Efficient Soft-Output GNSS Signal Parameter Estimation using Signal Compression Techniques

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ABSTRACT

The direct implementation of the maximum likelihood estimator for the time-delay estimation problem is practically intractable for navigation signals due to its complexity, especially when due to multipath reception several superimposed replica are taken into account. In particular, the complexity arises from the evaluation of the likelihood, which requires operations upon a data set of many thousand samples for each code period of the received signal. Thus within conventional navigation receivers tracking loops are used for estimating the signal parameters. Recently it has been shown that signal compression techniques can overcome the problem of computational complexity, as the likelihood function can be formulated efficiently upon a reduced data set of much smaller size compared to the original data, where the reduced data set forms a sufficient statistic for the estimated signal parameters. As a consequence, the evaluation of the likelihood becomes feasible with moderate cost, which might enable a paradigm change in terms of information handover within the receivers, as much more information can be handed over from the signal domain to the navigational part of the receiver. Within this paper a novel approach for integrating a conventional tracking loop with a likelihood evaluator is proposed. It is shown that the parameter information can be kept recursively in a data set of small size in a sequential estimation procedure by using a bank of first order IIR filters, such that an approximate posterior can be reconstructed from the filtered data set. The performance of the proposed method is assessed by computer simulations. The results show that the conventional delay lock loop is outperformed with respect to noise performance as well as with respect to the multipath bias whilst providing capabilities with respect to multipath monitoring and mitigation.

INTRODUCTION

From the viewpoint of navigation receiver algorithm design, satellite navigation can be regarded as a parameter estimation problem that requires the formulation of appropriate estimators within a given framework of real-time constraints. In general, any kind of estimator is designed to provide estimates for parameters based upon observations. The maximum-likelihood (ML) estimator provides the parameter estimate that maximizes the probability of the observations conditioned on the parameters. The maximum-aposteriori (MAP) estimator, which is similar to the MLestimator, but does also incorporate prior knowledge, provides the parameter estimate that is the most likely one conditioned on the observations. Another example is the minimum mean square error (MMSE) estimator, which provides the mean of the posterior density [1]. All these estimators have in common that they are based on operations that obtain a "hard" estimation value from a "soft" probability density function (PDF). As actually only the entire PDF comprises all the information that can be inferred from the observations, every "hard" parameter decision based upon the PDF discards some of the information contained within the PDF. Consequently, it is advantageous to keep PDFs instead of hard estimates in order to improve system performance. But unfortunately storage of PDFs can be complex, especially for general non-Gaussian PDFs. Indeed, in general the estimation of GNSS signal parameters produces non-Gaussian PDFs, having lead so far to receiver architectures that process hard estimates obtained by the tracking loops and assume the underlying PDF to be Gaussian. This is the case implicitly within every conventional navigation receiver and also, for instance, in a more sophisticated way for the vector DLL and all GNSS/INS hybridization schemes that are based on Kalman filtering [2]. Newer approaches, namely the particle filters [3], can overcome the Gaussian constraint by representation of the PDF by a set of samples, and thus are capable of general nonlinear non-Gaussian sequential estimation problems. However, particle filters are optimal only for an infinite number of particles and they are still rather a topic of scientific investigations and not used in practice very often.

Beyond these problems a major impairment has been so far the access to the signal-level likelihood function, which was not feasible due to real-time constraints. Recently it was shown that signal compression techniques can overcome this problem [4][5], as the likelihood function can be calculated from a reduced data set, which is provided by a bank of correlators and is thus of much smaller size compared to the original one. In the evolution of navigation receiver signal processing significant advance is expected to be gained, if receiver signal processing architectures start to evolve towards the desired "soft" PDF-based ones by considering the accessibility to the likelihood function due to the signal

compression. And even if not an entire PDF is provided, at least some more information from the received signal can be made available compared to the conventional tracking loop architecture.

The paper is structured as follows: After the introduction of the signal model the concept of sequential Bayesian estimation is covered briefly. Afterwards, an efficient approximation to a sequential Bayesian estimator is derived theoretically and a possible architecture for implementation is proposed subsequently. Finally simulation results are shown and discussed in order to conclude the paper.

