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The similar size of slums

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ABSTRACT

More than half of the world's population currently resides in urban areas. In the majority of developing countries slums are a defining part of the urban scape. Their supply with energy, basic infrastructure, among others is one of the main challenges of modern civilizations. To provide an optimal supply, the spatial patterns of slums in cities have to be explored. While most of current literature is focused on inter-urban dynamics, this paper is focused on intra-urban pattern (i.e. the spatial pattern of morphological slums within a city) and its link to the inter-urban ones. Therefore, census and remote sensing data are analyzed to create rank size distributions of morphological slums in different cities of developing countries. The observations were compared to rank size distributions of cities in a respective developing country. It is found that typical inter-urban pattern can be transferred to intra-urban pattern. Surprisingly is the fact that the size of slums is independent from city and global region in the analyzed cities. The slums in Mumbai, Manila, Rio de Janeiro and Cape Town have an average area of 0.016 km^2 with a standard deviation of only 0.004 km^2.

1. Introduction

In 2014, more than 54% of the world's population lived in cities. It is expected that this proportion will rise to a level of 66% by the year 2050 (United Nations, 2014). While the urban population increases per se, the number of very large agglomerations (megacities) with a significant proportion of slums is also increasing (United Nations, 2016). The rapid urbanization and growth of megacities can be observed especially in developing countries, which are mainly located in Africa, South America and Asia (Taubenböck et al., 2015a). The capital of Bangladesh (Dhaka) for example grows every hour by an estimated 50 people (Taubenböck et al., 2015a). This fact leads to a massive strain on the infrastructure of these cities and to underdeveloped water or energy supply systems (Jain, Knieling, & Taubenböck, 2015; Van der Bruggen, Borghgraef, & Vinckier, 2010). Furthermore, these megacities are characterized by a typical polar structure (Hoerning, 2016): a complex arrangement of slum areas form a complex pattern within the formal city where social gradients are present: poor people reside in close spatial vicinity next to rich ones (Marques & Saraiva, 2017). In most of the above mentioned cities, a big amount of these poor inhabitants live in areas outside of municipal planning efforts, called slums or informal settlements (Hofmann, Taubenböck, & Werthmann, 2015). These informal settlements are often characterized by the poor living conditions of their inhabitants, which are strongly related to the often poorly developed infrastructure, having a negative impact on different areas of life, such as health or education (Martnez, Mboup, Sliuzas, & Stein, 2008).

These slums are a significant part of urban regions and they form a highly complex pattern within the agglomeration. In cities such as Mumbai populations in slums feature 55% of the entire population and have become a defining part of the urban landscape (Taubenböck et al., 2015a). Political, social and economic mechanisms coupled with geographical constraints lead to different spatial patterns. Obviously, optimal supply systems for these topologies of slums depend on the observed urban pattern. To create holistic solution strategies to improve the bad living conditions of the urban poor (Martnez et al., 2008), it is important to understand these different urban patterns and their spatial distribution (Friesen, Rausch, & Pelz, 2017; Rausch, Friesen, Altherr, Meck, & Pelz, 2018; Hachmann, Jokar Arsanjani, & Vaz,).

There are many approaches describing urban systems and their spatial configurations. A common used tool are rank size distributions. In this method, the different elements of a system are ordered by size and the emerged distribution is analyzed. While there are many studies of describing inter-urban patterns (cities in a region or a country), rank-size distributions were rarely used in an intra-urban context. In this paper we will focus on the rank size distributions of a specific urban structural type within the city - namely morphological slums. We understand morphological slums as a characteristic physical appearance of the built environment, i.e. organic, complex spatial layouts of very high building density featuring very small and predominantly low

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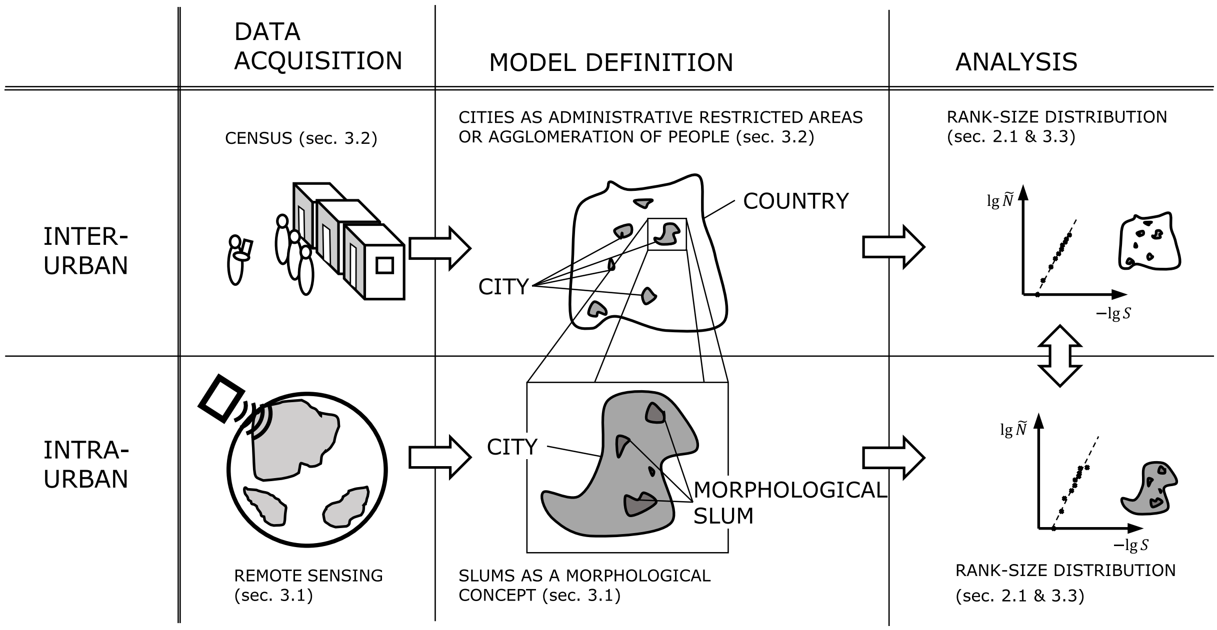

Fig. 1. Framework of this work.

buildings (Wurm and Taubenböck, 2018; Taubenböck, Kraff, & Wurm, 2018.). In addition, we also integrate for comparison census based slums (i.e. areas classified as slums based on household income) to allow for analyzing different measurement methods.

