

# Self-diffusion in single-component Yukawa fluids

Sergey Khrapak, Boris Klumov, and Lenaic Couedel

*Institute for Materials Physics in Space, DLR, Oberpfaffenhofen, Germany;  
Aix-Marseille-University, CNRS, Laboratoire PIIM, Marseille, France;  
Joint Institute for High Temperatures, RAS, Moscow, Russia.*



Knowledge for Tomorrow



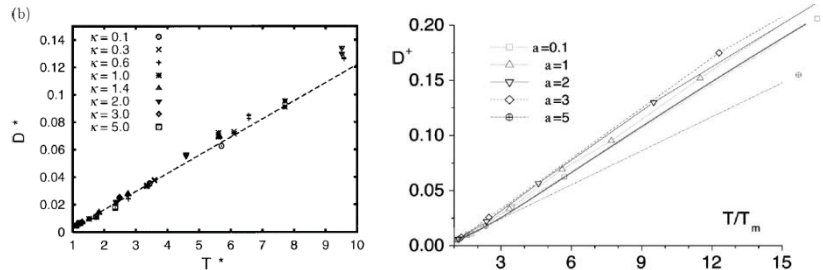
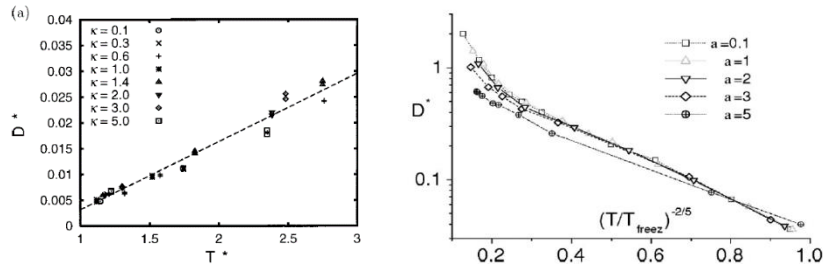
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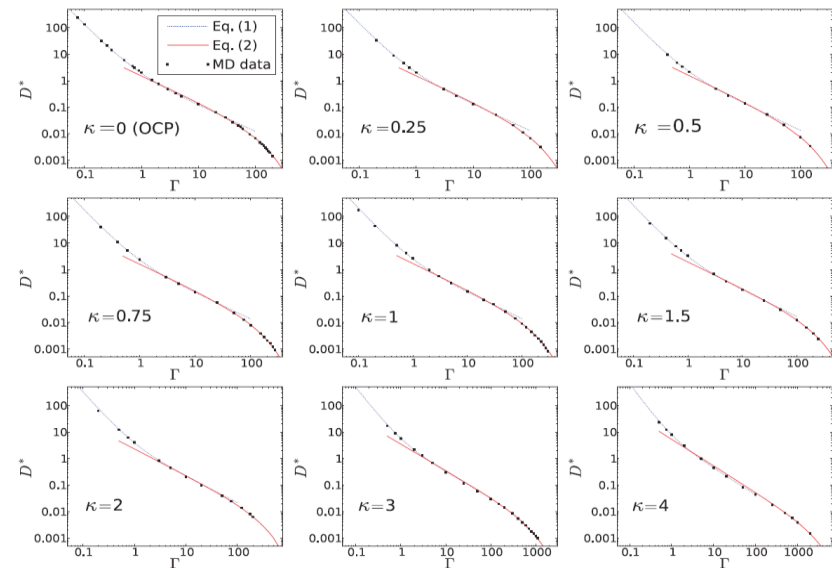
# Some previous results (single component Yukawa systems in three dimensions)

- Extensive MD simulation results tabulated by Ohta and Hamaguchi
- Rosenfeld: Excess entropy and freezing temperature scalings
- Daligault: MD simulations + fit



Ohta and Hamaguchi  
PoP (2000)

Rosenfeld PRE  
(2000)



Daligault PRE (2012)