PROBLEM FORMULATION

Assume that the complex valued baseband-equivalent received signal is equal to

$$y(t) = d(t) \sum_{i=1}^{N(t)} a_i(t) s(t - \tau_i(t)) + n(t)$$
(1)

where s(t) is the transmitted navigation signal, d(t) is the data signal modulated on the navigation signal, N(t) is the total number of paths reaching the receiver, and $a_i(t)$ and $\tau_i(t)$ are their individual complex amplitudes and time delays, respectively. The signal is disturbed by additive white Gaussian noise, n(t). Grouping blocks of *L* samples at times $(n+kL)T_s$, n=0,...,L-1, together into vectors \mathbf{y}_k , k=0,1,..., this can be rewritten as

$$\mathbf{y}_{k} = d_{k} \sum_{i=1}^{N_{k}} a_{k,i} \mathbf{s}(\tau_{k,i}) + \mathbf{n}_{k} = \mathbf{S}(\boldsymbol{\tau}_{k}) \mathbf{a}_{k} d_{k} + \mathbf{n}_{k}$$
(2)

In the compact form on the right hand side the samples of the delayed signals are stacked together as columns of the matrix $S(\tau_k)$, $\tau_k = (\tau_{k,1}, \dots, \tau_{k,N_k})$, and the amplitudes are collected in the vector $\mathbf{a}_k = (a_{k,1}, \dots, a_{k,N_k})$. It is assumed in (2) that the parameters τ_k , \mathbf{a}_k , N_k and d_k are constant within the corresponding time interval.

The objective is to estimate the parameters $\mathbf{\tau}_k$, \mathbf{a}_k , N_k and d_k for each time instance k in terms of PDFs, namely the posteriors $p(\mathbf{a}_k, \mathbf{\tau}_k, d_k, N_k | \mathbf{y}_k, ..., \mathbf{y}_0)$. These PDFs contain "soft" reliability information for each parameter instead of a "hard" estimate only.

SEQUENTIAL BAYESIAN ESTIMATION

A general framework for generating such PDFs is given by the sequential Bayesian estimation approach [1][6]. In principle the posteriors can be computed recursively with alternating calculation of the prediction (Chapman-Kolmogorov) equation

$$p(\mathbf{a}_{k}, \mathbf{\tau}_{k}, d_{k}, N_{k} | \mathbf{y}_{k-1}, \dots, \mathbf{y}_{0}) = \int p(\mathbf{a}_{k}, \mathbf{\tau}_{k}, d_{k}, N_{k} | \mathbf{a}_{k-1}, \mathbf{\tau}_{k-1}, d_{k-1}, N_{k-1}, N_{k-1}, N_{k-1} | \mathbf{y}_{k-1}, \dots, \mathbf{y}_{0}) d\mathbf{a}_{k-1} d\mathbf{\tau}_{k-1} dd_{k-1} dN_{k-1}$$
(3)

and the update equation (calculation of posterior pdf):

$$p(\mathbf{a}_{k}, \mathbf{\tau}_{k}, d_{k}, N_{k} | \mathbf{y}_{k}, \dots, \mathbf{y}_{0}) = \frac{p(\mathbf{y}_{k} | \mathbf{a}_{k}, \mathbf{\tau}_{k}, d_{k}, N_{k}) p(\mathbf{a}_{k}, \mathbf{\tau}_{k}, d_{k}, N_{k} | \mathbf{y}_{k-1}, \dots, \mathbf{y}_{0})}{p(\mathbf{y}_{k} | \mathbf{y}_{k-1}, \dots, \mathbf{y}_{0})}$$
(4)

An illustration is given in Figure 1.

Although (3) and (4) describe the exact solution to the estimation problem, in practice they provide only a conceptual solution in the sense that they cannot be determined analytically. Hence approximations or suboptimal algorithms have to be used instead [3].

In the following we propose a novel approach for an efficient approximation of a sequential Bayesian estimator for the problem formulated above and for its realization.