To do so, we analyze histograms, where the relative frequency of morphological slums sizes is plotted versus a size parameter. For identifying the morphological slums we deploy remote sensing data and classify slums as morphological objects using characteristic physical parameters of their settlement appearance. After presenting our conceptual background and a brief overview of the history of rank size distributions in the following section, we show the methods and data we used in section 3 before we present our results in section 4, discuss them in section 5 and finally conclude the paper with section 6.

2. Conceptual background

The aim of this paper is the analysis whether size distributions of morphological slums in different cities show similarities. To do so, we first present the method of rank size distributions in the following subsection (Fig. 1, right column). It was introduced in the last century and points out that most of the cities in different countries follow the so-called Zipf's law (Nitsch, 2005). In the main part of this paper, we investigate the intra-urban size distribution of morphological slums in four different cities (Fig. 1, lower row).

Beyond the analysis of size distributions of morphological slums, the analysis of census data allows in parallel an investigation on the influence of measurement methods as well as the question whether similarities between inter-urban and intra-urban size distributions exist.

2.1. Rank size distributions

Before applying the concept of rank size distributions to morphological slums, we present an overview of their use in the description of urban systems in general. Therefore, the following review is divided in two sections. The first section presents the use of distributions for analyzing and pointing out characteristics of cities within a region (e.g. country, state) while the second one presents the research of size distributions in intra-urban systems, i.e within a city.

2.1.1. City distributions within a region

In 1913, Felix Auerbach looked at the relation between rank $\widetilde{N}$ of a city belonging to a specific country and its size S measured in number of citizen: $\widetilde{N} = \widetilde{N}(S)$ (Auerbach, 1913). The rank represents the position of a city when ordering the cities by their size. He observed that the rank is proportional to the reciprocal size of the city. For the largest Brazilian cities this relation is plotted in Fig. 2 with census data from 2005 (de Geografia e Estatstica, 2017a).

This dependency was investigated in the following years for cities in different regions and countries (Jefferson, 1939; Singer, 1936). The most popular research was done by George Kingsley Zipf (Zipf, 1941; Zipf, 1949). The power law relation

$$\widetilde{N} \propto \left(\frac{1}{S}\right)^{\alpha} \tag{1}$$

sketched in Fig. 2 is known as Zipf's law. The empirical gained power coefficient is according to Zipf approximate one, $\alpha \approx 1$. This result was confirmed by different investigations in the following decades (e.g. (Nitsch, 2005; Rosen & Resnick, 1980; Soo, 2005, 2007)). In a distribution of this kind, also known as Pareto distribution, there is no typical rank and no typical size, i.e. there is no inherent scale. Such systems are therefore called scale-free.

Further empirical studies from Reed and Eeckhout have shown that Zipf's law only describes the largest elements of a distribution (Eeckhout, 2004; Reed, 2002). If one considers a larger number of cities, it can be observed that the rank of smaller cities can no longer be

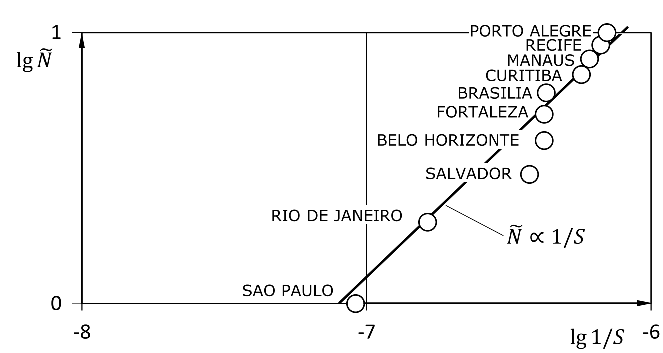

Fig. 2. Rank logarithm of $\widetilde{N}$ of a city vs. the logarithm of the reciprocal size $1/S$ measured in citizen for the ten largest Brazilian cities. The census data from 2005 (de Geografia e Estatstica, 2017a) confirm the known Zipf relation: the rank is proportional to the reciprocal size.

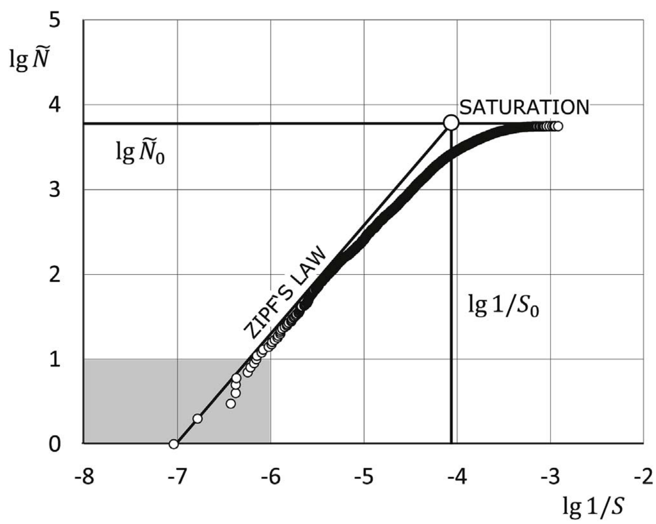

Fig. 3. Rank $\lg \widetilde{N}$ of a city vs. the reciprocal size $\lg 1/S$ measured in citizen for all Brazilian cities using census data for the year 2005 (de Geografia e Estatstica, 2017a). The grey area is shown as Fig. 2.

described with relation (1). Rather, a saturation occurs, where the order of magnitude of the rank no longer changes greatly.

This is seen in the rank size distribution of Brazilian cities if one considers not only the largest cities but all (Fig. 3).

