## RAIM Algorithms Analysis for a Combined GPS/GALILEO Constellation

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## BIOGRAPHY

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## ABSTRACT

How far a user can rely on his navigation system is a central question for safety of life applications like air navigation especially in approach phases. For Oceanic, En route or Non Precision Approach phases, the integrity requirements as defined by the ICAO (International Civil Aviation Organization) should be fulfilled by the future Galileo Safety of Life Service.

This paper presents the performances of RAIM algorithms using a covariance matrix of a single frequency absolute positioning receiver noise calculated using one year measurement data.

In the configuration of combined GPS/GALILEO constellation, the user will have the possibility to track at least 10 satellites at the same time. This high availability of satellites will provide a high availability of RAIM algorithms.

The original approach used in this paper is to use the IPRE

(Instantaneous Pseudo Range Error) developed in [1] as the input parameter of the RAIM algorithms. This concept provides a generalized covariance matrix of pseudo range noise taking into account correlations of pseudo range errors with close elevation and azimuth angles. Thanks to a Cholesky decomposition, it is always possible to use the classical  $\chi^2$  distribution to obtain the fault detection threshold. The advantage of generalizing the RAIM methods is not only in the simplicity of the algorithm, but it is also in its efficiency thanks to lower protection levels obtained.

## INTRODUCTION

The IPRE (Instantaneous Pseudo Range Error) provides not only a level of instantaneous error but also a "look up table" from which a covariance matrix of satellites on visibility can be extracted the method will be detailed later. This means that at each time step, the user is able to calculate the real covariance matrix of the visible satellites based on one year measurements from an IGS station. The predefined look up table is defined regionally. This means that the satellite configuration (which repeats itself almost every day for GPS) is supposed not to change within a radius of 50 km around the covariance measurement site. We will see in the next section why it is so important to be inside this area. A first part of this paper will consist of modeling the covariance matrix of pseudo range noise for a given location. A second part will be dedicated to the description of the RAIM algorithms used in this paper with a special emphasize on the integration of a generalized noise model. In a third part, we will present the simulation test cases that have been chosen. In a last part, we will present the results and we will draw some conclusions of the impact of a GPS/Galileo constellation on RAIM availability and performances.

# COVARIANCE MATRIX OF PSEUDO RANGE NOISE

The model of covariance matrix usually taken as input for the majority of RAIM algorithms is a simple but usually said representative model of pseudo range error. As given in the introduction, one of the aims of this work is to make this covariance matrix as much representative as we can of the real pseudo range noise faced by a single frequency receiver. We do not only take into account the variations according to the elevation angle but also the cross correlations of the pseudo range errors derived from two satellites with different azimuth and elevation angles. In this section, we will first introduce the error measurements using the concept of IPRE (Instantaneous Pseudo Range Error) developed in [1] and then the practical method used to develop the regional covariance matrix of pseudo range noise. We will end this section by giving an Example of covariance matrix calculated at a given epoch at Oberpfaffenhofen near Munich (Germany).

### The concept of IPRE

Given the general observation equation of a single frequency absolute positioning receiver, it is possible to derive the fundamental error equation by making a Taylor expansion to the first order, considering the error relatively small with respect to the pseudo ranges. The second step is to consider the deviations of the measurements from a reference. This instantaneous error measurement is the base of our statistical analysis. [2]:

$$\Delta \rho = c \cdot (-\Delta B + \Delta I + \Delta T + \nu) + \epsilon \cdot \left(\widehat{R} - \widehat{P}\right) + \mathbf{D} \cdot \Delta R$$
(1)

where  $\Delta \rho \equiv IPRE$  is the vector of instantaneous pseudorange errors corresponding to the observable satellites,

 $c \cdot \Delta B \equiv Clk$  is the vector of satellite clock errors,

 $\epsilon \cdot \left(\widehat{R} - \widehat{P}\right) + \mathbf{A} \cdot \Delta R \equiv Eph$  is the vector of ephemeris errors,

 $\epsilon$  is a matrix containing the errors in unit vectors of user to satellites,

 $\widehat{R}$  is the vector of estimated position of satellites,

P is the vector of estimated position of the user,

**D** is a matrix containing the unit vectors of user to satellites,

 $\Delta R$  is the vector of the satellite position error,

 $c \cdot \Delta I \equiv Iono$  is the vector of ionospheric errors,

 $c \cdot \Delta T \equiv Trop$  is the vector of tropospheric errors,

 $c \cdot \nu \equiv MN$  is the vector of multipath and receiver noise errors,

For more details see [1].

