# Moving from PS to Slowly Decorrelating Targets: A Prospective View.

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## Abstract

During the last 8 years Permanent Scatterer interferometry (PSInSAR) has been widely used as a powerful tool for surface deformation monitoring both for scientific and commercial projects. A few years after the introduction of this technology, it was highlighted how the constraints on the stability of the radar signature of the targets used as opportunistic measurement points could be too tight. Forcing the PS to be coherent in all interferometric pairs used in the analysis could cause the loss of information coming from scatterers exhibiting PS behavior only in a subset of SAR images. Another concern was related to the estimation of the DEM reconstruction: the problem here is to take advantage of the PS framework trying to extend the set of image pixels where an estimation of the elevation is possible. Apart from geometrical decorrelation and variations of the Doppler centroid values of the acquisitions, temporal decorrelation phenomena strongly limit the coherence values of many scatterers within the area of interest. The question arising is whether the presence of radar scatterers exhibiting high or moderate coherence levels at low temporal baselines (typically one month) can be useful for real life applications and, in case of positive answer, which algorithms are most suited to extract useful information from slowly decorrelating scatterers. Although this topic is still the subject of extensive research activities, this paper summarizes some facts that, in the authors' opinion, will be the base of any future algorithm. In particular, the analysis of the coherence matrix computed on a pixel-by-pixel basis is shown to be a key-element of any study of decorrelating targets. Its modeling can be extremely useful to extract feature parameters for image segmentation.

## 1 Introduction

Since the launch of ERS-1 Satellite by the European Space Agency in 1991 a great amount of SAR data has been available for interferometric processing. One of the main application of radar interferometry is deformation monitoring with an unprecedented combination of scale and precision. The joint exploitation of large SAR datasets was made possible principally by Permanent Scatterer Interferometry (PSI) techniques, first proposed 8 years ago (see [1]). The main idea was to limit the interferometric analysis to targets with a radar signature particularly stable in time, called Permanent Scatterers or PSs.

Even in the presence of severe decorrelation by traditional interferometric standards the selection of PSs allowed to recover and process a useful phase signal. The resulting measurement accuracy was proved to be in the order of millimeters and the covered area can be as wide as thousands of square kilometers, even though the measures are taken on a sparse grid on points.

This is precisely the main limitation of PSInSAR with respect to the full potential of SAR data. Only targets that are shining constantly for years can be effectively treated, whereas targets with more transient coherence capabilities are usually discarded. In the former cathegory we find many man-made structures (steel, concrete), rocky formations etc., in the latter agricultural fields and possibly many kinds of natural terrains. Today and in the near future we will be assisting to the availability of SAR data taken by platform or constellations with a short revisit time, a few days instead of a few weeks. With such a short revisit time there will be a huge number of targets that stay coherent for a few takes before decorrelating (see [2]). The information they carry should be exploited and fully integrated in PSI techniques.

## 2 The coherence matrix

We think that one of the main challenges is to identify the presence of interferometric coherence in each location for each possible interferograms. A synthetic view of the interferometric capabilities of a target is given by the coherence matrix. Each element is the coherence of a particular interferogram and all possible $N(N-1)/2$ interferogram are represented ($N$ is the number of images in the dataset). A mathematical definition of the generic element of the coherence matrix is the following:

$$\gamma_{n,k} = \frac{|\,\mathrm{E}[i_n \bar{i}_k]|}{\sqrt{E[|i_n|^2]\, E[|i_k|^2]}} \quad (1)$$

where $i_n$ is the value of a certain pixel in image $n$. Potentially, we could have a different matrix for each target in the scene.

We think that techniques aiming at working with decorrelating targets need to acquire the knowledge of the target coherence form the data themselves, since a reliable independent modeling of coherence is unlikely to suffice. The coherence can be estimated as usual by averaging the interferograms in the space dimension. Let $P$ be a neighborhood of a give target, we compute

$$\hat{c}_{n,k} = \frac{1}{N_p} \sum_{p \in P} i_n(p) \bar{i}_k(p) \quad (2)$$

as the generic covariance matrix element. The coherence matrix element is the same but for a normalization factor:

$$\hat{\gamma}_{n,k} = \frac{\sum_{p \in P} i_n(p) \bar{i}_k(p)}{\sqrt{\sum_{p \in P} |i_n(p)|^2 \sum_{p \in P} |i_k(p)|^2}}. \quad (3)$$

# 3 ML estimates and other possibilities

The coherence matrix can characterize a dataset interferometric potentials, but how to exploit such information? Over the last years we have found that some potential is given by a Maximum Likelihood estimator (see [3]) that we briefly describe here. It is not a definitive solution given a number of issues but it shows nicely how to build from theoretical principles an estimator that adapts itself to the local coherence situation.