Due to saturation, the system is no longer scale-free. Rather, a scale or typical size can be read in the distribution, which is for example the point of intersection between the straight line of Zipf's law and the maximum rank $\widetilde{N}_0$. Instead of using the intersection, it is advantageous to use the geometric mean

$$S_0 = \sqrt[\widetilde{N}_0]{\prod_{j=1}^{\widetilde{N}_0} S_j} \tag{2}$$

as typical size, where $\widetilde{N}_0$ is the number of observed cities.

In terms of statistics, $P: = 1 - N$ is the cumulative distribution function associated with Figs. 2 and 3 with the normalized rank $N: = \widetilde{N}/\widetilde{N}_0$ (Eeckhout, 2004). Hence, the probability density distribution is the negative derivative of the rank size distribution.

$$p = \frac{dP}{dS} = -\frac{dN}{dS} \tag{3}$$

Reed and Eeckhout (Eeckhout, 2004; Reed, 2002) therefore propose the logarithmic normal distribution (log-normal distribution)

$$p = \frac{1}{S}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2\sigma^2}\log^2 S/S_0\right) \tag{4}$$

with the standard deviation

$$\sigma = \sqrt{\frac{1}{\widetilde{N}_0}\sum_{i=1}^{\widetilde{N}_0}\log^2 S_i/S_0} \tag{5}$$

as a suitable measure (Fig. 4).

This becomes more pronounced when plotting $\lg N$ vs. $-\lg S$ and the related density distribution p, Fig. 4 (i).

Different investigations in the last years showed that both the population data, as well as the area of the different cities were nearly log-normal distributed (Decker, Kerkhoff, & Moses, 2007). This is indeed confirmed by Fig. 4 (ii), where the cities of Brazil are shown with the connected relative frequency distribution. The data were collected from the *Instituto Brasileiro de Geografia e Estatstica* (*IBGE*). The population statistic includes all cities recorded in the census. The spatial extents of the cities was collected via satellites data from the CBERS-2B satellite (de Geografia e Estatstica, 2017b). In the dataset only cities with more than 100 000 inhabitants are considered, according to the population estimate by the IBGE for the year 2005.

The results confirm the statements above. Both distribution are scale free for small ranks with a power close to unity, confirming Zipf's law. The histograms in Fig. 4 show, that the log-normal distribution describes both the population and the area size distribution of cities in Brazil very well. Although the standard deviation differs for area and population data, the form of the distribution is similar.

Comparing the geometric mean S_0 of population data of different countries it became obvious: the geometric means differ from country to country. Looking at the data provided by GonzlezVal, Ramos, SanzGracia, & VeraCabello, (2015) the geometric mean of Brazil is of the same magnitude as of Japan but is one order of magnitude greater than of the US or Germany (Table 1). There is no uniform geometric mean of the city size in different countries.

Summing up the above, the analysis of size distributions is a commonly used tool to describe urban systems. Although the discussion on which distribution (Pareto, log-normal, loglogistic, …) fits the data best is ongoing (Eeckhout, 2009; Giesen, Zimmermann, & Suedekum, 2010; GonzlezVal et al., 2015; Levy, 2009; Reed & Jorgensen, 2004), we will not contribute to this debate with this paper. Since studies of urban areas as well as population sizes show that cities can be described approximately by log-normal distributions (Decker et al., 2007; Eeckhout, 2009), and we want to compare them with intra-urban size distributions of morphological slums, we use the log-normal distribution to fit the data and calculate quantitative values to compare the slum systems in different cities.

2.1.2. *Urban cluster distributions within a city*

The inter-urban size distributions of urban systems have been investigated often. In contrast, we will focus in this paper on intra-urban rank-size distributions, which have only been targeted in few cases so far. Intra-urban size distributions have direct practical relevance, as e.g. the spatial patterns determine the types and organization of supply systems.

In 1998, Schweitzer and Steinbrink performed the first systematic investigation on rank size distribution on urban clusters (Schweitzer & Steinbink, 2002; Schweitzer & Steinbrink, 1998) observing the urban areas of Berlin, Munich, Daegu, Moscow and Philadelphia and distinguish between built-up and unbuilt units. They sorted the different clusters according to their size and found a very similar relation like those known from inter-urban distributions. Fragikas and Seto did a very similar approach using remote sensing data, investigating rank size distributions of urban clusters (Fragkias & Seto, 2009). Chen and Wang investigated rank size distributions of urban areas and the connections between the urban space-filling process and the rank size relationships (Chen & Wang, 2014). Taubenböck et al. revealed intra-urban rank size distributions for urban mass concentrations which are used as proxy for centers and subcenters (Taubenböck, Standfuss, Wurm, Krehl, & Siedentop, 2017). Beside the mentioned studies, no study is known which examines the rank size distribution of specific thematic structural types in general, and for morphological slums in particular. Barros and Sobreira performed an analysis about self organization across the scales but did not investigate the topology of a whole city (Barros & Sobreira, 2002; Sobreira & Gomes, 2001). As globally, however, a large proportion of inhabitants of cities live in slums (the UN assume that 33% of the urban population lives in slums (United Nations, 2014)), an analysis of these structures is essential and remote sensing seems to be one essential tool in the framework (Mahabir, Crooks, Croitoru, & Agouris, 2016).