Figure 1: Sequential Bayesian estimation principle

APPROXIMATION OF THE SEQUENTIAL BAYESIAN ESTIMATOR

Efficient representation of likelihood

The data size in a typical navigation system is huge. To reduce the influence of noise, the received signal typically has to be observed over several codeword lengths, which can result in vectors \mathbf{y} containing several millions of samples. This means that even a single numerical evaluation of the likelihood function requires a large computational burden, making such an approach infeasible in a real-time application. This problem can be approached by the reduced complexity techniques suggested in [4] [5]. The large vector containing the received signal samples is transformed into a vector \mathbf{y}_c of much smaller size before the actual evaluation takes place. The goal is a systematic approach to achieve such a reduction with a negligible performance loss.

The likelihood function for the considered problem is given by:

$$p(\mathbf{y}_{k} | \boldsymbol{\tau}_{k}, \mathbf{a}_{k}, d_{k}, N_{k}) = \frac{1}{(2\pi)^{L/2} \sigma^{L}} \exp\left[-\frac{1}{2\sigma^{2}} (\mathbf{y}_{k} - \mathbf{S}(\boldsymbol{\tau}_{k}) \mathbf{a}_{k} d_{k})^{H} (\mathbf{y}_{k} - \mathbf{S}(\boldsymbol{\tau}_{k}) \mathbf{a}_{k} d_{k})\right]$$
(5)

With the data size reduction techniques presented in [4][7] the long vector \mathbf{y}_k can be transformed into a subspace of much smaller size according to

$$\mathbf{y}_{c,k} = \mathbf{Q}_c^H \mathbf{y}_k, \qquad \mathbf{S}_c(\mathbf{\tau}_k) = \mathbf{Q}_c^H \mathbf{S}(\mathbf{\tau}_k)$$
(6)

$$\mathbf{Q}_{c}^{H}\mathbf{Q}_{c} = \mathbf{I} \text{ and } \mathbf{Q}_{c}\mathbf{Q}_{c}^{H} = \mathbf{I},$$
⁽⁷⁾

such that the likelihood can be expressed as

$$p(\mathbf{y}_{k} | \boldsymbol{\tau}_{k}, \mathbf{a}_{k}, d_{k}, N_{k}) = C \cdot \exp\left[-\frac{1}{2\sigma^{2}}\left(-\mathbf{y}_{c,k}^{H}\mathbf{S}_{c}(\boldsymbol{\tau}_{k})\mathbf{a}_{k}d_{k} - d_{k}^{*}\mathbf{a}_{k}^{H}\mathbf{S}_{c}(\boldsymbol{\tau}_{k})^{H}\mathbf{y}_{c,k} + d_{k}^{*}\mathbf{a}_{k}^{H}\mathbf{S}_{c}(\boldsymbol{\tau}_{k})^{H}\mathbf{S}_{c}(\boldsymbol{\tau}_{k})\mathbf{a}_{k}d_{k}\right)\right]$$
(8)

All calculations can now be performed on this reduced size model, as it follows from the Neyman-Fisher factorization [1] that $\mathbf{Q}_c \mathbf{y}$ is a sufficient statistic for the parameters. In other words, there is no information loss if the parameters are estimated after correlation with the matrix \mathbf{Q}_c . If \mathbf{Q}_c is a square matrix, then the transformation above is simply a rotation. In order to reduce complexity one needs to find a rectangular matrix that has a small number of columns and, hence, compresses the data size. Hereby, the conditions above should be satisfied as closely as possible if loss in performance is to be avoided. For real-time applications the signal compression becomes relevant as it can be build upon a bank of correlators that are spaced by a grid of τ_b with respect to the delay domain. Furthermore the evaluation of the likelihood becomes feasible with arbitrary parameter resolution due to interpolation techniques. A detailed description on the data size reduction and the associated interpolation techniques is given in [7].