For our needs, let's define  $ODTS \equiv Clk - Eph$  ODTS stand for Orbit Determination and Time Synchronization

error

In this section, the convention used to define the error is as follow:  $\Delta X = X - \hat{X}$ =reference - estimate.

## The statistical analysis

The results from one year measurements of the IPRE vs. time of all visible satellites for each time step is the basis to generate the covariance matrix of error. We have a statistical process were 3 variables are considered: the IPRE of a given satellite, its elevation angle (El) and its azimuth (Az). An approach per class is adopted here. This means that we consider classes of elevation angle, classes of azimuth angle and classes of IPRE. In a first step we distribute all the measurement samples into classes.

Variable	Lower bound	Upper bound	Class width
IPRE	-15m	15m	0.25m
Elevation	$5^{o}$	$90^{o}$	$5^{o}$
Azimuth	$0^o$	360 <sup>o</sup>	$10^{o}$

#### Table 1 Field of variables

We want to generate the covariance matrix of the variable IPRE / El,Az: The IPRE given the elevation angle and the azimuth. The covariance study will be done with respect to (El,Az).

 $n_{ijk}$  is the number of points belonging to the class i of elevation angle, the class j of azimuth and the class k of IPRE.

Lets take two satellites on visibility A and B whose elevation and azimuth classes are (i,j) and (i',j'), the covariance between these satellite errors is calculated as follow:

$$Cov(A, B) = \frac{1}{N_{iji'j'}} \sum_{k} \sqrt{n_{ijk} n_{i'j'k}} \left( IPRE_k - \overline{IPRE_{ij}} \right) \left( IPRE_k - \overline{IPRE_{i'j'}} \right)$$
(2)

with

$$\begin{split} N_{iji'j'} &= \sum_{k} \sqrt{n_{ijk} n_{i'j'k}} \\ \overline{IPRE_{ij}} &= \frac{1}{N_{ij-}} \sum_{k} n_{ijk} IPRE_{k} \\ N_{ij-} &= \sum_{i} n_{ijk} \end{split}$$

This approach differs from the classical covariance of time series variables. The reason for that is that considering time series rather than the approach per IPRE class would have induced a huge period of measurement to have a representative statistics. This expression provides a 4D

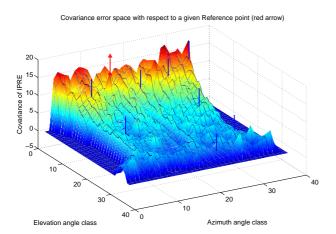


Fig. 1 Example of surface of covariance

look up table that will be used to constitute the covariance matrix of observations for each epoch.

The figure 1 represents in z axis the value of the covariance term of a considered (Elevation, Azimuth) satellite with respect to the red arrow. This look up table is available for a region around the considered IGS station for which the constellation of satellites at every epoch is considered the same. This "regional" covariance matrix of noise observables depends also on the correction algorithms considered in the model of single frequency receiver. In our case we considered the Klobuchar model for ionosphere delay correction and the MOPS model for the troposphere delay correction.

## DESCRIPTION OF RAIM ALGORITHMS USED

The RAIM algorithms used to monitor the performances of combined GPS/Galileo receiver are the one described in [3]. We present here a brief description of the weighted RAIM.