3. Data and methods

Our work flow is represented in the following Fig. 5. We first introduce the classification of morphological slums using data from remote sensing, then explain our study sites selection and finally present

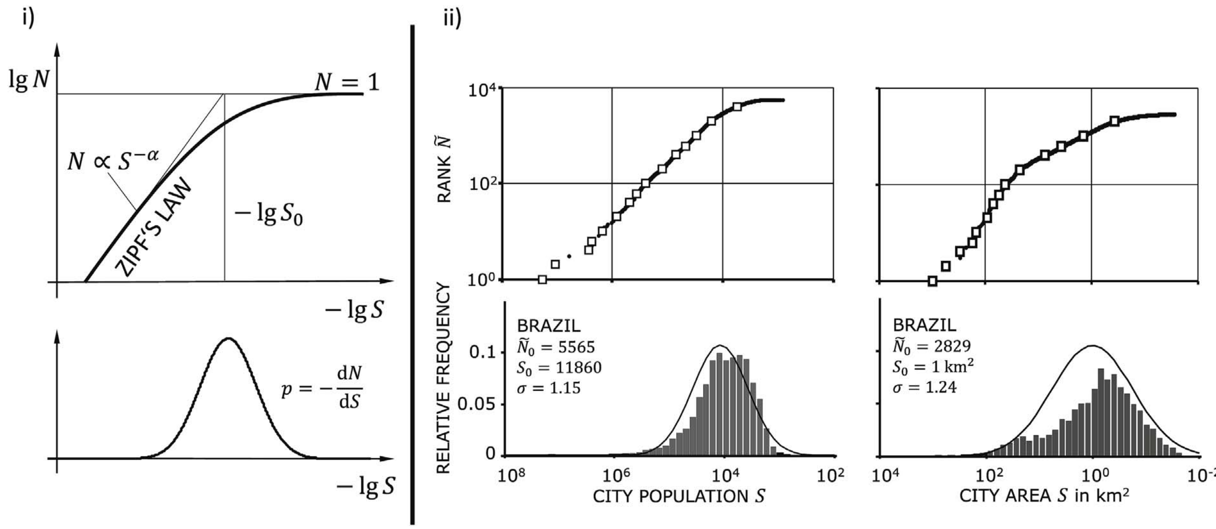

Fig. 4. (i) Rank $\lg N$ of a city vs. the reciprocal size $\lg 1/S$ and related probability density function. (ii) Rank size distribution of Brazilian cities ordered by population estimation from 2005, rank size distribution of Brazilian cities ordered by area using satellite data from 2005, size distribution and log-normal fit of Brazilian cities by population and size distribution and log-normal fit of Brazilian cities by area (de Geografia e Estatstica, 2017b).

Table 1
Data of different countries and values for log-normal distributions. GON represents data from GonzlezVal et al. (2015), C represents Census data (sec. 3.2).

Country (Source)	$\widetilde{N}_0$	S_0 in people	σ
US Population (GON)	28664	1249	1.83
German Population (GON)	11292	1845	1.51
Japanese Population (GON)	2102	9321	1.24
Brazil Population (C)	5565	11860	1.15

the methods applied for analyzing the size distributions.

3.1. Remote sensing for mapping morphological slums

The terms squatter settlements, slums or informal settlements are frequently used in literature for describing housing and living conditions of the urban poor (e.g. (Graesser et al., 2012; Kuffer, Pfeffer, Sliuzas, & Baud, 2016; Taubenböck & Kraff, 2015)). In this paper, we approach the localization of the urban poor using the term morphologic slums. We do so, as we detect and spatially delimit slums based on very high resolution optical Earth observation (EO) data allowing only a physical approach towards the localization of these particular settlements. The literature review reveals that many studies find that formal and slum settlements show significant differences in urban morphology (e.g. (Baud, Kuffer, Pfeffer, Sliuzas, & Karuppannan, 2010; Flores-Fernandez, 2011; Smollich, 2015), (Kuffer et al., 2016b)). In dependence on this, we base our morphologic approach onto an ontology defining typical characteristics of morphologic slums in EO data (Wurm, Taubenböck, Weigand, & Schmitt, 2017). Along the ontology framework developed by Kohli et al. (Kohli, Sliuzas, Kerle, & Stein, 2012), we determine physical parameters on the suggested settlement level and the object level for the systematic classification of morphologic slums. On the settlement level, we aim to capture the structure by the parameters high settlement density and organic, complex layout of buildings. On the object level we capture the characteristics of the individual buildings using the parameters small building sizes and low building heights (compare, e.g. Taubenböck & Kraff, (2014)). The assessments whether all these parameters are de facto given are seen relative to the entire morphology of the respective city and its direct neighborhoods. We follow this protocol systematically for all study cities.

In this study, we focus on the intra-as well as inter-urban rank size distributions of morphologic slums; for a reliable data base, we decided to digitize the settlement structures of interest using the cognitive perception of an interpreter. We do so, as automatic classification approaches still have shortcomings regarding accuracy (e.g. compare the reviews by Kuffer et al., (2016a) and Mahabir, Croitoru, Crooks,

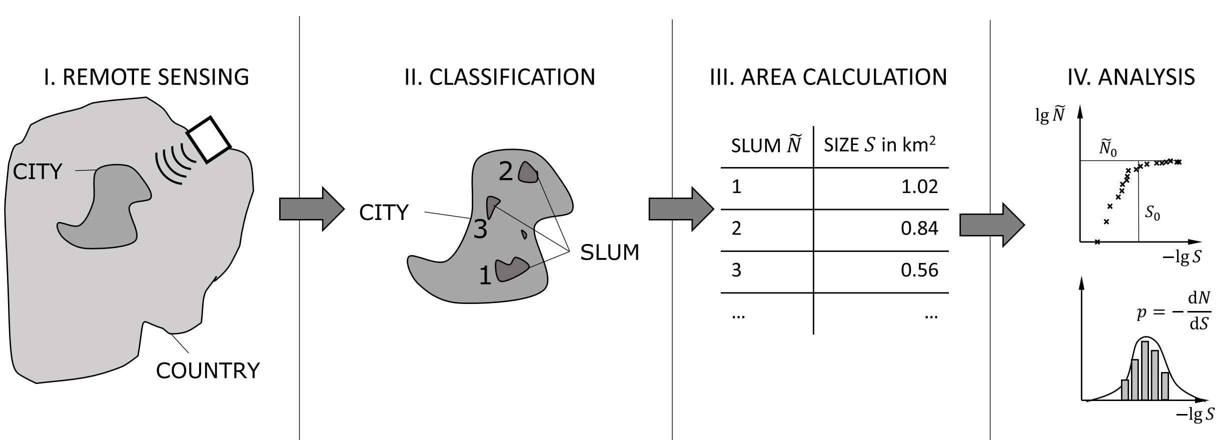

Fig. 5. Work flow to analyze the size distribution of morphological slums detected by remote sensing.