## de Gennes approach

- De Gennes (1959) related the **self-diffusion** coefficient in classical atomic liquids to the pairwise **interaction potential  $\phi(r)$**  and liquid structure in terms of the **radial distribution function (RDF)  $g(r)$**

$$D = \sqrt{\frac{\pi}{2}} \frac{v_{\text{T}}^2}{\Omega_{\text{E}}},$$

with the characteristic frequency (**Einstein frequency**)

$$\Omega_{\text{E}}^2 = \frac{n}{3m} \int_0^{\infty} dr g(r) \Delta\phi(r),$$

- Einstein frequency can thus be identified as a rough measure of momentum transfer (friction) rate in liquids
- The approach is not expected to be exact, but suggests a **useful normalization**

$$D_E = D (\Omega_{\text{E}}/v_{\text{T}}^2)$$



# Motivation

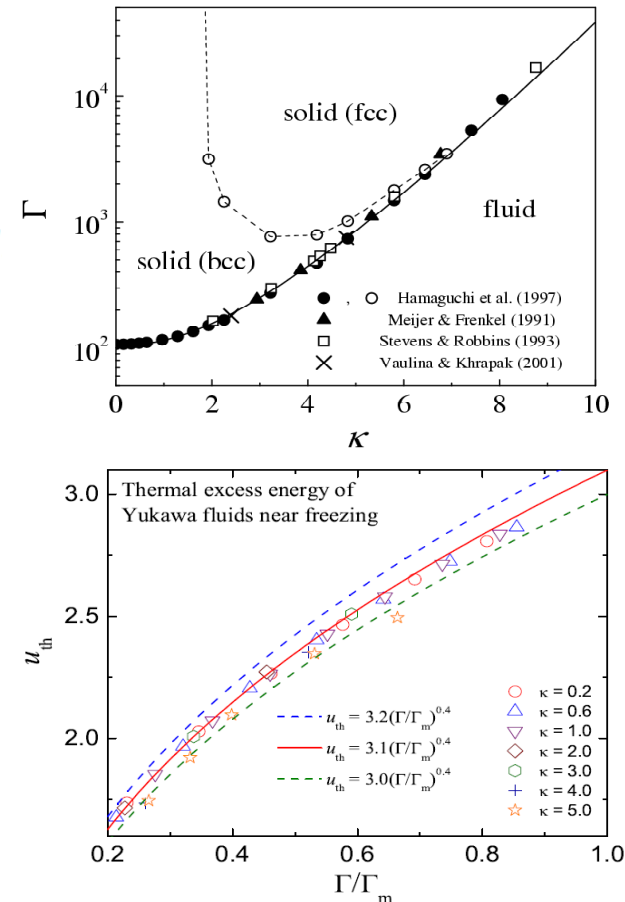
- Quantitative test of de Gennes prediction (using Yukawa systems)
- New data for self-diffusion of Yukawa melts
- Compare results for Yukawa melts with those for liquid metals and other related systems at the melting temperature
- Provide simple explanation of the observed quantitative similarity



# Background information

- Yukawa potential  $\phi(r) = (Q^2/r) \exp(-r/\lambda)$
- Phase state determined by the coupling  $\Gamma = Q^2/\hat{a}T$  and screening  $\kappa = a/\lambda$  parameters
- Thermodynamics of Yukawa fluids and crystals is well understood
- Einstein frequency is trivially related to the excess internal energy

$$\Omega_E^2 = \frac{2}{9} \frac{\kappa^2}{\Gamma} \omega_p^2 u_{\text{ex}} = \frac{2}{3} \frac{\kappa^2}{a^2} v_T^2 u_{\text{ex}}$$



For thermodynamics see e.g. Khrapak et. al. PRE (2015), JCP (2015)

