Removing Hanging Faces from Tree-Based Adaptive Meshes for Numerical Simulations – an implementation in t8code

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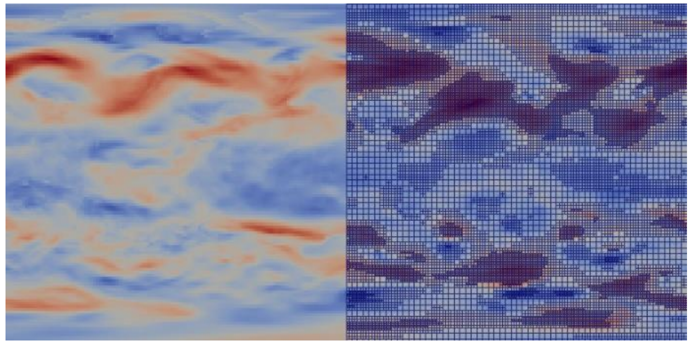
Johannes Holke (German Aerospace Center | High-Performance Computing)

DLR

Adaptive Mesh Refinement (AMR) in Numerical Applications with t8code

numerical application with AMR



Why AMR?

- Many mathematical applications are based on mesh structures
- The element size correlates with the accuracy (but also with the computational costs)
- AMR as a compromise of high accuracy and computational efficiency

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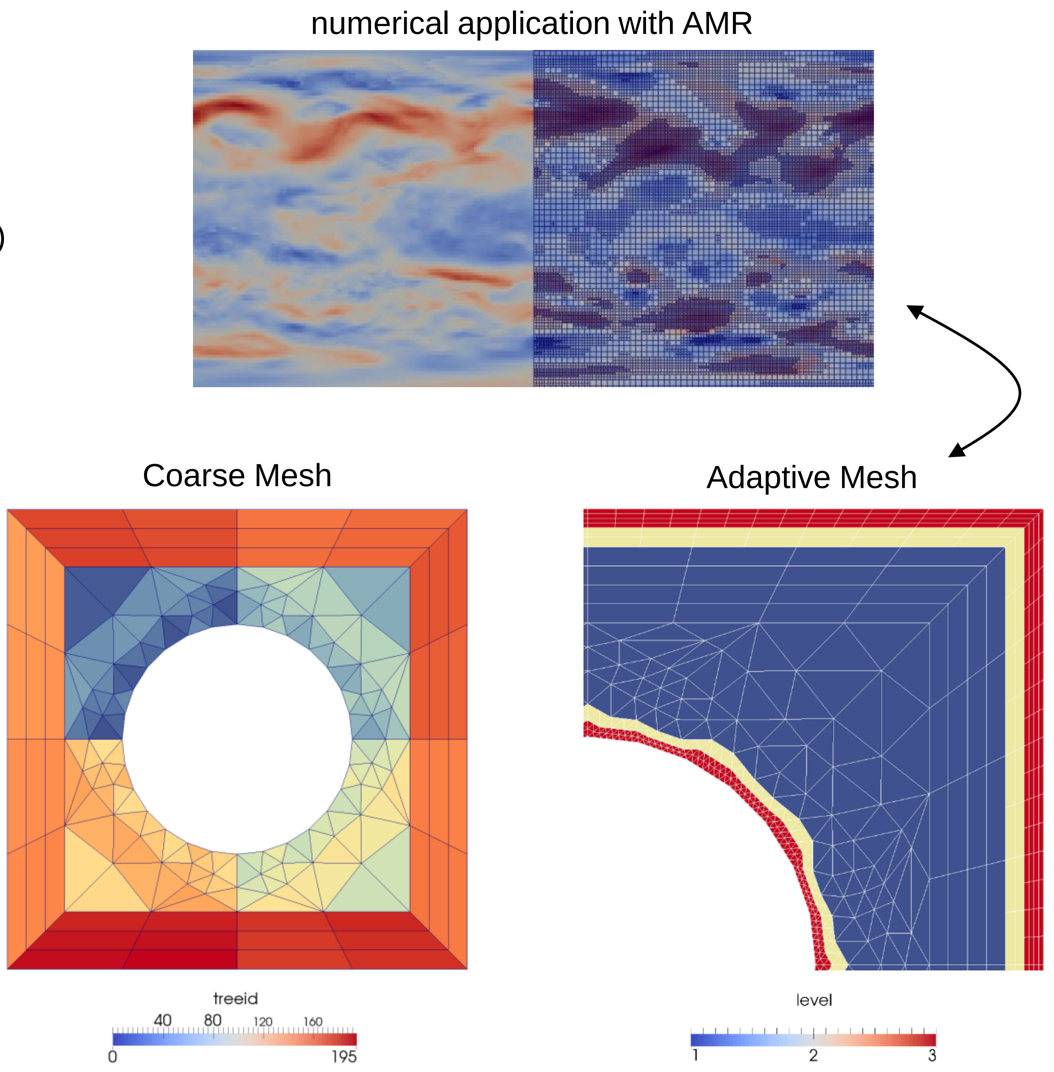
- A open-source C/C++ library for parallel AMR [4]
- Tree-based hybrid meshes
- Adaptive refinement of each tree
- A modular Space-filling Curve Index (SFC Index) for element enumeration