#### **Position determination**

We start our analysis with the well known linearized navigation equation

$$Y = \mathbf{H}X + \varepsilon$$

where X is the four (or five, when using Galileo) dimensional position vector expressed in WGS84 ECEF<sup>1</sup> coordinates, Y is a N dimensional vector that contains the pseudorange measurements for the N satellites in view, H is the direct cosine matrix (also often called the observation matrix), and  $\varepsilon$  is the measurement noise vector. Let us denote W the noise covariance matrix,

$$\mathbf{W} = E[\varepsilon \varepsilon^T]$$

The least squares estimation of the user position  $\hat{X}$  is given by

$$\hat{X} = (\mathbf{H}^T \mathbf{W}^{-1} \mathbf{H})^{-1} \mathbf{H} \mathbf{W}^{-1} Y = \mathbf{G} Y$$

## Position accuracy and dilution of precision

The position error made when using the least squares algorithm is given by

$$e_{WGS84} = X - X$$

Because X and  $\hat{X}$  are given in the WGS84 ECEF referential, the error is defined in this referential too.

$$e_{WGS84} = \mathbf{G}Y - X$$
$$e_{WGS84} = \mathbf{G}(HX + \varepsilon) - X$$
$$e_{WGS84} = \mathbf{G}\varepsilon$$

Since it is more convenient to work in the user local geodetic coordinate system ENU (East North Up), we use the transformation matrix  $\mathbf{T}_{WGS84}^{ENU}$ <sup>2</sup> in order to obtain

$$e_{ENU} = \mathbf{T}_{WGS84}^{ENU} e_{WGS84}$$

Thus

$$e_{ENU} = \mathbf{T}_{WGS84}^{ENU} \mathbf{G}\varepsilon$$
$$e_{ENU} = \mathbf{M}\varepsilon \tag{3}$$

With  $\mathbf{M} = \mathbf{T}_{WGS84}^{ENU} \mathbf{G}$ . This expression allows us to define both  $e_{ENU}^{h}$  the horizontal error vector and  $e_{ENU}^{v}$  the vertical error vector such that

$$e_{ENU}^{h} = \left(\begin{array}{c} e_{ENU}(1)\\ e_{ENU}(2) \end{array}\right)$$

And

$$e_{ENU}^v = e_{ENU}(3)$$

Moreover it comes,

$$\mathbf{Cov}(\mathbf{e_{ENU}}) = \mathbf{T}_{WGS84}^{ENU} \mathbf{GWG}^T \mathbf{T}_{WGS84}^{ENU^T}$$

And we note

$$\mathbf{Cov}(\mathbf{e_{ENU}}) = \begin{pmatrix} \sigma_{e_E}^2 & \sigma_{e_Ee_N} & \sigma_{e_Ee_U} & \sigma_{e_Ee_dt} \\ \sigma_{e_Ne_E} & \sigma_{e_N}^2 & \sigma_{e_Ne_U} & \sigma_{e_Ne_dt} \\ \sigma_{e_Ue_E} & \sigma_{e_Ue_N} & \sigma_{e_U}^2 & \sigma_{e_Ue_dt} \\ \sigma_{e_dte_E} & \sigma_{e_dte_N} & \sigma_{e_dte_U} & \sigma_{e_dt}^2 \end{pmatrix}$$

Since the measurement noise covariance matrix is consider to be general, we can no longer separate the expected positionning errors into dilution of precision parameters and UERE. We choose to express new confidence parameters,

<sup>&</sup>lt;sup>1</sup>ECEF stands for Earth-Centered Earth Fixed referential

<sup>&</sup>lt;sup>2</sup>The matrix  $\mathbf{T}_{WGS84}^{ENU}$  is fully defined by knowing the true position coordinates. Of course in real operational situation this cannot be achieved. The transition matrix used for the computation is approximated by the one at the estimated position.

• the expected global accuracy is:

 $\sqrt{\sigma_{e_E}^2 + \sigma_{e_N}^2 + \sigma_{e_U}^2 + \sigma_{e_{dt}}^2}$ 

• the expected position accuracy is:

$$\sqrt{\sigma_{e_E}^2 + \sigma_{e_N}^2 + \sigma_{e_U}^2}$$

• the expected horizontal accuracy is:

$$\sqrt{\sigma_{e_E}^2 + \sigma_{e_N}^2}$$

• the expected vertical accuracy is:

 $\sigma_{e_U}$ 

• the expected time accuracy is:

$$\sigma_{e_{dt}}$$

#### **Errors detection [4]**

To monitor errors occurrence we have to choose one observable statistic that depends strongly on pseudorange measurement noise. We will use, like in the RAIM method developed in [5], the least square (LS) residuals that can be expressed by:

$$w = Y - \hat{Y}$$

Where we call  $\hat{Y}$  the LS estimation of the measurement vector, defined by  $\hat{Y} = \mathbf{H}\hat{X}$ . It can be shown quite easily that it exists one matrix  $\mathbf{Q}$  such that

$$w = \mathbf{Q}\varepsilon$$

It has been demonstrated that if  $\mathbf{W} = \sigma^2 \mathbf{I}$  the square norm of the residuals follows a  $\chi^2$  law with N - 4 degrees of freedom. The problem in our case is that the diagonal elements of the covariance matrix of noise are different for each satellite and that the error sources are correlated. Thus the assumption that  $||w||^2$  is following a  $\chi^2$  law is no longer true. But, since the covariance matrix  $\mathbf{W}$  is by definition defined positive, we can do a Cholesky decomposition of it. So it exists one unique non-singular square matrix  $\mathbf{A}$  such that:

$$\mathbf{W} = \mathbf{A}\mathbf{A}^T$$

According to this property, we can define a new vector of measurement noise,  $\varepsilon'$ , following a normal  $\mathcal{N}(0, I_N)$  law such that  $\varepsilon$  follows the same law as  $\mathbf{A}\varepsilon'$ . Moreover it comes:

$$Y = \mathbf{H}X + \varepsilon$$
$$Y = \mathbf{H}X + \mathbf{A}\varepsilon'$$
$$\mathbf{A}^{-1}Y = \mathbf{A}^{-1}\mathbf{H}X + \varepsilon$$
$$Y' = \mathbf{H}'X + \varepsilon'$$

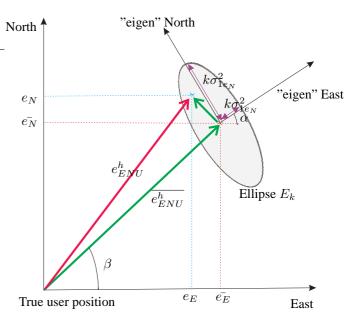


Fig. 2 Failure impact on the horizontal user position

From this new measurement model, where  $Cov(\varepsilon') = I$ we can form new LS residuals,

$$w' = Y' - \hat{Y'}$$
$$w' = \mathbf{A}^{-1}w$$

Thanks to the definition of  $\varepsilon'$ , the square norm of the random variable w' is following a  $\chi^2$  law with N - 4 degree of freedom. We are now able to select the error detection threshold  $T_d$  by inverting the cumulative density function (cdf) of  $||w'||^2$ .

### **Protection levels**

The protection levels are used in order to guarantee that the solution error will not be larger than a given value. To compute these protection levels we have to see first how an error affects the LS residuals vector w',

$$w' = \mathbf{A}^{-1} \mathbf{Q} \varepsilon$$
$$\|w'\| = \varepsilon^T \mathbf{Q}^T \mathbf{A}^{-1T} \mathbf{A}^{-1} \mathbf{Q} \varepsilon$$
$$\|w'\| = \varepsilon^T \mathbf{Q}_{w'} \varepsilon$$

We note  $E[\varepsilon] = \theta$  the expectation value of the random pseudorange noise vector. If there is a failure on one<sup>3</sup> of

<sup>&</sup>lt;sup>3</sup>We only consider one satellite failure in our analysis, this hypothesis is of course a bit restrictive because in real operational conditions the environment may induce several biases on measurements.