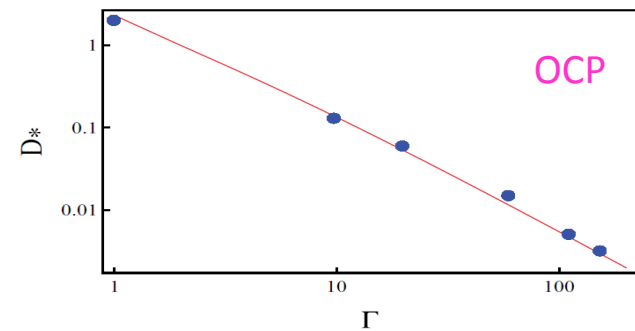
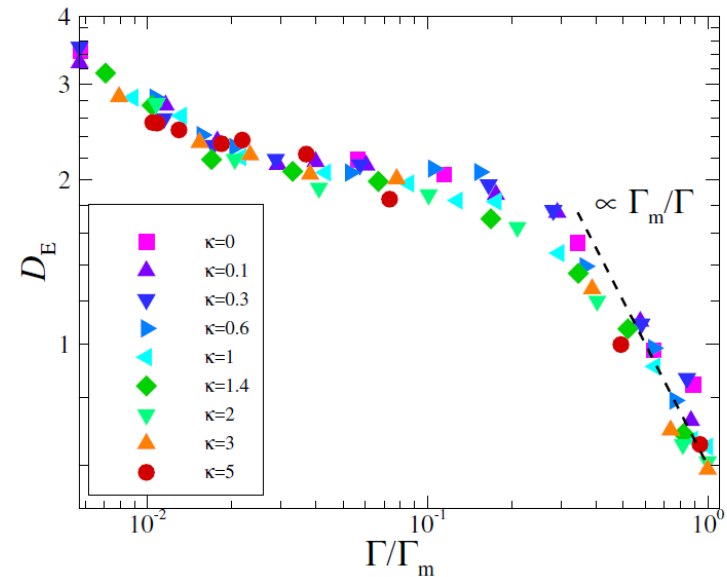


# Self diffusion coefficient of Yukawa fluids

- When plotted vs  $\Gamma/\Gamma_m$  self-diffusion shows no systematic dependence on  $\kappa$
- Weak dependence of self-diffusion on coupling – success of de Gennes approach
- A simple estimate is accurate to within a factor of two in the fluid regime

$$D \simeq v_T^2 / \Omega_E$$

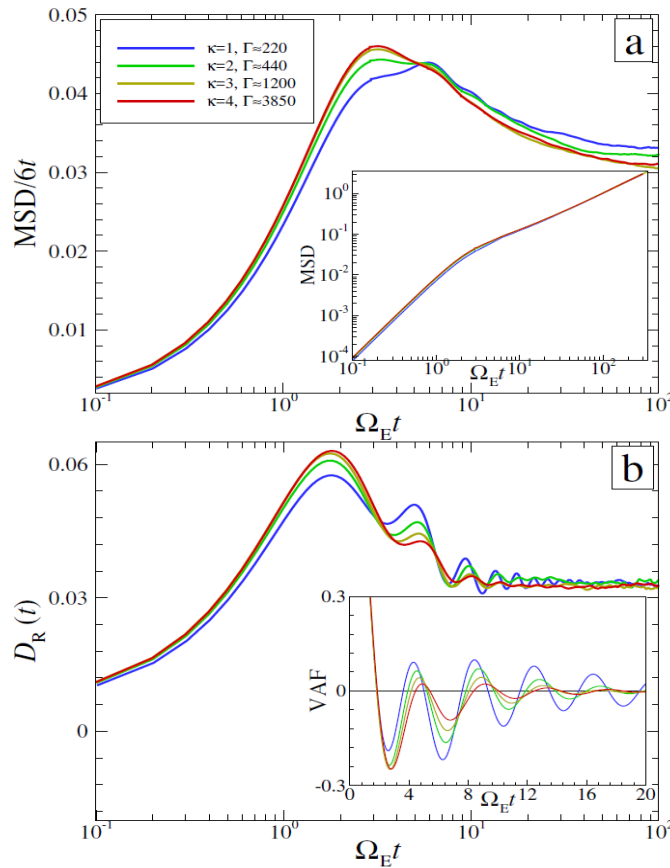
- **Three regimes of self-diffusion** can be identified (weak coupling regime, plateau-like behavior, approach to freezing)