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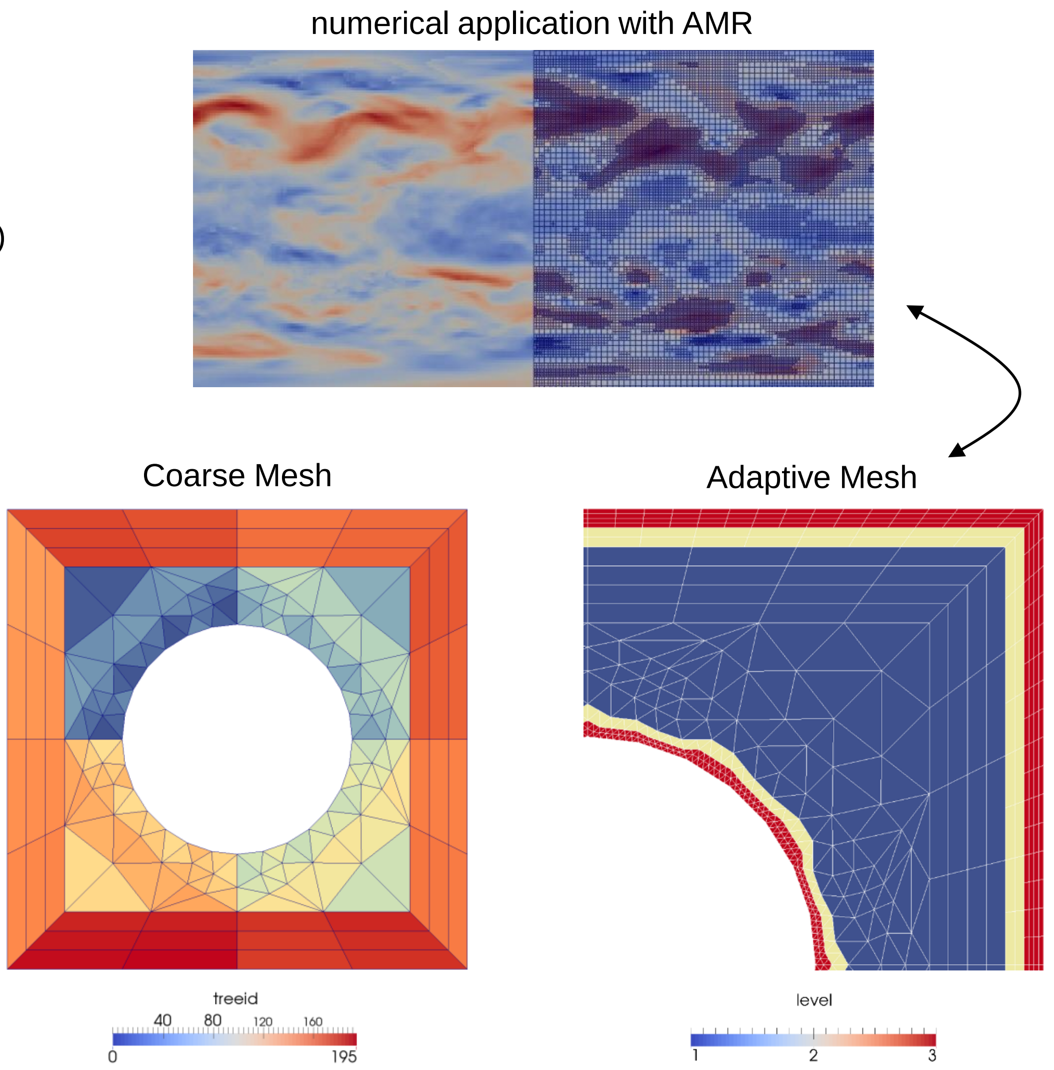
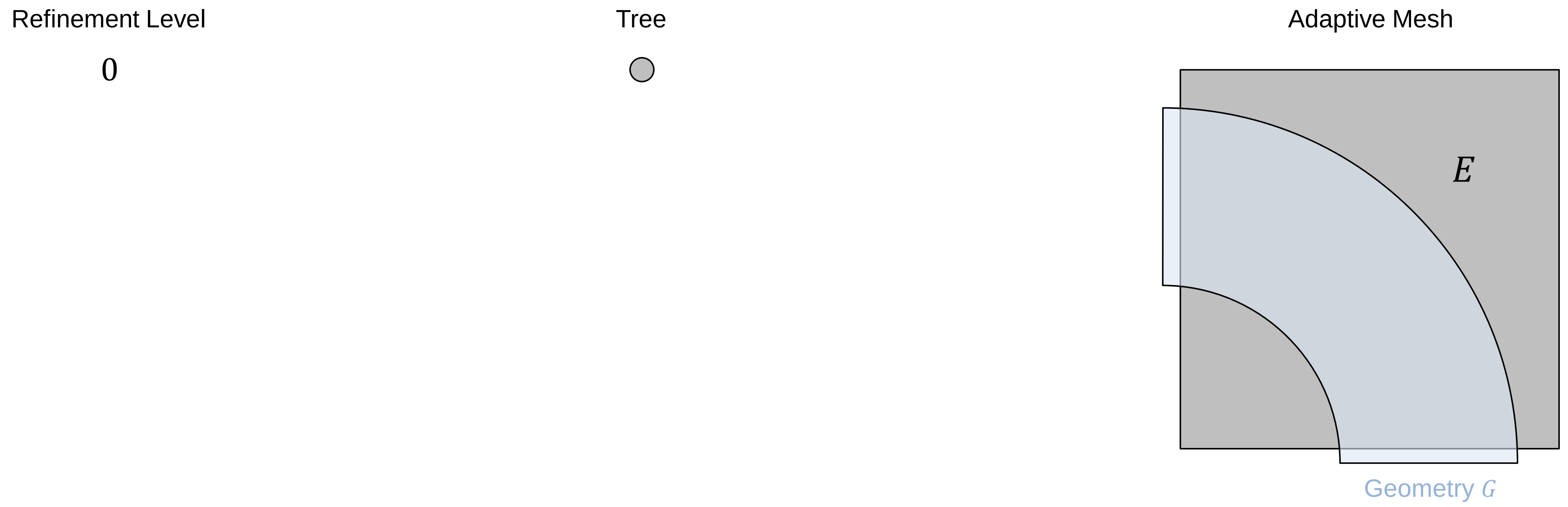
How does tree-based AMR work?



Tree-based AMR

- **Refinement criterion**: refine an element E in the mesh if $E \cap G \neq \emptyset$
- **Refinement:** the replacement of an element E by its children: $E \to \{C_0, C_1, C_2, C_3\}$ with $\cup_i C_i = E$



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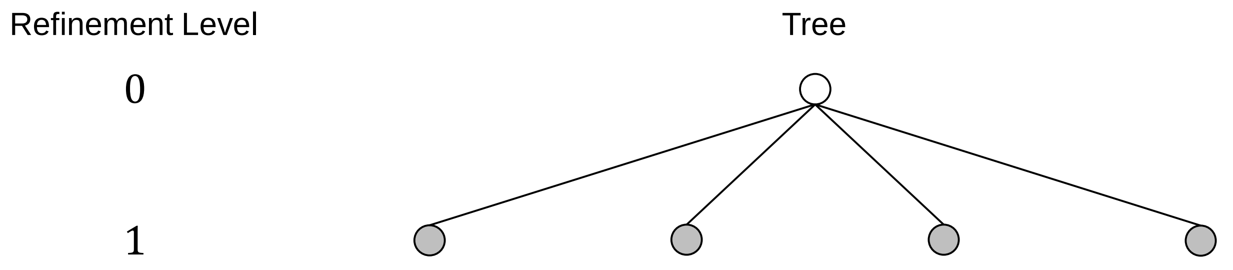

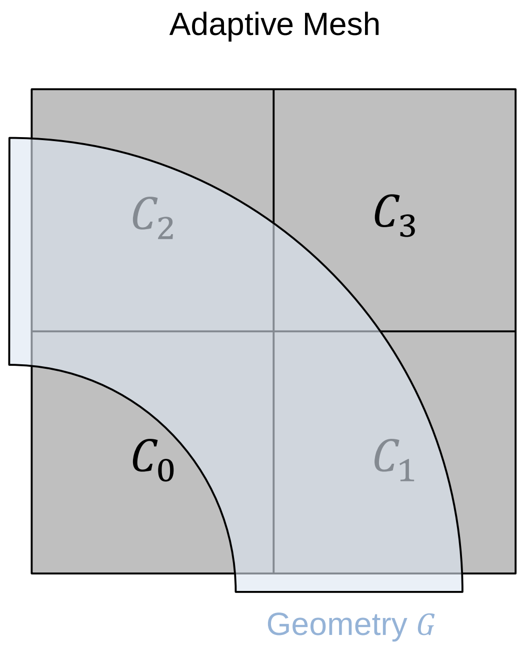

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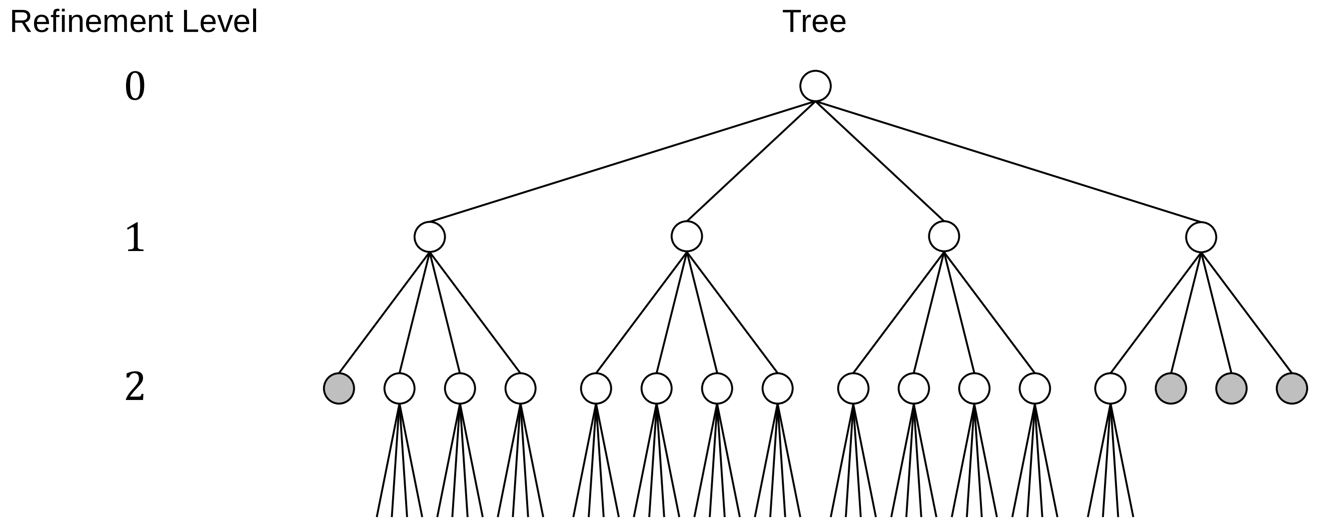

