tt_e, l_e) + \varepsilon_{eni}\big]\right]_{\xi_s \in C_{sni}}$$

(A2)

which is replaced in equation (A1) to obtain:

$$\begin{aligned} P_{ni}(s, \xi)_{s \in C_{ni}, \xi \in C_{sni}} &= \Pr ob\left\{\sum_{e \in s}\big[U_{ni}(ta_e)\big] + \varepsilon_{sni} \geq Max\left[\sum_{e \in s}\big[U_{ni}(ta_e)\big] + \varepsilon_{sni}\right]_{s \in C_{ni}}\right\} \\ &\quad \cdot Prob\left[\sum_{e \in \xi_s}\big[U_{ni}(tt_e, l_e) + \varepsilon_{eni}\big] \geq Max\left[\sum_{e \in s}\big[U_{ni}(tt_e, l_e) + \varepsilon_{eni}\big]\right]\right]_{\xi_s \in C_{sni}} \qquad \text{(A3)} \\ &= P_{n,i}(s \in C_{ni}) \cdot P(\xi \in C_{sni}) \end{aligned}$$