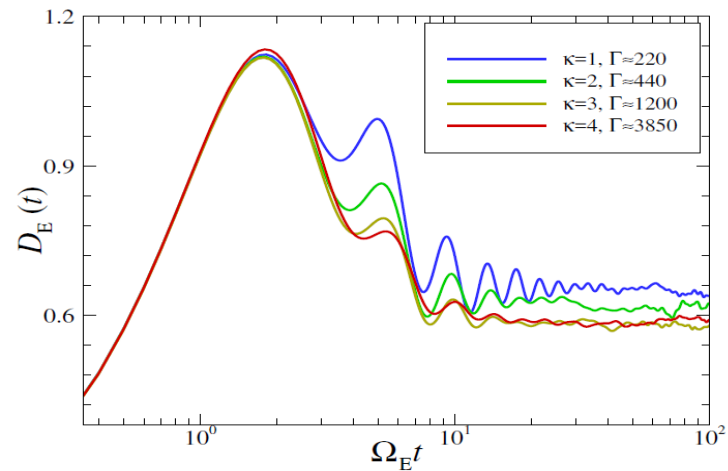
MD data for fluids by Ohta and Hamaguchi (2000); melts – present work



# Self-diffusion in Yukawa melts



- Two possibilities to determine SD coefficient
  - MSD
  - Green-Kubo relation
- Two normalizations:  $D_R = Dn^{1/3}/v_T$  and  $D_E$
- At freezing  $D_R \simeq 0.03$  and  $D_E \simeq 0.6$



Simulations by Klumov&Couedel





# Understanding the value of the self-diffusion coefficient at freezing

- Near the **melting point**

$$\frac{1}{2}m\Omega_E^2\langle\delta r^2\rangle\sim\frac{3}{2}T.$$

- According to the **Lindemann melting rule**  $\langle\delta r^2\rangle\sim L^2\Delta^2$ , where  $L\sim 0.1$
- From this the following relation between different normalizations emerges

$$D\simeq 0.6\frac{v_T^2}{\Omega_E}\simeq\frac{0.6L}{\sqrt{3}}v_T\Delta\simeq 0.03v_Tn^{-1/3}$$

- Coincides with what we have documented for Yukawa melts



# How universal is self-diffusion at freezing?

- Similar values  $D_R \simeq 0.03$  observed for several other **simple fluids** (Hertz, GCM, IPL)
- **Liquid metals** demonstrate the same magnitude (see Table)
- In the **OCP limit**  $D_R \simeq 0.031$
- In the **HS limit**  $D_R \simeq 0.02$
- Universality for soft enough interactions?

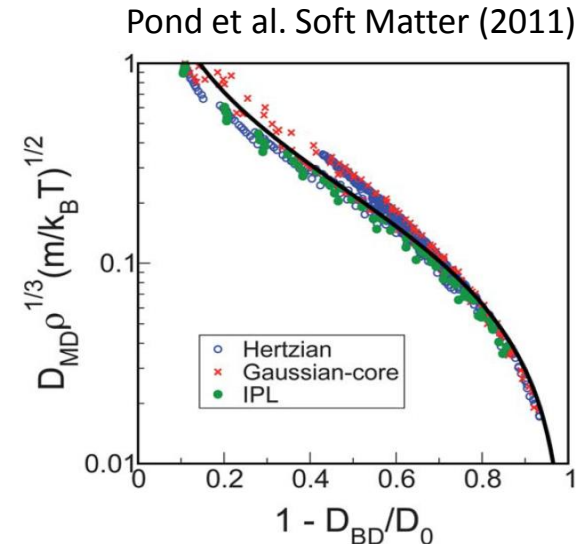


TABLE I. Reduced diffusion constants  $D_R$  of ten liquid metals at the corresponding melting points as calculated from the data summarized in Ref. 45.

Li	Na	K	Rb	Cu	Ag	Pb	Zn	In	Hg
0.029	0.033	0.032	0.034	0.040	0.031	0.035	0.027	0.032	0.034

Recalculated from the data summarized in March and Tosi book



# Conclusion

- De Gennes prediction is inexact, but useful
- Constancy of self-diffusion at freezing -> **Dynamic freezing criterion for simple soft atomistic systems?**
- Application: Momentum transfer in strongly coupled plasmas (complex plasmas)
- Other transport properties (e.g. viscosity)





# Thank you for your attention!

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