Approximation of prediction stage

Consider, without loss of generality, the single path estimation problem, i.e., $N_k=1$. Then the likelihood function is given by

$$p(\mathbf{y}_k \mid a_k, \tau_k, d_k) = C \cdot \exp\left[\frac{\Re\{d_k a_k \mathbf{y}_{c,k}^H \mathbf{s}_c(\tau)\}}{\sigma^2} - |a_k|^2 |d_k|^2 \frac{\mathbf{s}_c^H(\tau_k) \mathbf{s}_c(\tau_k)}{2\sigma^2}\right]$$
(9)

In the prediction stage the Chapman-Kolmogorov equation (3) combines this likelihood with the parameter transition model, represented by $p(a_k, \tau_k, d_k | a_{k-l}, \tau_{k-l}, d_{k-l})$. Consider the posterior from the last time

$$p(a_{k-1}, \tau_{k-1}, d_{k-1} | \mathbf{y}_{k-1}, ..., \mathbf{y}_{0}) = C \cdot \exp\left[\frac{\Re\{d_{k-1}a_{k-1}\overline{\mathbf{y}}_{c,k-1}^{H}(d_{k-1})\mathbf{s}_{c}(\tau_{k-1})\}}{\sigma^{2}} - W_{k-1} | a_{k-1} |^{2} | d_{k-1} |^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k-1})\mathbf{s}_{c}(\tau_{k-1})}{2\sigma^{2}}\right]$$
(10)

At first the prediction with respect to the data is considered

$$p(a_{k},\tau_{k},d_{k},a_{k-1},\tau_{k-1} | \mathbf{y}_{k-1},...,\mathbf{y}_{0}) = \sum_{d_{k-1}} p(a_{k},\tau_{k},d_{k} | a_{k-1},\tau_{k-1},d_{k-1}) p(a_{k-1},\tau_{k-1},d_{k-1} | \mathbf{y}_{k-1},...,\mathbf{y}_{0})$$

= $p(a_{k},\tau_{k} | a_{k-1},\tau_{k-1}) \sum_{d_{k-1}} p(d_{k} | d_{k-1}) p(a_{k-1},\tau_{k-1},d_{k-1} | \mathbf{y}_{k-1},...,\mathbf{y}_{0})$ (11)

where the right hand side can be expressed as

$$\sum_{d_{k-1}} p(d_{k} | d_{k-1}) p(a_{k-1}, \tau_{k-1}, d_{k-1} | \mathbf{y}_{k-1}, ..., \mathbf{y}_{0}) = \\ = C_{0} \sum_{d_{k-1}} p(d_{k} | d_{k-1}) \cdot \exp\left[\frac{\Re\{d_{k-1}a_{k-1}\overline{\mathbf{y}}_{c,k-1}^{H}(d_{k-1})\mathbf{s}_{c}(\tau_{k-1})\}}{\sigma^{2}} - W_{k-1} | a_{k-1} |^{2} | d_{k-1} |^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k-1})\mathbf{s}_{c}(\tau_{k-1})}{2\sigma^{2}}\right] \\ = C_{0} \sum_{d_{k-1}} \frac{1}{M} \cdot \exp\left[\frac{\Re\{d_{k-1}a_{k-1}\Delta d_{k,k-1}\overline{\mathbf{y}}_{c,k-1}^{H}(d_{k-1})\mathbf{s}_{c}(\tau_{k-1})\}}{\sigma^{2}} - W_{k-1} | a_{k-1} |^{2} | d_{k-1}\Delta d_{k,k-1} |^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k-1})\mathbf{s}_{c}(\tau_{k-1})}{2\sigma^{2}}\right] \quad (12) \\ \approx C_{1} \max_{d_{k-1}} \exp\left[\frac{\Re\{d_{k-1}a_{k-1}\Delta d_{k,k-1}\overline{\mathbf{y}}_{c,k-1}^{H}(d_{k-1})\mathbf{s}_{c}(\tau_{k-1})\}}{\sigma^{2}} - W_{k-1} | a_{k-1} |^{2} | d_{k-1}\Delta d_{k,k-1} |^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k-1})\mathbf{s}_{c}(\tau_{k-1})}{2\sigma^{2}}\right] \\ = C_{1} \exp\left[\frac{\Re\{d_{k}a_{k-1}\widehat{\mathbf{y}}_{c,k-1}^{H}(d_{k})\mathbf{s}_{c}(\tau_{k-1})\}}{\sigma^{2}} - W_{k-1} | a_{k-1} |^{2} | d_{k} |^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k-1})\mathbf{s}_{c}(\tau_{k-1})}{2\sigma^{2}}\right]$$