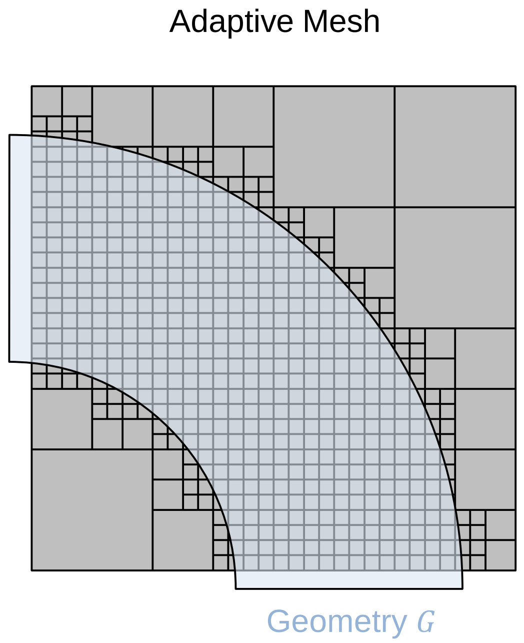

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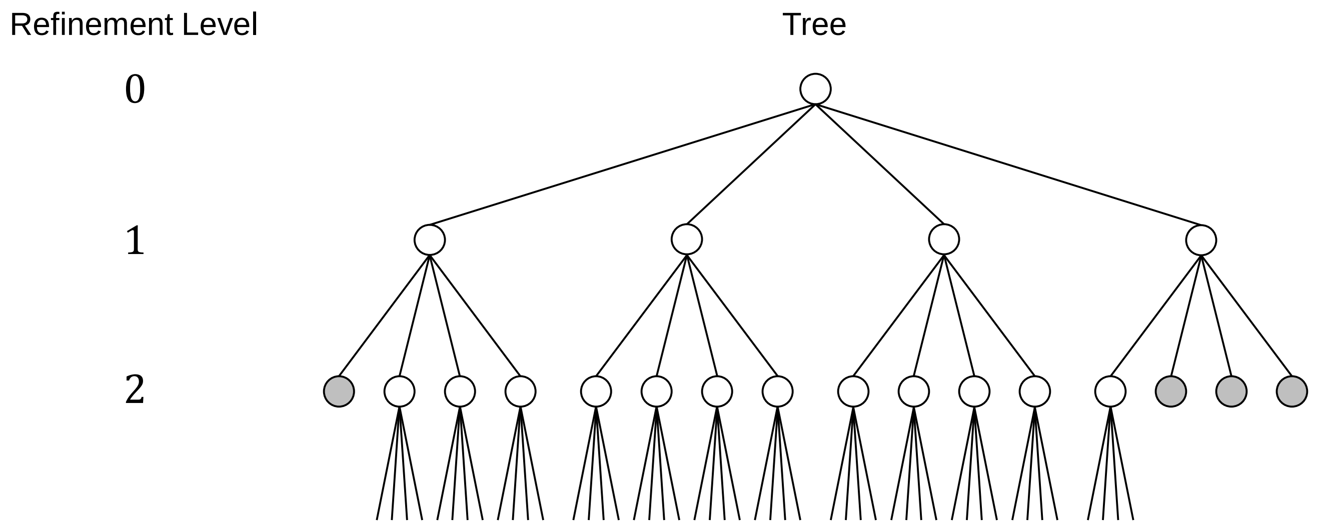

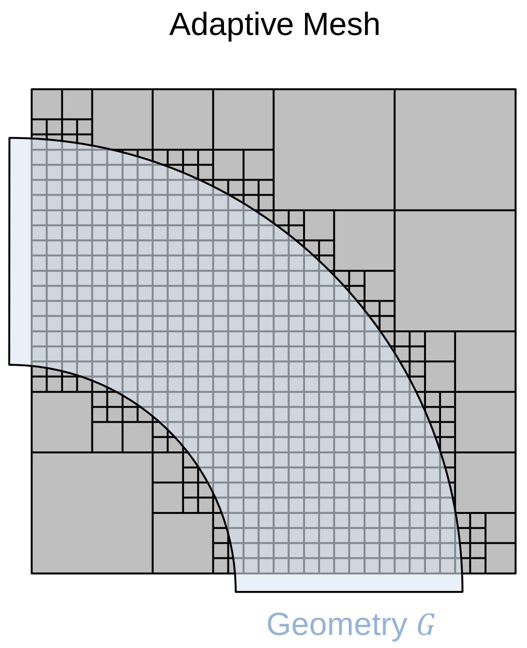

- **Hanging Faces:** the intersection of an elements vertex with an other elements face

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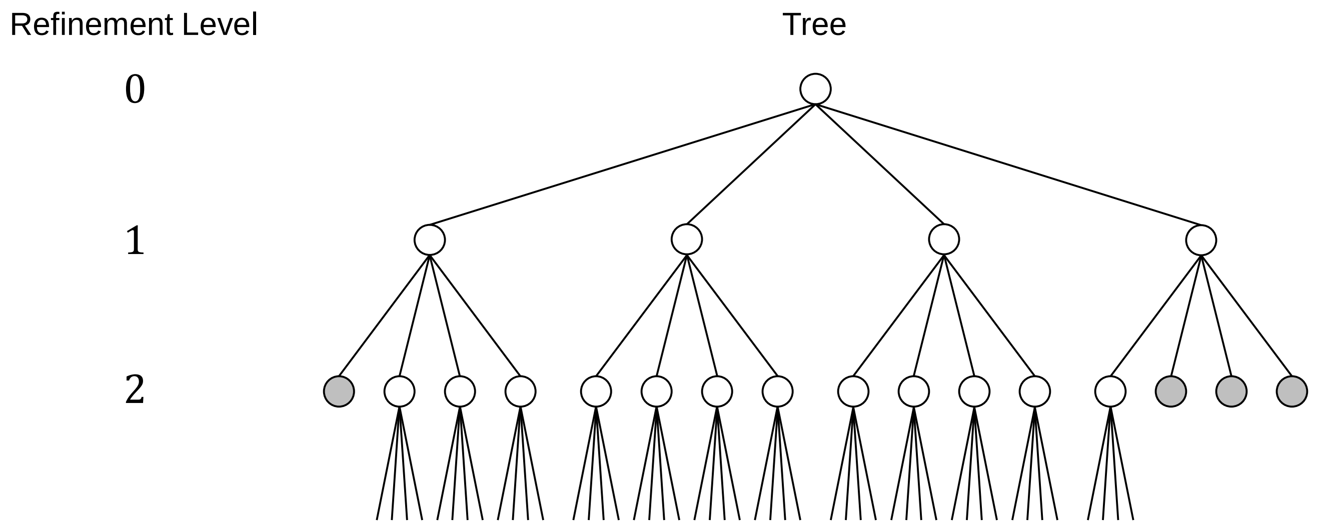
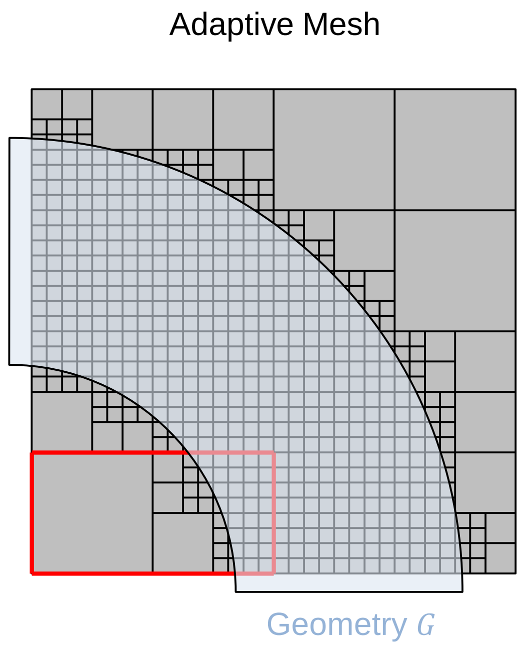