Agouris, & Stefanidis, (2018)). Visual image interpretation performed by interpreters familiar with the ontology as well as with local conditions provides a flexible and useful approach mapping morphologic slums. The cognitive perception and the local knowledge of the interpreters allow due to the geometric resolution of VHR optical EO-data to basically derive an individual polygon per slum area following a standardized digitization protocol. This protocol allows for an unbiased and consistent mapping approach of morphologic slums across cities. This is a challenging task for e.g. squatter settlements, as their typical characteristic is the lacking land tenure (e.g. Dovey & King, (2011)), which cannot be derived from EO-data. In this case, our morphologic approach using EO-data will not allow their classification. The protocol for mapping morphologic slums defines a consistent scale for digitizing at 1:1000, the classification of areas of highest settlement density, small buildings, and organic, complex layout of buildings, the classification of slums at block level, i.e. areas which are usually circumscribed by street networks or natural borders, the definition of a minimum distance of 10 m for blocks being treated as individual spatial entities (compare (Wurm & Taubenböck, 2018)). We project the classified slum areas onto the settlement areas dervied from the Global Urban Footprint (Esch et al., 2012).

3.2. Study site selection

The cities investigated in this paper are Metro Manila (Philippines), Mumbai (India), Rio de Janeiro (Brazil) and Cape Town (South Africa). We choose these cities because of their different geographical location in different continents, the different cultural and economic influences of the countries the cities lie in and the different geographical topologies. Beyond, all four cities feature a significant share of slums within their urban environment. Mumbai has 12.5 million (2011), Metro Manila 12.9 million (2015), Rio de Janeiro 6.4 million (2013) and Cape Town 3.7 million inhabitants (2011) according to the last available census data. The first three cities respectively their metropolitan regions (Rio de Janeiro) are megacities, located in Asia and South America.

For all four cities, we analyze the rank size distributions of classified morphological slums. However, in two cases we use additional geoinformation: In Cape Town we analyze the rank size distribution of morphological slums combined with townships. The townships feature a more planned structure (and thus, cannot be unambiguously classified using the EO-approach) but are considered slum-like areas for the urban poor. The morphologic differences of the built environment are visualized in Fig. 6c. We do that in the case of Cape Town because the administrative authorities have taken a great influence on the settling behavior of the poor inhabitants in the last century by establishing townships (Sodemann, 1986) and we want to investigate the influence on the resulting size distribution.

In Rio de Janeiro, we analyze the rank size distributions of census slums to identify differences of measurement methods between the census and the EO-approach. This data was collected in the census 2010 (Demogrfico). In the Brazilian census the favelas or slums were called subnormal agglomerations or *Aglomerados subnormais*. One of these units has at least 51 houses or shacks and has to fulfill some other criteria like land occupation of others and a generally disordered and dense mannered arrangement (Demogrfico). The spatial delineations of census slums, which rely on the spatial entity of given administrative units and morphologic slums, which follow the built up structure of the protocol introduced above differ significantly (Fig. 6a). Both investigations considered only slums within the administrative area of Rio de Janeiro.

3.3. Method applied for rank-size distributions

To create the rank size distributions, we calculate the area of the morphological slums using GIS (geoinformation systems). The elements are ordered by size and plotted in a double logarithmic way. In a second step, we use the log-normal distribution to fit the data using equations (2) and (5). Therefore, we analyze the relative frequency distribution and present the results using histograms with logarithmic bin sizes next to the fitted log-normal distribution. While the geometric mean S_0 can be interpreted as a typical number of citizens when using population data, for the spatial data we use the typical length

$$l := \sqrt{S_0}. \tag{6}$$

If the shape of an element is assumed to be square, l can be interpreted as its edge length, thus representing a characteristic length in the considered system.

4. Results

In the results section, we first show the size distribution of the area of morphological slums in Cape Town, Metro Manila, Mumbai and Rio de Janeiro, collected with remote sensing. Second, we compare the dataset of Rio de Janeiro created with remote sensing with one collected in the Census 2010 by survey. Third, we also compare one data set of Cape Town with and without the consideration of townships.

4.1. Intra-urban distributions

Fig. 7 (left) shows the similar size S_0 of morphological slums in Manila, Mumbai, Rio de Janeiro and Cape Town. Even though $\widetilde{N}_0$ differs by an order of magnitude (Manila 1350 slums, Mumbai 1003, Rio de Janeiro 633 and Capetown 123 slums), the cumulative distributions are of surprising similarity and saturation occurs for nearly the same size for all considered cities.

This comes more pronounced when plotting the normalized rank N against the area S (Fig. 7, right).

All distributions fall on one curve and reach saturation in the vicinity of $\overline{S}_0$. We calculate the arithmetic mean $\overline{S}_0$ of the different geometric means $S_{0,i}$ with

$$\overline{S}_0 := \sum_{i=1}^{K} S_{0,i}/K = 0.0158\ \mathrm{km}^2 \tag{7}$$

and

$$\overline{\sigma}_0 := \sqrt{\sum_{i=1}^{K} (\overline{S}_0 - S_{0,i})^2/K} = 0.0036\ \mathrm{km}^2 \tag{8}$$

with the number $K = 4$ of the observed cities. This yields to an average slum length of $\bar{l} := \sqrt{\overline{S}_0} = 126.5$ m.

The distributions for Manila, Mumbai and Rio de Janeiro are a little steeper than the inter urban distributions (cf. Fig. 4) and also show a saturation. This is little different for Cape Town, where the number of slums $\widetilde{N}_0$ is much lower and the distribution flatter.

This similarity of the slum sizes can also be seen when observing the size distributions shown in the following Fig. 8.