We start from the probability density function of a pixel. Let $\mathbf{i}$ be the vector collecting the stochastic variable representing the pixel values through the dataset. We assume that $\mathbf{i}$ is Gaussian distributed which is likely to be the best option for distributed targets where a dominant scatterer is not present:

$$f(\mathbf{i}) = \frac{\exp(-\mathbf{i}^H \mathbf{C}^{-1} \mathbf{i})}{\det(\mathbf{C}) \pi^N}. \quad (4)$$

with $\mathbf{C} = \mathrm{E}[\mathbf{i}\,\mathbf{i}^H]$ being the covariance matrix.

To pass from the probability density function to the ML estimator we introduce some modeling to the phase term contained in the covariance matrix. In our case we consider that the phase term depends on the target elevation $h$ and line-of-sight velocity $v$. We split the $\mathbf{C}$ matrix into three terms highlighting the dependence on the parameters:

$$\mathbf{C}(h,v) = \mathbf{\Phi}(h,v)\, \mathbf{C_0}\, \mathbf{\Phi}^H(h,v). \quad (5)$$

The matrix $\mathbf{C_0}$ is a real-valued covariance matrix and we estimate it from the data. The matrix $\mathbf{\Phi}$ is a diagonal matrix with elements

$$\varphi_n(h,v) = \exp\left(-j\frac{4\pi\, t_n}{\lambda} v\right) \exp\left(-\frac{4\pi\, b_n}{\lambda \sin\vartheta_0\, r} h\right). \quad (6)$$

The Maximum Likelihood estimator is obtained by maximization of the pdf (4) substituting for $\mathbf{i}$ the observed values.

After some manipulations it can be shown that the expression to maximize is the following:

$$F(h,v) = -\mathbf{i}^H \mathbf{\Phi}(h,v) \mathbf{C_0}^{-1} \mathbf{\Phi}(h,v)^H \mathbf{i} \quad (7)$$

or, in a more explicit form:

$$F(h,v) = -\sum_{n,k} a_0(n,k) i_n \bar{i}_k \exp[-j(\varphi_n(h,v) - \varphi_k(h,v))] \quad (8)$$

$a_0(n,k)$ is defined as the $(n,k)$ element of the inverse matrix of $\mathbf{C_0}$.

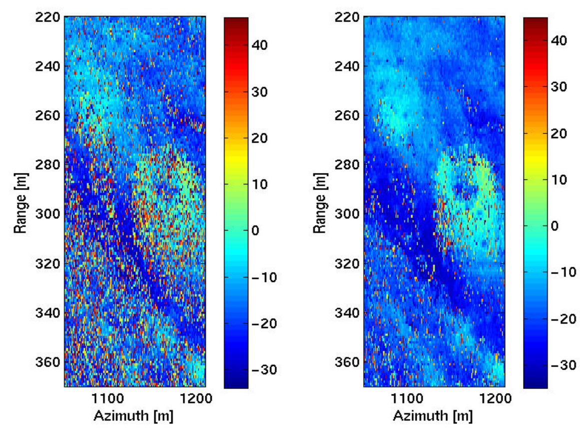


**Figure 1:** Topographic correction. Left: PS processing (coherence is ignored); Right: ML processing (coherence information is exploited).

## 3.1 A general paradigm

Expression (8) can be considered a general paradigm for joint (coherent) estimates from a multi-image dataset. Basically it searches for an agreement between all interferograms, weighted by the $a_0(n,k)$'s. Summing over half the interferograms would be possible taking the real part of the summation. Different choices for $a_0(n,k)$ yield to different estimators.

One possibility would be to assign the weights $a_0(n,k)$ according to some decorrelation model instead to derive them from the data.

- The typical PS estimator is usually presented as a sum over images and not over interferograms. However one can easily show that it can be rewrited into a sum over interferograms with all weights equal. In other words a PS estimator gives each interferograms the same relevance, regardless of the temporal or geometric baseline. The same result can be found choosing the $a_0(n, k)$ according to the inverse of the covariance matrix $\mathbf{C}_0$, when the same coherence is given to each pair, which is a possible definition of a PS.

- With an exponential decorrelation model ($\gamma_{n,k} = \gamma_0 \exp(-|t_n - t_k|/\tau)$) the ML estimator would consider only interferograms between subsequent images. In the particular case when the time sampling is regular and we are looking only for the average velocity the ML estimator would sum those interferograms with an operation which is know as *interferogram stacking*.

- Another possibility is to limit the summation to interferograms with temporal and/or geometric baselines under a certain threshold. This is often a good compromise between speed and reliability.

# References


[1] A. Ferretti, C. Prati, and F. Rocca. Permanent scatterers in SAR interferometry. *IEEE Trans. Geosci. and Remote Sensing*, 39(1):8–20, 2001.

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