Here the following relations have been used:

$$\Delta d_{k,k-1} = \frac{d_k}{d_{k-1}} \quad \Delta d^{(i)} = \frac{d^{(i)}}{d^{\max}}, \quad i = 1, \dots, M$$

$$d^{\max} = \arg \max_{d^{(i)}} p(a_{k-1}, \tau_{k-1}, d_{k-1} = d^{(i)} | \mathbf{y}_{k-1}, \dots, \mathbf{y}_0)$$

$$\approx \arg \max_{d^{(i)}} \overline{\mathbf{y}}_{c,k-1}(d_{k-1} = d^{(i)})$$
(13)

$$\hat{\mathbf{y}}_{c,k-1}(d_k = d^{(i)}) = \overline{\mathbf{y}}_{c,k-1}(d_{k-1} = d^{\max}) \cdot \Delta d^{(i)}$$

In order to complete the prediction with respect to *a* and τ we need to perform

$$p(a_k, \tau_k, d_k | \mathbf{y}_{k-1}, ..., \mathbf{y}_0) = \int p(a_k, \tau_k, d_k, a_{k-1}, \tau_{k-1} | \mathbf{y}_{k-1}, ..., \mathbf{y}_0) da_{k-1} d\tau_{k-1}$$
(14)

We simplify this stage by a mean shift with respect to *a* and τ of the function

$$\exp\left[\frac{\Re\{d_{k}a_{k-1}\hat{\mathbf{y}}_{c,k-1}^{H}(d_{k})\mathbf{s}_{c}(\tau_{k-1})\}}{\sigma^{2}} - W_{k-1} |a_{k-1}|^{2} |d_{k}|^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k-1})\mathbf{s}_{c}(\tau_{k-1})}{2\sigma^{2}}\right]$$
(15)

which is achieved by the following operation on the sample points that represent this posterior:

$$\tilde{\mathbf{y}}_{c,k-1}(d_k) = \Delta a \, \mathbf{M}_{s_c} \operatorname{diag}[\mathbf{\Phi}(\Delta \tau)] \mathbf{M}_{s_c}^{-1} \hat{\mathbf{y}}_{c,k-1}(d_k)$$
(16)

This expression is based on a signal interpolation according to

$$\mathbf{s}_{c}(\tau) = \mathbf{M}_{s_{c}} \mathbf{\Phi}(\tau) \tag{17}$$

with a convolution matrix \mathbf{M}_{s_c} and the Vandermonde vector function $\Phi(\tau)$ [7]. If it is known that the data is not changing from time *k*-1 to *k* the data prediction does not have to be carried out and

$$\tilde{\mathbf{y}}_{c,k-1}(d_k) = \Delta a \; \mathbf{M}_{s_c} \operatorname{diag}[\mathbf{\Phi}(\Delta \tau)] \mathbf{M}_{s_c}^{-1} \overline{\mathbf{y}}_{c,k-1}(d_k) \tag{18}$$

In practice (16) and (18) can be reduced to

$$\tilde{\mathbf{y}}_{c,k-1}(d_k) = \begin{cases} \overline{\mathbf{y}}_{c,k-1}(d_k) & \text{if } d_k = d_{k-1} \text{ by definition} \\ \hat{\mathbf{y}}_{c,k-1}(d_k) & \text{otherwise} \end{cases}$$
(19)

when the shifts $\Delta \tau$ and Δa are performed by means of equivalent delay and phase shifts of the correlator reference signals.

Furthermore a widening by some factor w < 1 is performed, resulting in

$$p(a_{k},\tau_{k},d_{k} | \mathbf{y}_{k-1},...,\mathbf{y}_{0}) = C \cdot \exp\left[w\left(\frac{\Re\{d_{k}a_{k}\tilde{\mathbf{y}}_{c,k-1}^{H}(d_{k})\mathbf{s}_{c}(\tau_{k})\}}{\sigma^{2}} - W_{k-1} | a_{k} |^{2} | d_{k} |^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k})\mathbf{s}_{c}(\tau_{k})}{2\sigma^{2}}\right)\right]$$
(20)

This expression for the prior pdf replaces the ideal prior given in (3).