- **Hanging Faces:** the intersection of an elements vertex with an other elements face

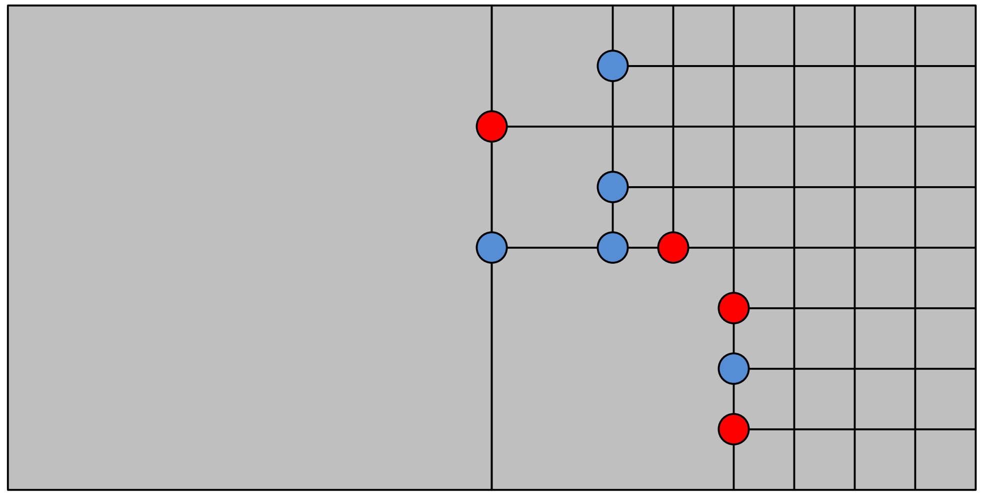
How can we make the mesh conformal after adaptation?

Removing Hanging Faces via Transition Cells of Triangular Subelements

Removing hanging faces is a two-step procedure*:

1. Remove red nodes: balance the mesh
2. Remove blue nodes: use transition cells of triangular subelements

Adapted Mesh



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1. Balancing:

In balanced meshes, the maximum level difference of neighboring elements is 1.

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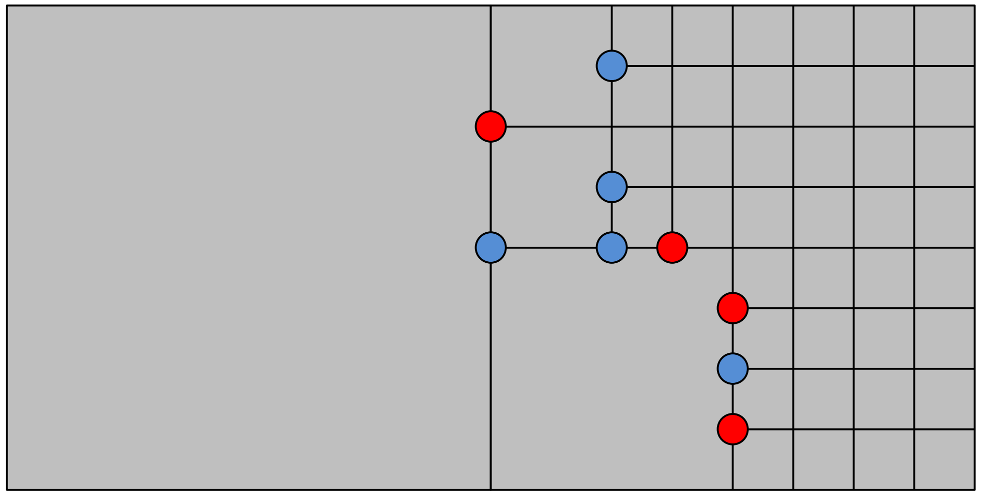

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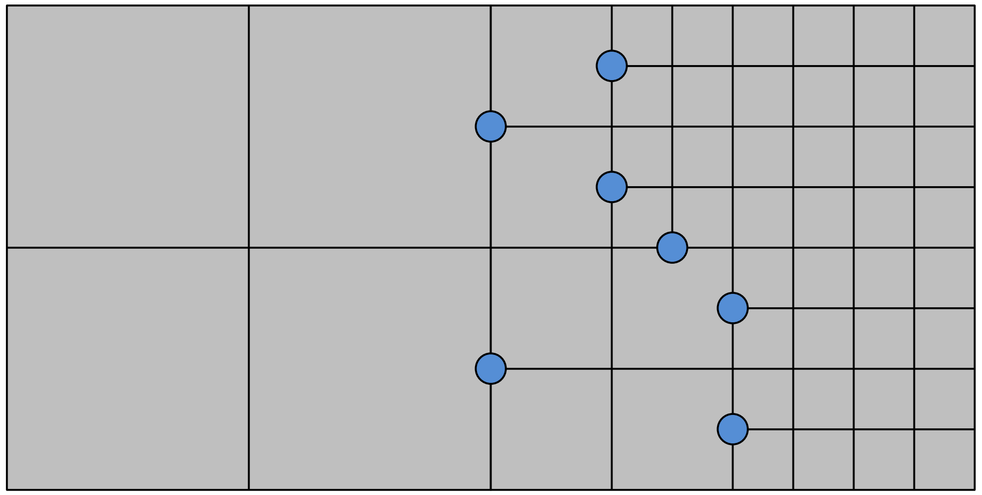

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A transitioned mesh is a mesh that is conformal due to the insertion of **transition cells** T - families of **triangular subelements** $\{S_0, S_1, \ldots, S_n\}$.

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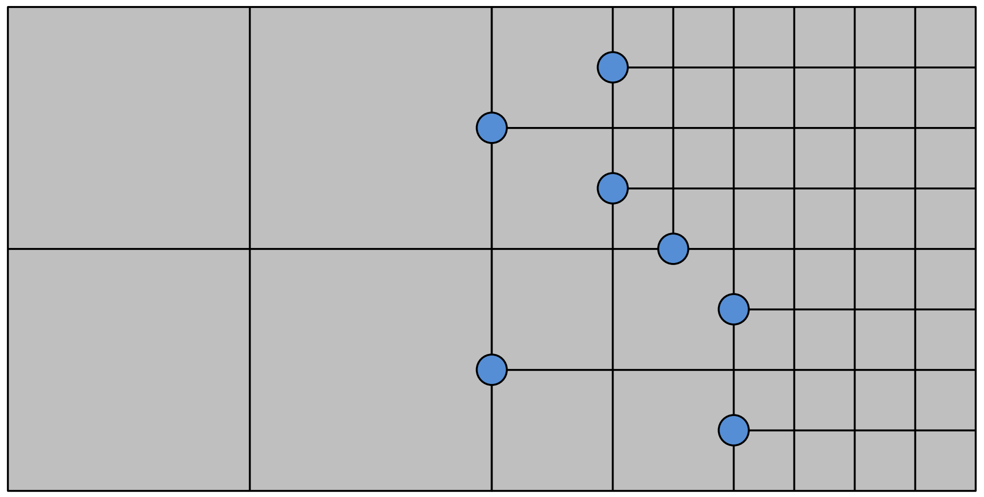