All distributions have a geometric mean of nearly $S_0 \approx 10^{-2}$ km². 91.5% of all considered slums have a size between 10^{-3} km² and 10^{-1} km². In Rio 90.1% of the slums, in Manila 93.6%, in Mumbai 91.3%, in Cape Town (without townships) 82.9% and in Cape Town (with townships) 80.6% lie within this range. While the lower limit 10^{-3} km² corresponds approximately to the size of a penalty area in a soccer field, the upper limit 10^{-1} km² is in the order of 10 soccer fields.

Since the slum size appears globally uniform based on the four cities, supply systems can be developed which are designed specifically for this spatial expansion. All results are summarized in Table 2.

4.2. The influence of different slum classifications on the distributions - examples from Cape Town and Rio de Janeiro

In this subsection, we focus on the influence of varying slum

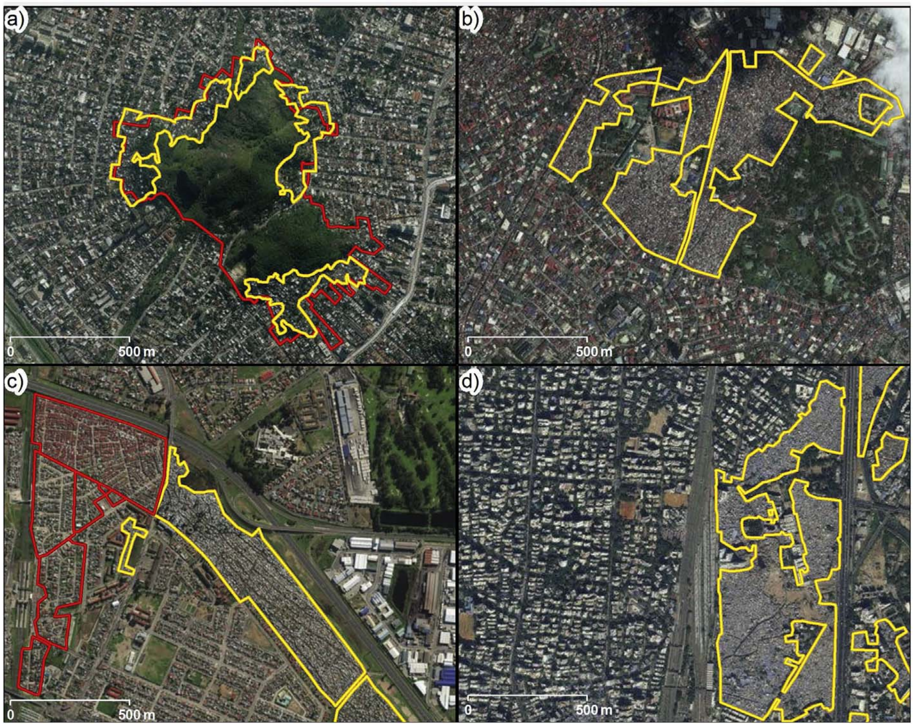

Fig. 6. Spatial delineation of morphologic slums (Yellow) across the globe in relation to surrounding formal structures: a) Rio de Janeiro: in contrast: Spatial delineation of census slums (Red); b) Manila; c) Cape Town; alternative morphologic structures of townships (Red); d) Mumbai. Source background image: ESRI basemap. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

classification data and methods onto rank size distributions. The two data sets capturing slums of Rio de Janeiro were collected with two different methods. While the first one was collected by detecting slums from EO-data in 2015, the second one was collected by Census tracts in 2010. We investigate the influence of the different approaches onto rank-size distributions within the official administrative boundaries of Rio de Janeiro.

We compare two distributions of the settlements in Cape Town. One distribution with and one without considering townships, which by their regularity do not correspond to the usual organic pattern of slums detected by the ontology used in the physical approach (see Fig. 9).

4.2.1. Rio de Janeiro

The geometric mean S_0 is different in both data sets. While the Census data from 2010 provides a value $l = 193$ m ($S_0 = 0.0374$ km^2) the remote sensing from 2015 provides $l = 141$ m ($S_0 = 0.0198$ km^2). This effect can be reasoned to the administrative units for slums used in the census; the artificial administrative units do not always correspond with the urban morphology but integrate a larger variety of morphological structures. On the other hand, the standard deviation of the EO-data set $\sigma = 1.273$ is a little higher than the one of the census data set $\sigma = 1.212$. The total number of slums is differing between both data sets and the total area of all slum areas in the EO-data is half as big as in the census data (59.1 km^2 Census and 25.5 km^2 EO-data). The difference in the total number of slums can be explained by the fact that some of the big slum areas were treated as one object in the one classification and as more than one object in the other classification. This fact can be seen in the following example in the upper right corner of the detail view in Fig. 10. While the big slum was classified as one object in the census data, it was treated as two slums in the remote sensing data.

4.2.2. Cape Town

In the case of Cape Town, we find that although the number of observed objects is almost tenfold (123 without and 1208 with townships), the typical length is nearly identical in both distributions ($l = 132$ m and $l = 135$ m). However, the size distribution when considering the townships loses its stochastic character, looks more regularly and corresponds visually rather a log-normal distribution. In the

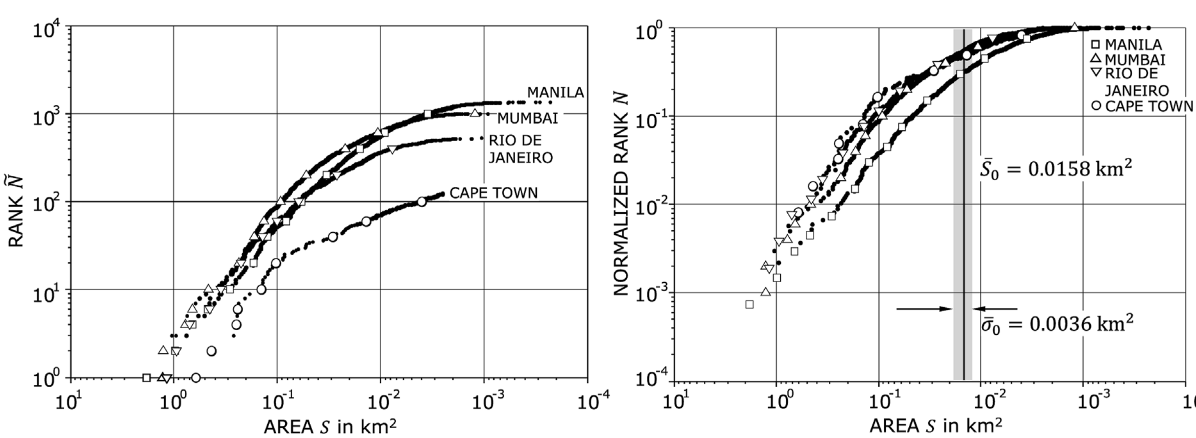

Fig. 7. Rank size distributions (left) and normalized rank size distributions (right) of morphological slums in Manila, Mumbai, Rio de Janeiro and Cape Town (without Townships) with $\bar{S}_0$ and $\bar{\sigma}_0$.