Simplified update stage

Using (9) and (20) within (4) gives the posterior pdf

$$p(a_{k},\tau_{k},d_{k} | \mathbf{y}_{k},...,\mathbf{y}_{0}) = C \cdot p(\mathbf{y}_{k} | a_{k},\tau_{k},d_{k})p(a_{k},\tau_{k},d_{k} | \mathbf{y}_{k-1},...,\mathbf{y}_{0})$$

$$= C \cdot \exp\left[\frac{\Re\{d_{k}a_{k}\mathbf{y}_{c,k}^{H}\mathbf{s}_{c}(\tau)\}}{\sigma^{2}} - |a_{k}|^{2}|d_{k}|^{2}\frac{\mathbf{s}_{c}^{H}(\tau_{k})\mathbf{s}_{c}(\tau_{k})}{2\sigma^{2}}\right]$$

$$\cdot \exp\left[w\left(\frac{\Re\{d_{k}a_{k}\tilde{\mathbf{y}}_{c,k-1}^{H}(d_{k})\mathbf{s}_{c}(\tau_{k})\}}{\sigma^{2}} - W_{k-1}|a_{k}|^{2}|d_{k}|^{2}\frac{\mathbf{s}_{c}^{H}(\tau_{k})\mathbf{s}_{c}(\tau_{k})}{2\sigma^{2}}\right)\right]$$
(21)

Using the notation

$$\overline{\mathbf{y}}_{c,k}^{H}(d_k) = \mathbf{y}_{c,k}^{H} + w \widetilde{\mathbf{y}}_{c,k-1}^{H}(d_k)$$
(22)

and

$$W_k = \sum_{i=0}^k w^i \tag{23}$$

the posterior in (21) can be written as

$$p(a_{k},\tau_{k},d_{k} | \mathbf{y}_{k},...,\mathbf{y}_{0}) = C \cdot \exp\left[\frac{\Re\{d_{k}a_{k}\overline{\mathbf{y}}_{c,k}^{H}(d_{k})\mathbf{s}_{c}(\tau_{k})\}}{\sigma^{2}} - W_{k} |a_{k}|^{2} |d_{k}|^{2} \frac{\mathbf{s}_{c}^{H}(\tau_{k})\mathbf{s}_{c}(\tau_{k})}{2\sigma^{2}}\right]$$
(24)

which is the time k equivalent to (10).

The sequential estimator will finally reach a steady state with respect to the variance when k reaches the equivalent integration time. In the steady state the magnitude of the filtered data points will not increase any more and the value W_k will converge to the infinite geometric series

$$\lim_{k \to \infty} W_k = \lim_{k \to \infty} \sum_{i=0}^k w^i = \frac{1}{1 - w}$$

$$\tag{25}$$

Due to the approximations described above an efficient realization of the estimator is possible. Namely, the recursive estimation process given in Figure 1 can be transformed into the logarithmic domain in terms of a nonlinear recursive filtering of \mathbf{y}_c , like illustrated in Figure 2. The correlator transforms the received sampled signal \mathbf{y}_k into the reduced space as defined by the data size reduction given in (6). The output $\mathbf{y}_{c,k}$ enters the recursive relation (22) for the different data hypotheses. According to (19) the data prediction is performed as described by (13) whenever required. The output posterior in (24) can now be evaluated from $\overline{\mathbf{y}}_{c,k}(d^{(i)})$ at any desired time, whilst it is not mandatory that this calculation is performed for each time instance k. From practical implementation aspects it can be stated that the estimator just extends the coherent integration period, what is a common technique today, and thus uses a filtered data set before estimation, resulting in a noise level improvement. We have shown that it is possible to reconstruct a true posterior from these filtered data samples and that the estimator makes optimal use of the underlying data.