the satellites it comes

$$\theta = \begin{pmatrix} 0 \\ \vdots \\ b \\ \vdots \\ 0 \end{pmatrix}$$

Thus

$$\|w'\| = \mathbf{Q}_{w'}(i,i)b^2$$

#### **Horizontal Protection Level**

If we consider the horizontal solution error  $e_{ENU}^h$ , it can be shown that (equation 3)

$$||e_{ENU}^{h}||^{2} = (\mathbf{M}(1,i)^{2} + \mathbf{M}(2,i)^{2})b^{2}$$

Thus

$$\|e_{ENU}^{h}\|^{2} = \frac{\mathbf{M}(1,i)^{2} + \mathbf{M}(2,i)^{2}}{\mathbf{Q}_{w'}(i,i)} \|w'\|^{2}$$

where  $\mathbf{M} = \mathbf{T}_{WGS84}^{ENU} \mathbf{G}$  The quantity  $\sqrt{\frac{\mathbf{M}(1,i)^2 + \mathbf{M}(2,i)^2}{\mathbf{Q}_{w'}(i,i)}}$  is called the horizontal Slope (HSlope) and the *i* index tells us that it is dependent on the satellite we have supposed to fail. So, with the no noise assumption ( $\varepsilon = \theta$ ) the horizontal value that we can protect is

$$HPL = \max_{i=1...N} HSlope(i) \times T_d$$

But in reality the noise will spread the horizontal solution around the previous value. This spreading effect is represented on the figure 2 by the ellipse. The definition of the HPL induces that there is a missed alert when the horizontal solution error is lower than the HPL value because of a too noisy environment. Let  $P_{md}$  denote this probability of missed detection.

$$P_{md} = 1 - p$$

Where p is the probability to be *inside* the ellipse. Thus the value of HPL should be set to the distance between the user true position and the furthest point from the origin inside the ellipse. This means that the HPL should theoretically be the radius of the circle centered at the true position which is tangent to the ellipse. Analytically, this value is complicated to obtain and we use the estimation proposed in [6] in order to approximate it.

The idea is to be a little bit more conservative than by considering the tangent to the ellipse and take, instead of the furthest point inside the ellipse, the furthest point of the smallest rectangle that contains the noise ellipse. This rectangle is defined on the figure 3. The value of HPL is

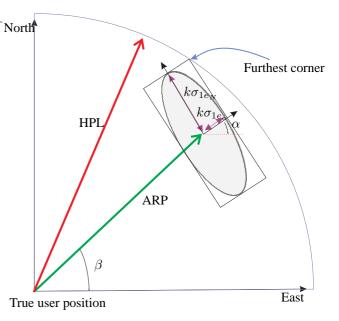


Fig. 3 Horizontal Protection Level practical value [6]

found by a vectorial decomposition of the  $\mathcal{OM}_1$  distance  $(OM_1 = \|\overrightarrow{OM_1}\|).$ 

$$\overrightarrow{OM_1} = \overrightarrow{OC} + \overrightarrow{CA} + \overrightarrow{AM_1} \tag{4}$$

Thus,

$$\|OM_1\|^2 = \|\overrightarrow{OC}\| + \|\overrightarrow{CA}\| + \dots$$
$$\dots + \|\overrightarrow{AM_1}\| + 2\overrightarrow{OC}.\overrightarrow{CA} + \dots$$
$$\dots + 2\overrightarrow{OC}\overrightarrow{AM_1} + 2\overrightarrow{CA}\overrightarrow{AM_1}$$

With(see figure 3):

00

• 
$$OC = ARP$$

• 
$$CA = k\sigma_{1e_N}$$

- $AM_1 = k\sigma_{1e_e}$
- $\overrightarrow{OC} \cdot \overrightarrow{CA} = OC \cdot CA \sin(\beta \alpha)$

• 
$$\overrightarrow{OC} \cdot \overrightarrow{AM_1} = OC \cdot AM_1 \cos(\beta - \alpha)$$

•  $\overrightarrow{CA} \cdot \overrightarrow{AM_1} = 0$ 

So,

$$HPL = \sqrt{\begin{array}{c} ARP^2 + k^2 \sigma_{1e_e}^2 + k^2 \sigma_{1e_N}^2 + \dots \\ \dots + 2kARP[\sigma_{1e_N}|\sin(\beta - \alpha)| + \dots \\ \dots + \sigma_{1e_e}|\cos(\beta - \alpha)|] \end{array}}$$