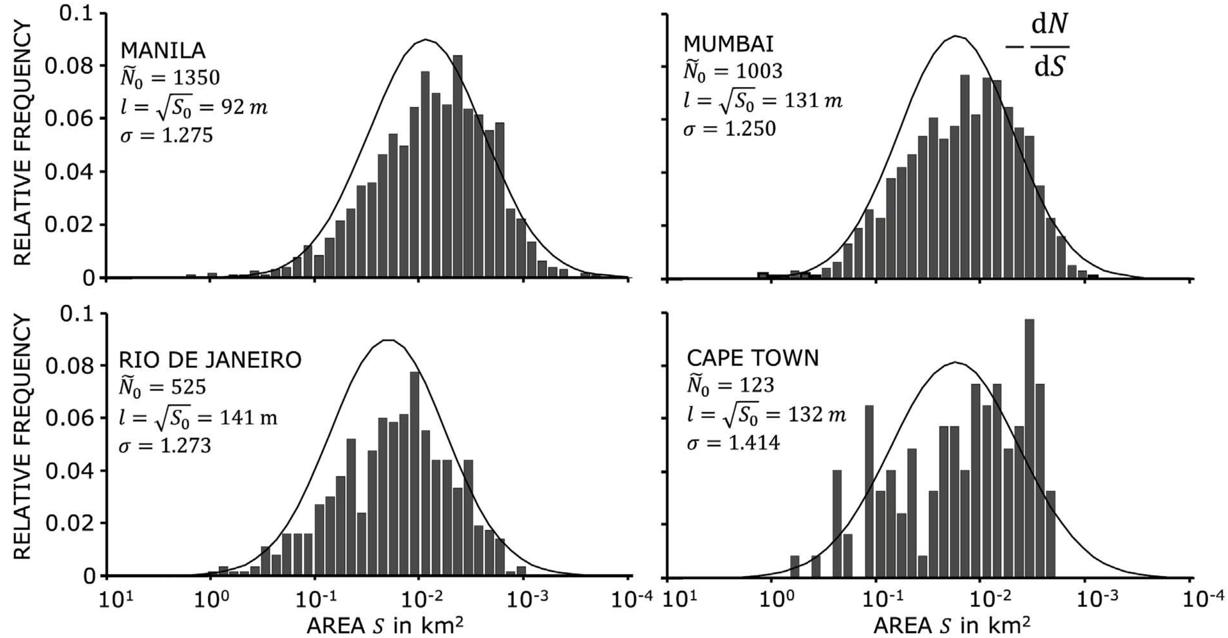

Fig. 8. Histograms and log-normal distribution of morphological slums in Manila, Mumbai, Rio de Janeiro, and Cape Town (without Townships).

Table 2
Data of different cities and values for log-normal distributions. C represents Census data (sec. 3.2) and RS represents data collected with remote sensing (sec. 3.1). In the case of Cape Town T represents data with and NT data without townships.

City (Source)	$\widetilde{N}_0$	S_0 in km²	σ	$l = \sqrt{S_0}$ in m
Mumbai, India (RS)	1003	0.0171	1.250	131
Manila, Philippiens (RS)	1350	0.0085	1.275	92
Cape Town, South Africa (RS/NT)	123	0.0174	1.414	132
Rio de Janeiro, Brazil (RS)	525	0.0198	1.273	141
Rio de Janeiro, Brazil (C)	763	0.0374	1.212	193
Cape Town, South Africa (RS/T)	1208	0.0181	1.626	135

case with townships, σ increases from $\sigma = 1.414$ to $\sigma = 1.626$ without considering townships. This lets us presume that in the specific case of Cape Town the planned building of townships had significant influence on the pressure of slum development and the respective distribution of morphological slums.

5. Discussion

As known from literature, both, the area and the population numbers of cities in different countries show similar size distributions (nearly log-normal) with different geometric means (Giesen et al., 2010; GonzlezVal et al., 2015). As shown, this is different when considering morphological slums in different cities of different parts of the