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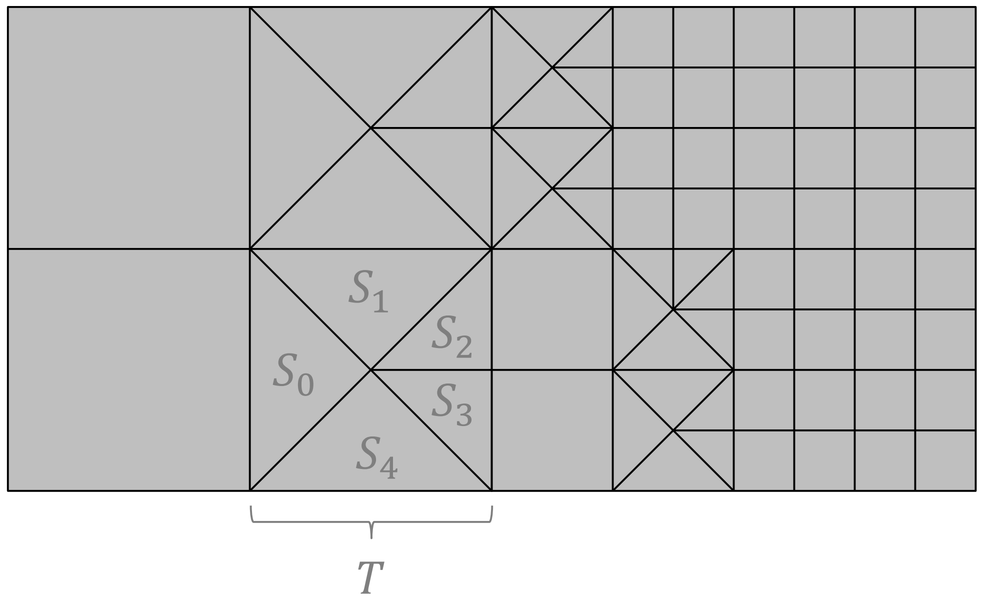

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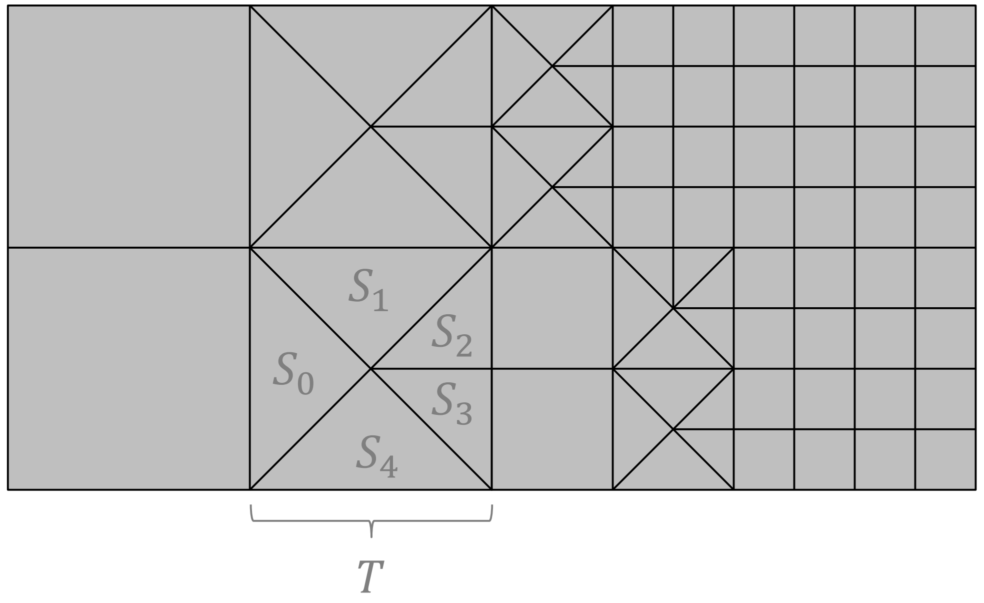

How can we find neighbors in transitioned meshes? Lets find a suitable SFC Index!

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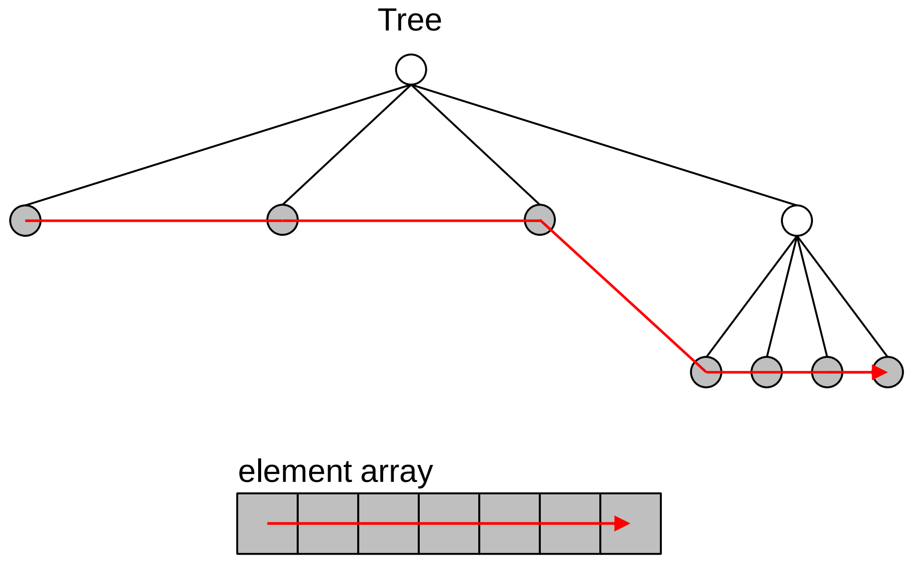
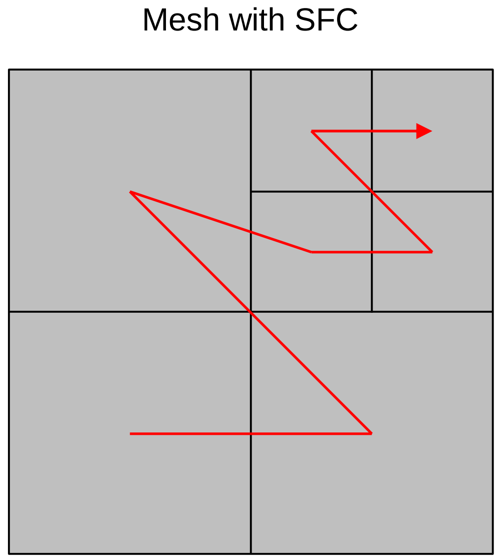
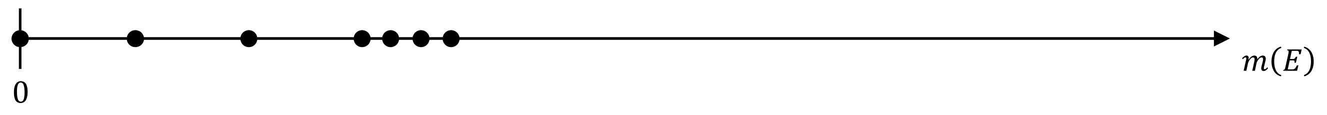
A Space-filling Curve (SFC) Index for transitioned meshes

- t8code uses the **Morton-Index** $m(E) \in \mathbb{N}_0$ to enumerate its elements