With,

$$k = \sqrt{-2\ln(1-p)}$$

## **Vertical Protection Level**

In similar way we can define a VSlope linking the test statistic to the norm of the vertical solution error,

$$\|e_{ENU}^v\| = VSlope(i)\|w_n\|$$

Where  $VSlope(i) = \sqrt{\frac{(M(3,i)^2)(N-4)}{Q_{w'}(i,i)}}$ . The upper-bound value for  $\|e_{ENU}^v\|$  is,

$$\|e_{ENU}^v\| \le \max_{i=1...N} VSlope(i)$$

The VPL value in a no noise environment can be obtained thanks to,

$$VPL = VSlope_{max} \times T_d$$

In considering the noise, the VPL value is much easier to evaluate than the HPL because the vertical error is onedimensionnal Gaussian variable.

$$e_{ENU}^{v} \sim \mathcal{N}(\overline{e_{ENU}^{v}}, \sigma_{e_{U}}^{2})$$

In our case we fix

$$\overline{e_{ENU}^v} = VSlope_{max} \times T_d \tag{5}$$

Thus the expression of VPL can be defined by

$$VPL = VSlope_{max} \times T_d + \alpha(P_{md}) \times \sigma_{e_U}$$
 (6)

where  $\alpha(p)$  represents the threshold for which we have all realization of a  $\mathcal{N}(0,1)$  law below this threshold with a probability p. Thus we have,

$$p = Pr(X \le \alpha) = \int_{\infty}^{\alpha} p df(x) dx \tag{7}$$

where pdf(x) is the probability density function for a  $\mathcal{N}(0,1)$  law:

$$pdf(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$
 (8)

Thus,

$$p = \int_{\infty}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \tag{9}$$

The probability of missed detection is the probability to have the VPL value below the vertical error. This means that,

$$p = 1 - P_{md}$$

We have,

$$P_{md} = 1 - \int_{\infty}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \qquad (10)$$

$$P_{md} = \frac{1}{2} - \int_0^\alpha \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \qquad (11)$$

$$1 - 2P_{md} = 2\int_0^\alpha \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$
 (12)

$$1 - 2P_{md} = erf(\frac{\alpha}{\sqrt{2}}) \tag{13}$$

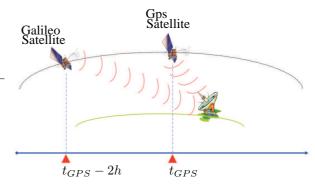


Fig. 4 Virtual Galileo Constellation [7]

where  $erf(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-t^2} dt$ . Hence we found the expression of  $\alpha(P_{md})$ :

$$\alpha(P_{md}) = \sqrt{2}erf^{-1}(1 - 2P_{md})$$
(14)

And finally,

$$VPL = VSlope_{max} \times T_d + \dots$$
$$\dots + \sqrt{2}erf^{-1}(1 - 2P_{md}) \times \sigma_{e_U}$$

## SIMULATION SCENARIOS

The error model has been generated for a single frequency absolute positioning receiver using the MOPS tropospheric correction model at Oberpfaffenhofen (near munich) during the year 2003, no filtering of pseudo range has been made. We define 3 test cases used as input of RAIM algorithm:

- TC1=Generalized covariance matrix with measured variances and covariances
- TC2=Diagonal covariance matrix with measured variances for all satellites in view
- TC3=Diagonal covariance matrix with constant variance (maximal value of all satellites in view)

We use 2 constellation scenarios:

- C1=GPS alone
- C2=GPS+ Virtual Galileo Constellation

The Virtual Galileo Constellation (VGC) is adopted in our study. It consists of the GPS constellation observed at two different time epochs (offset of 2 hours) [7]. The advantage of this scenario for Galileo is the possibility to use the same look up box to generate the covariance matrix of observables (see figure 4).

### **RESULTS OF THE SIMULATIONS**

In this section, we are going to comment the HPL and VPL obtained for different test cases and different constellations configurations as defined above. We considered one day of measurements with a sampling period of 1 minute. A RAIM simulator (RaimSim) developed by DLR under a C/C++ environment has been used to process the data. The graphics are obtained using MATLAB.