Figure 2: Tracking loop with soft-output extension

PERFORMANCE ASSESSMENT

For the performance assessment of the proposed efficient sequential estimator simulations have been carried out. By means of the noise performance of the MAP estimate, obtained using a Newton-Raphson gradient method [7], the soft estimator was compared to the corresponding Cramer Rao lower bound (CRLB) according to [1] and to the generic

incoherent DLL for a correlator spacing of 1.0 chips, 0.5 chips, 0.3 chips and 0.1 chips, respectively. The signal scenario corresponded to a GPS C/A code of 10 MHz one-sided bandwidth. In the simulation the DLL was running with a bandwidth of 2 Hz. In order to ensure a fair comparison between the DLL and the MAP estimator, the MAP estimator used also a 2 Hz filter in the correlator domain. However, it has to be noticed that due to the nonlinearity of the estimation problem a 2 Hz filter in the correlator domain is not completely equivalent to a 2 Hz DLL, which approximately implements also a 2 Hz filter in the delay domain. Nevertheless, for a fair comparison 2 Hz filtering in both domains seems to be an adequate choice for the DLL and the MAP estimator. Since the delay prediction within the MAP estimator is obtained by a parallel running, optionally carrier-aided DLL, much smaller correlator filter bandwidths than 2 Hz may be realized in practice, where due to the carrier-aiding there may be much more "trust" in the prediction.



Figure 3: Efficient single path MAP vs. DLL and CRLB

From the simulation results, which are depicted in Figure 3, it can be derived that the MAP approach outperforms the DLL. It attains the CRLB for high C/N_0 and diverges from the bound for low C/N_0 similar to the single-path in-the-Loop-MLE proposed in [7].

In order to address the multipath performance, the estimator was running in a first scenario with DLR's aeronautical channel model [8] and in a second scenario using an artificially created long-time correlated multipath channel. Since it turned out during the simulations that for the nominal scenario of $C/N_0 = 45$ dB-Hz the multipath events caused by the aeronautical channel do not cause such severe multipath conditions that the multipath bias becomes apparent in the noise, the ground reflection was increased artificially with respect to timely correlation as well as with respect to magnitude, which might correspond to a situation when the aircraft is approaching the runway over a water surface. The estimation error is shown for the DLL and the introduced MAP estimator (conditioned on 1-path and 2-path model respectively). Additionally, the log-likelihood ratio is plotted, which indicates the matching of the 2-path model at the most likely point in relation the 1-path model at the most likely point.



Figure 4: Aeronautical ground multipath channel, estimation error of DLL, 1-path and 2-path estimators with associated log-likelihood ratio

It can be seen from the results in Figure 4 that the higher order 2-path model is always able to match the data vector better than the 1-path model due to the additional degrees of freedom. Thus, if model selection shall be applied in practice the higher order model has to be penalized as it is done within the approaches where information criteria like the Akaike or Bayesian information criterion (AIC, BIC) are used in order to determine the appropriate model. From the simulation results it can be seen that the periods of strong multipath, which affect the DLL, become detectable from the log-likelihood ratio. Since the multipath event is still insufficient to cause a significant bias, a further simulation was

carried out using a severe long-time correlated multipath channel, whose influence cannot be smoothed out by the 2 Hz filtering. The results are plotted in Figure 5. Again DLL and the MAP estimation error (conditioned on 1-path and 2-path model respectively) are shown along with the multipath amplitude and the log-likelihood ratio.



Figure 5: Artificial long-time correlated multipath channel, estimation error of DLL, 1-path and 2-path estimators with associated log-likelihood ratio

Like before the multipath becomes detectable from the log-likelihood ratio, and due to the strong multipath the visibility becomes even better. The DLL is in this scenario significantly disturbed by the multipath event, such as also the 1-path estimator, whereas the 2-path estimator remains unbiased.

CONCLUSIONS

An implementation for a soft-output-capable sequential signal parameter estimator was introduced and assessed by computer simulations. The architecture consists of a generic tracking loop which runs in parallel to a signal compression based soft estimator. The proposed architecture is designed to offer a number of advantages for real-time implementation: The generic loop architecture is kept unmodified as fallback solution, if required, and the complex multipath estimation computations can be carried out at navigation rate due to pre-filtering in the reduced space, which is shown to be lossless and nearly optimal for the single path estimation problem. Advantages are to be expected in all kind of scenarios where the multipath is the dominant source of error compared to the noise, such as for long time-correlated multipath channels which are not mitigate-able by conventional smoothing techniques. Nevertheless, due to complexity reasons the suggested method addresses primarily safety critical applications like multipath mitigation and monitoring for ground station monitoring receivers, e.g. GBAS, and/or for receivers that are in use onboard of aircraft for high performance landing applications.

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