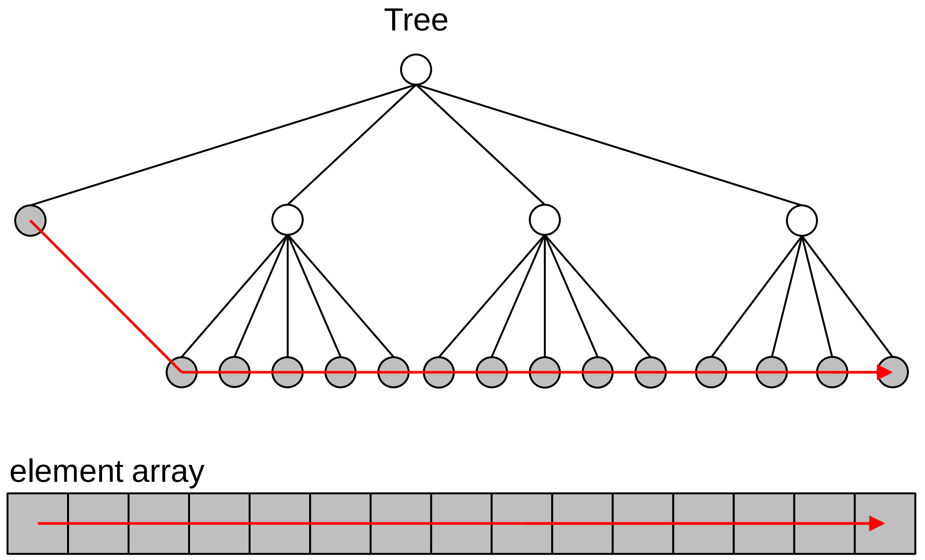


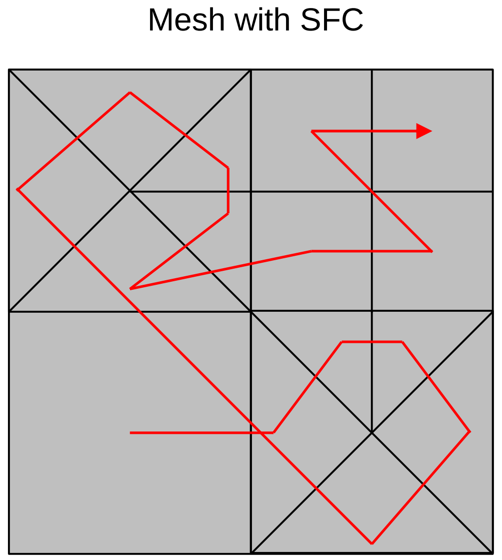


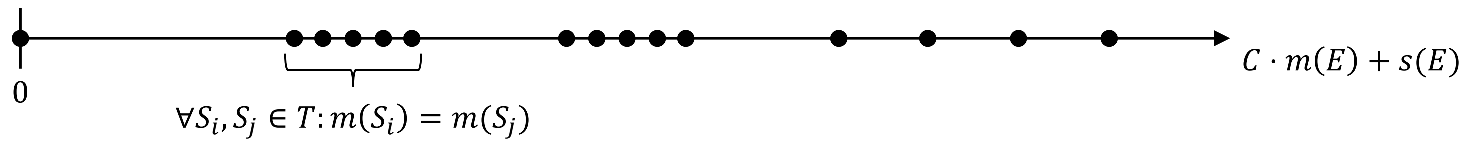


A Space-filling Curve (SFC) Index for transitioned meshes

- In transitioned meshes, we extend the Morton-Index by adding the **Subelement-Index** $s(E) \in \mathbb{N}_0$







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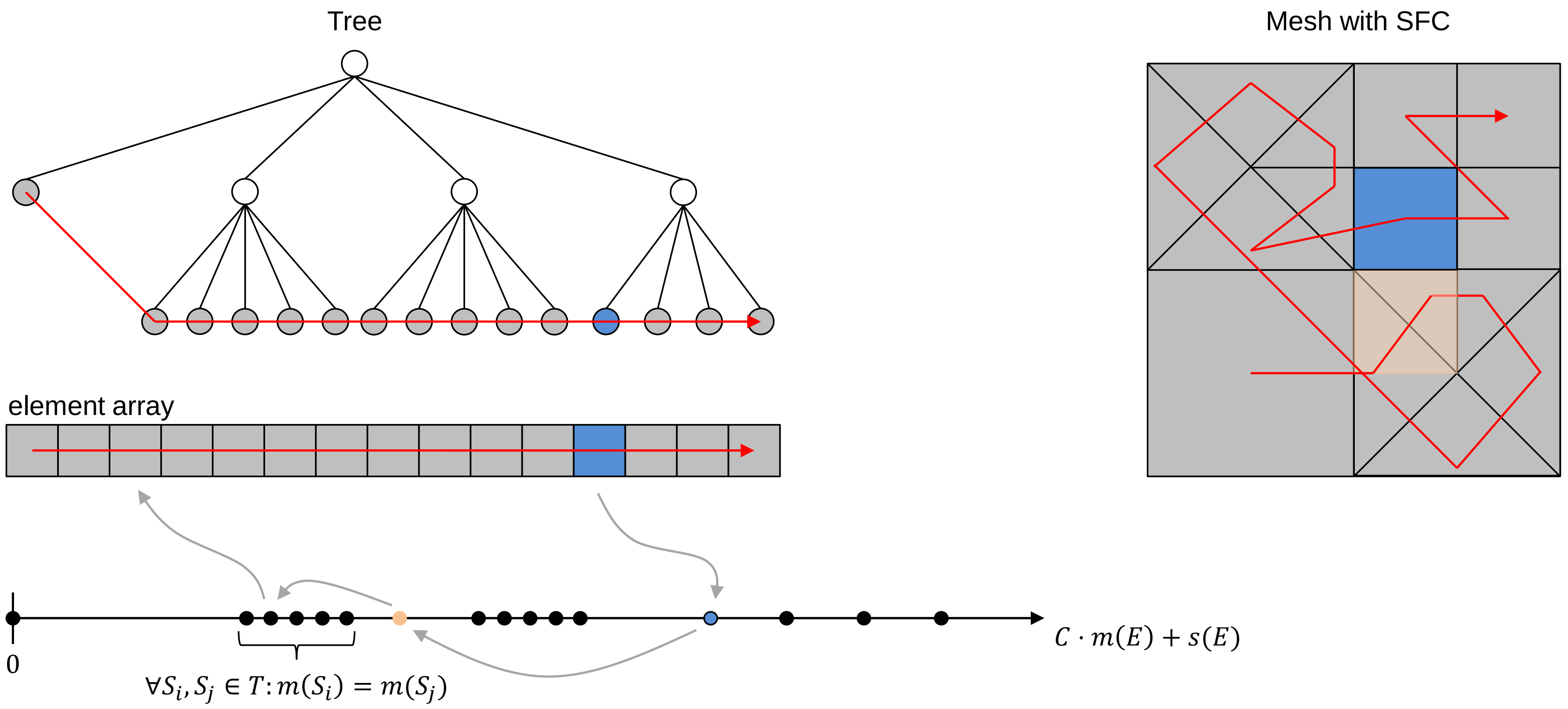

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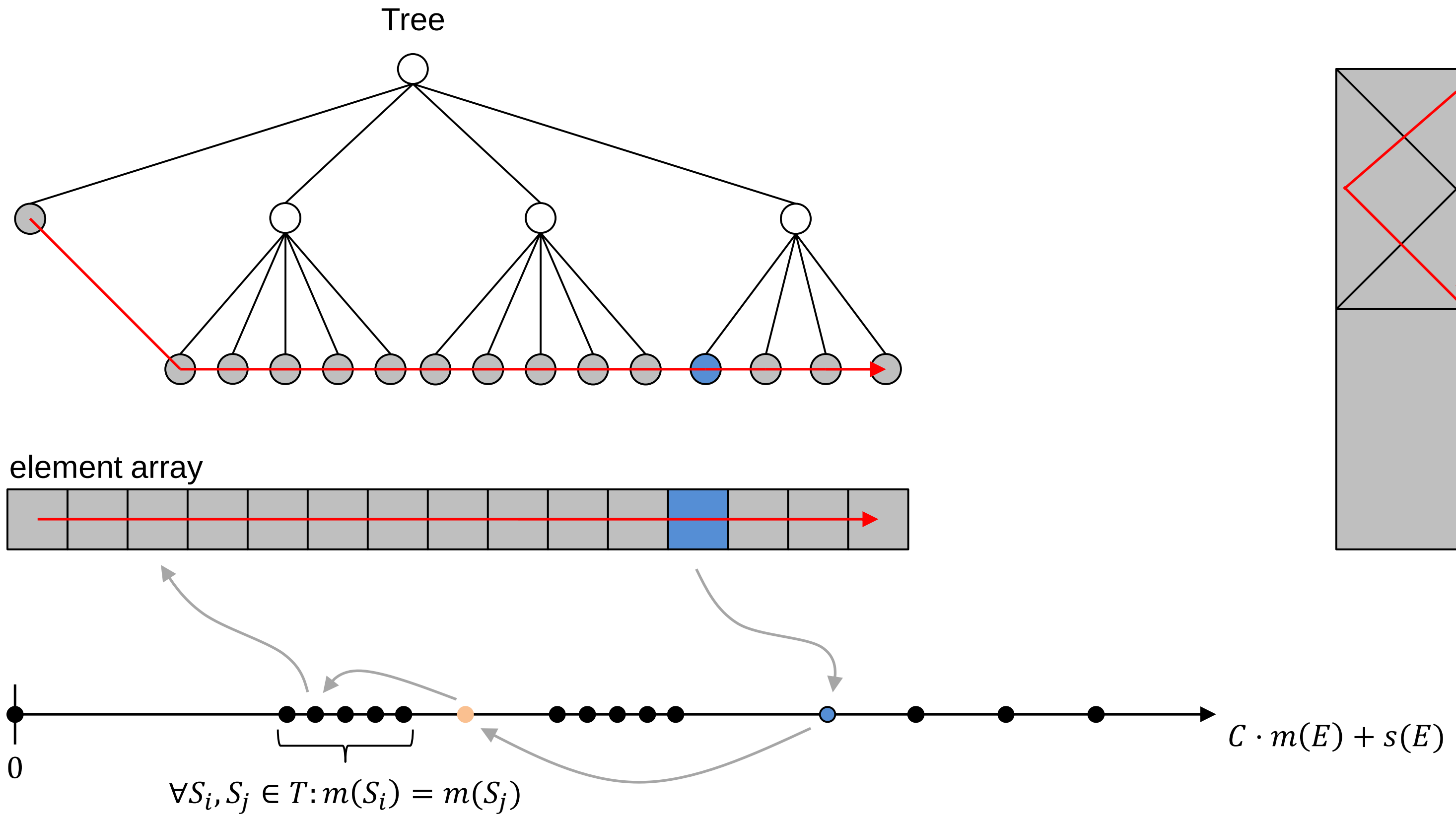
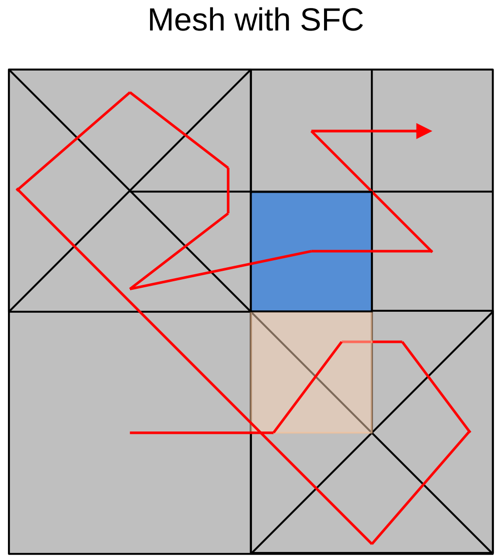