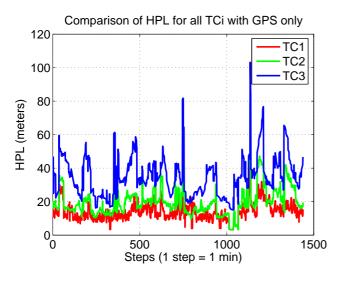


Fig. 5 HPL for all test cases and using GPS only

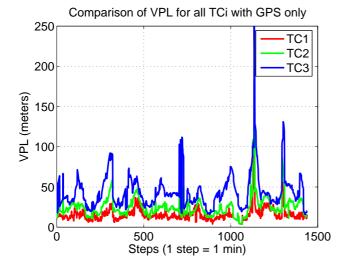


Fig. 6 VPL for all test cases and using GPS only

The figures 5 and 6 shows clearly an improvement of the protection levels. The Vertical component is of particular interest since this parameter is generally the most critical one because the air navigation requirements from APV1 to CAT. III are always considering very stringent vertical alarm limit.

The figure 7 shows for a combined GPS+Galileo constellation that the improvement from TC2 to TC1 is not so

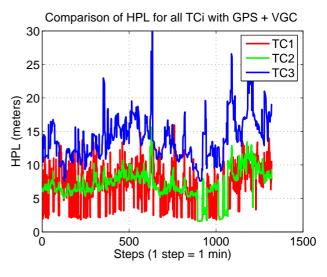


Fig. 7 HPL for all test cases and using GPS + VGC

obvious. The high level of fluctuations for TC1 is showing a limit of our model. A test using a different time offset (between GPS and VGC) should be done to state whether the covariance matrix or the geometry matrix are badly conditioned.

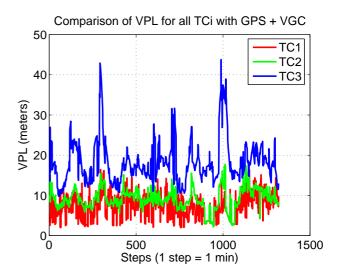


Fig. 8 VPL for all test cases and using GPS + VGC

The figure 8 shows the same type of behavior as previously. Nevertheless the use of a diagonal matrix with an elevation dependency is giving good results in comparison with TC3 which is obviously a too conservative case.

In figures 9 and 10 we can observe the impact of augmenting the number of visible satellites in the protection levels. As expected C2 gives better results for both HPL and VPL. The C2 curve is shifted with comparison to the C1 curve, this was done to take into account the 2 hours offset between GPS and VGC.

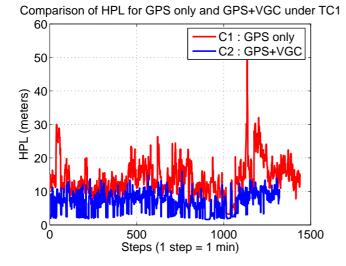
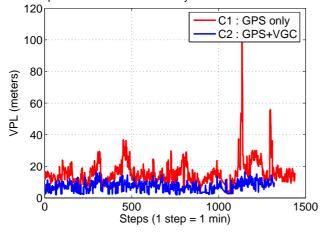


Fig. 9 HPL for TC1 using GPS only and GPS + VGC



Comparison of VPL for GPS only and GPS+VGC under TC1

Fig. 10 VPL for TC1 using GPS only and GPS + VGC

## CONCLUSIONS

The results obtained are encouraging in the sens that a better knowledge of the error behavior of pseudo range observables implies a better estimation of the protection levels. This better knowledge of the error is a result of a measurement campaign of the covariance matrix during one year. In the actual form of our 4D look up table, The availability of this matrix can't exceed a certain area. That's why we choose to denominate it as a "regional" covariance matrix. By taking a dual constellation GPS and Galileo, and considering that Galileo observables are facing the same level of pseudo range noise, the protection levels are decreasing just by considering more satellites on visibility. The hypotheses considering only one faulty satellite could be discussed when 2 constellations are taken into account. In fact the probability of a multi failure is higher and thus should be considered in a more precise study. It would be interesting to consider a lower level

of noise for Galileo as specified in [8]. In that case, a combined covariance matrix of noise has to be set. In any case the generalized RAIM algorithms using a Cholesky decomposition of the covariance matrix is a promising technique and is ready to take the advantages of using the Galileo constellation.

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