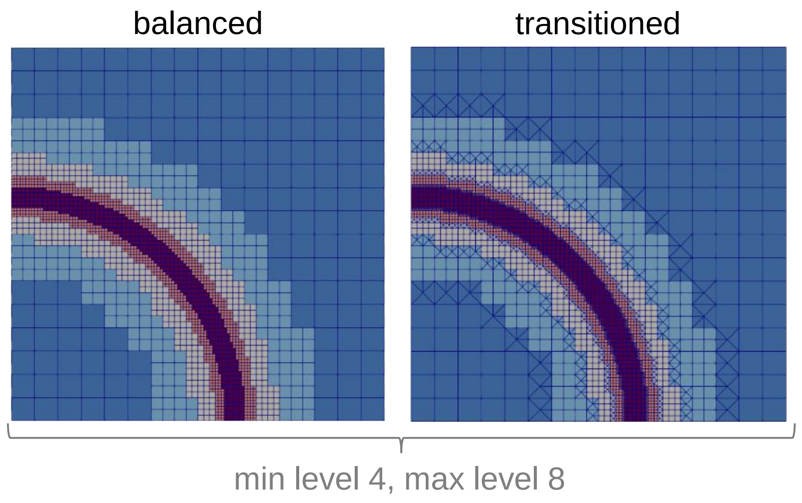


But is this approach efficient?

Benchmarks

Testcase – comparing balanced and transitioned meshes:

- Construct a balanced mesh and its transitioned version
- Compare the LFN runtime of these meshes



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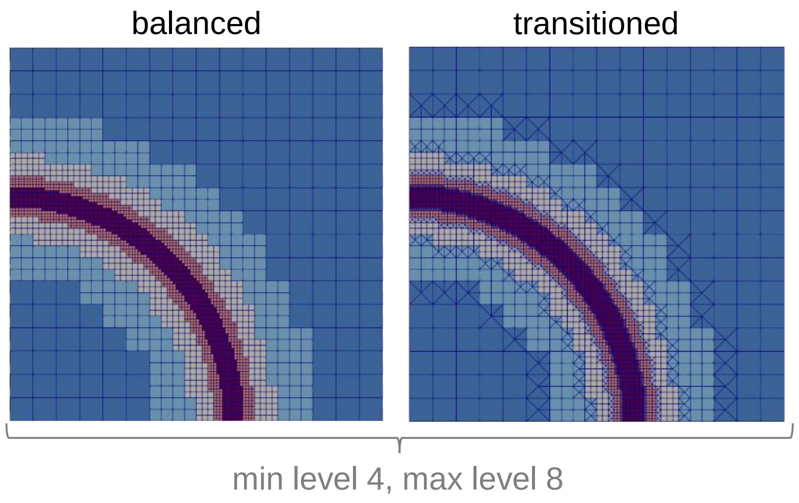
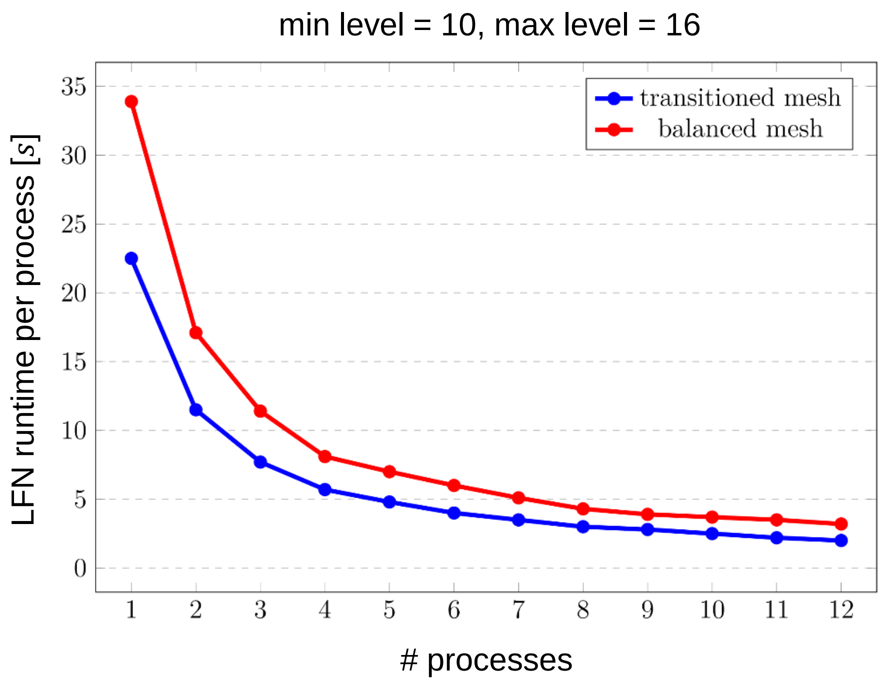


Results:



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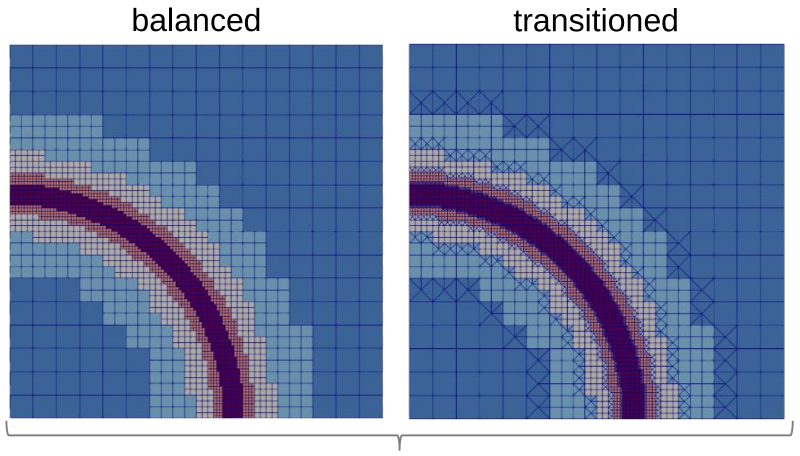

min level 4, max level 8

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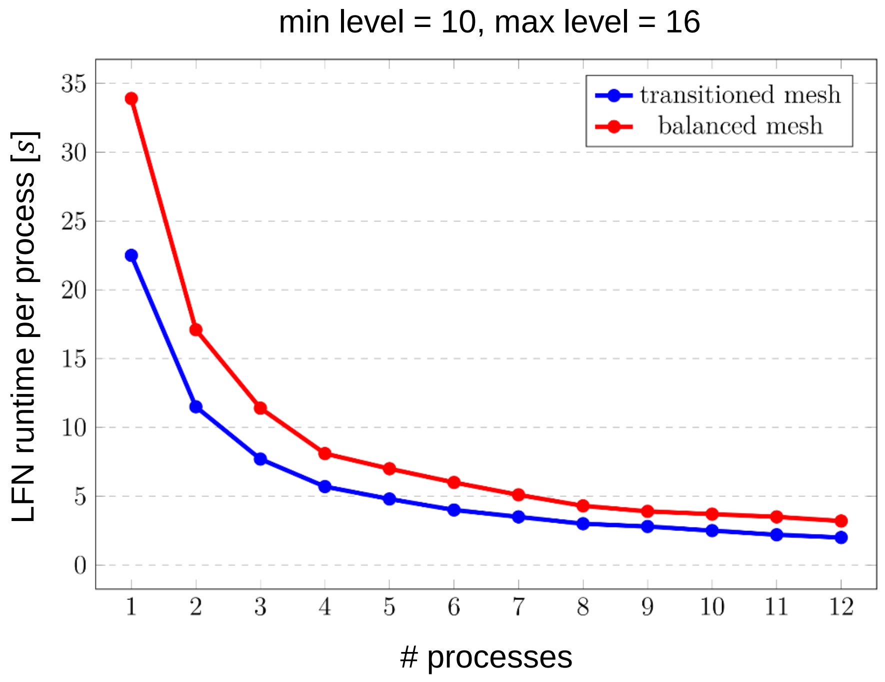
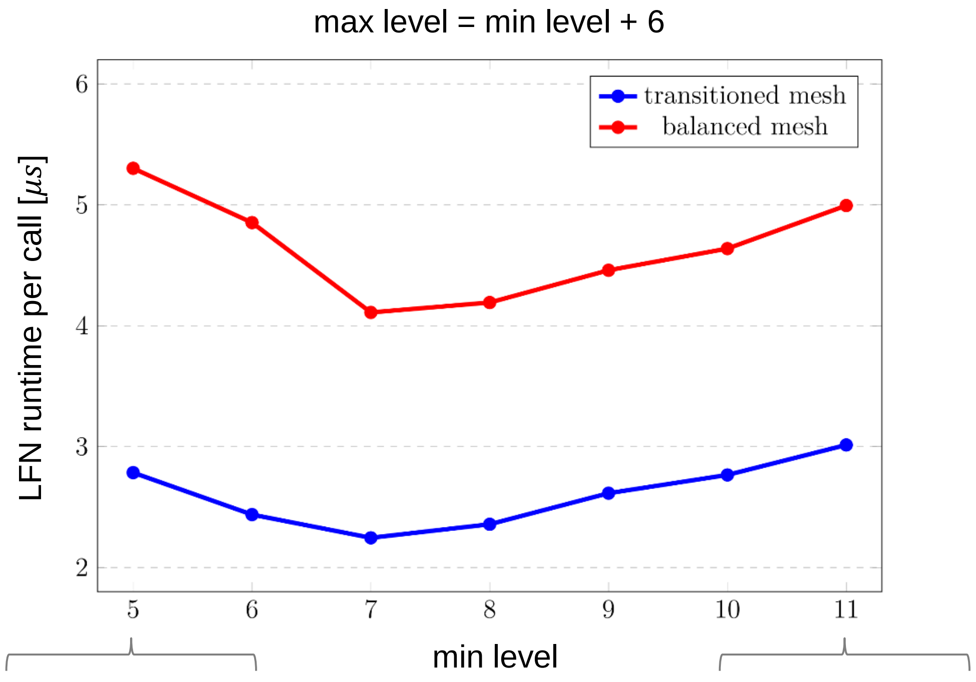
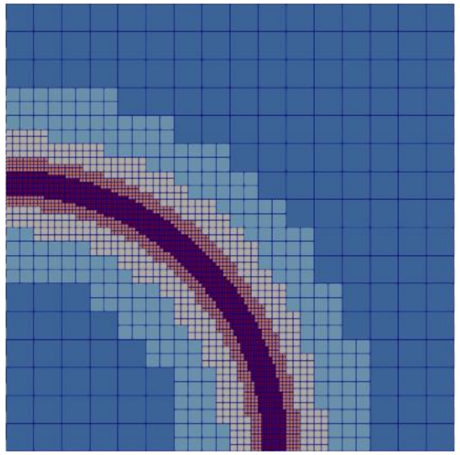


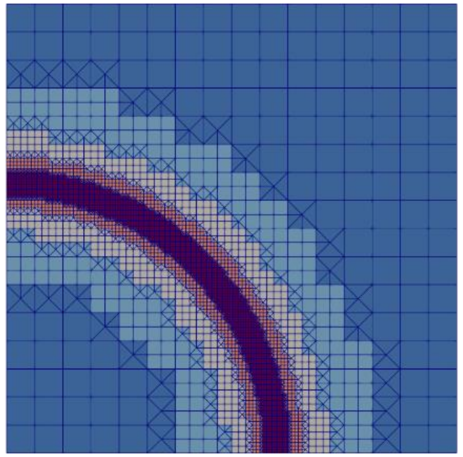


≈50k elements
≈200k LFN calls

≈7m elements
≈ 28m LFN calls

Conclusion and Outlook





Conclusion:

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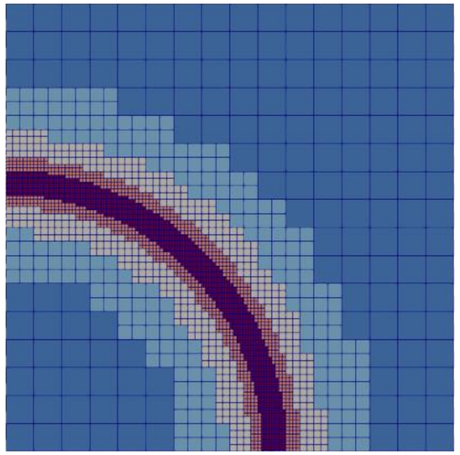

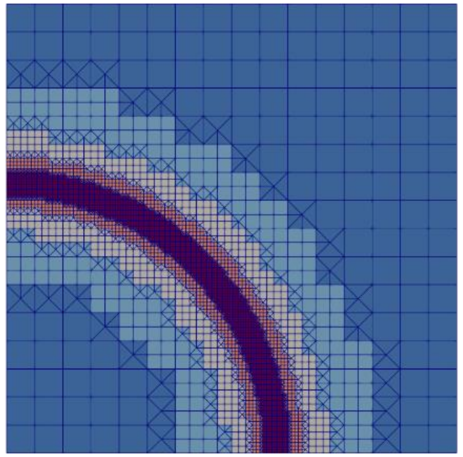

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Outlook/TODOs:

- Extension to 3D?
- Large scale runtime tests
- Merging the new software features into the main code base of t8code

Thank you

Special thanks to Johannes Holke and Gregor Gassner for making this project possible.

Questions?

References

1. Florian Becker. ”Removing hanging faces from tree-based adaptive meshes for numerical simulations”. Master thesis. Universität zu Köln. 2021. https://elib.dlr.de/146849/1/RemovingHangingFacesFromTreeBasedAMR.pdf.
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   https://github.com/holke/t8code.git.
5. Robert Schneiders. "Refining quadrilateral and hexahedral element meshes“. 1996.