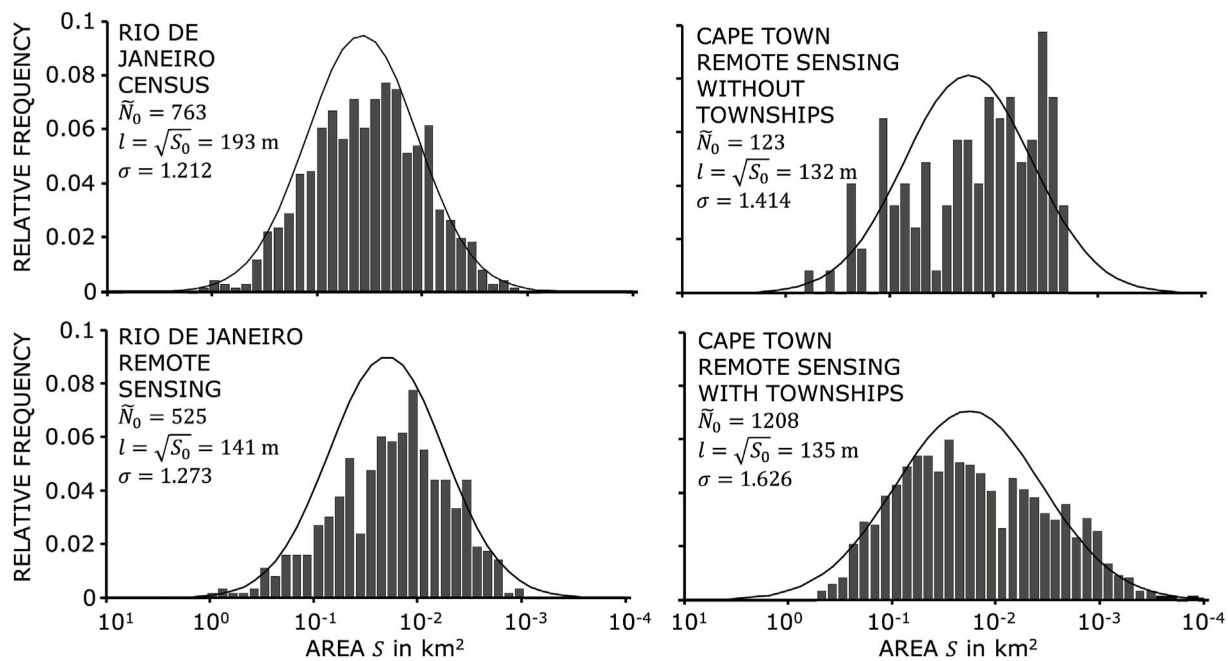

Fig. 9. Histograms and log-normal distribution of morphological and census collected slums in Rio de Janeiro, Brazil and Cape Town, South Africa with and without townships sorted by area.

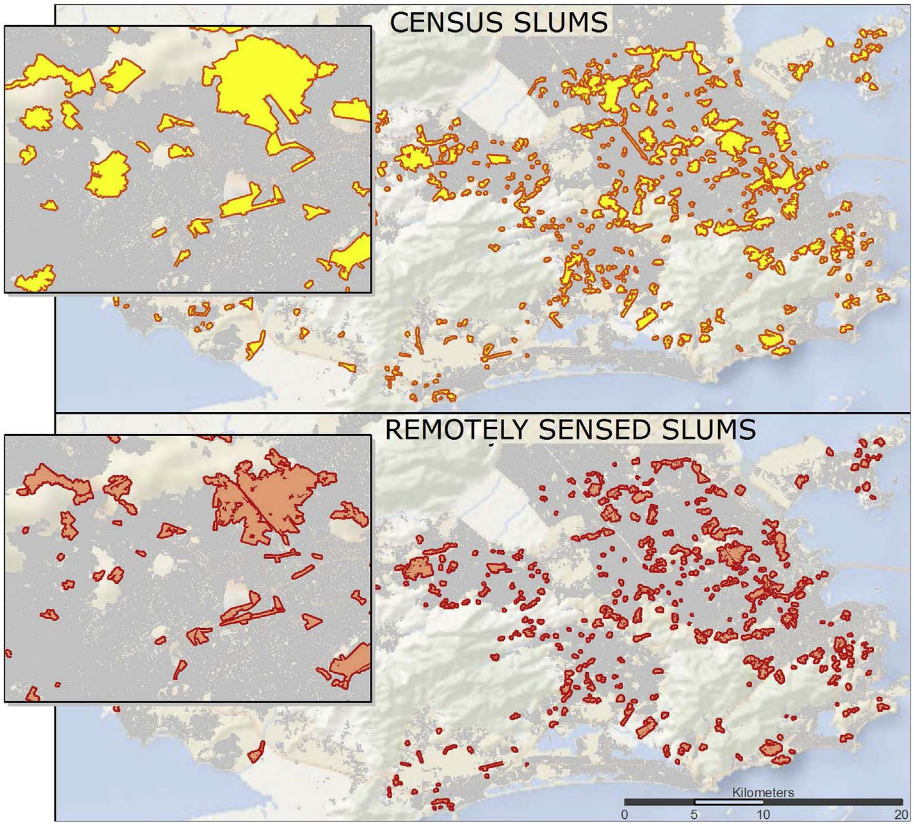

Fig. 10. Spatial outlines of census slums versus morphological slums in Rio de Janeiro, Brazil derived from remote sensing data projected onto the Global Urban Footprint.

world: Both, cities with a small number of morphological slums such as Cape Town ($\widetilde{N}_0 = 123$), as well as cities with topologies of more than 1000 morphological slums, such as Manila, show a similar size of morphological slums. The geometric mean is $S_0 \approx 10^{-2}$ km^2. A large proportion of the slums have a size between 10^{-3} and 10^{-1} km^2. In the case of a square base area, this corresponds to an edge length of 31.6 m and 316 m. Although the investigated cities have different historical backgrounds, other cultural and economic boundaries, the mean area of morphological slums in these different urban systems is $1.58 \cdot 10^{-2}$ km^2 corresponding to an edge length of 126.5 m. For the development of uniform supply structures these spatial dimensions this is crucial geoinformation, as they must be adapted to the respective topology of cities and their patterns.

The question arises as to what effects the findings made here have on the planning of the basic infrastructure (water, sanitary facilities, electrical energy, etc.). The study shows that, in addition to the large slums, which are often observed, there are many small slum units with a similar size. The typical slum size ($\bar{l}$ = 126.5 m) corresponds approximately to the size of a football field. For these frequently occurring but relatively small units, it may be advantageous to use decentralized supply structures. In the case of water supply, for example, these could be smaller filling stations supplied by trucks or, in the case of energy supply, concepts such as solar kiosks for charging mobile phones. It seems to be necessary that planning is not just focused on the well known large slum areas, than on the much more frequent occurrence of small slums.

In comparison to the other cities, Cape Town shows for the morphological slums a different behavior. The distribution shows a rather random structure instead of the expected log-normal distribution. We presume this different behavior relates to the planned townships for the urban poor reducing the pressure on illegal land occupation. However, if the planned townships are also considered we find the expected log-normal distribution of shelters for the urban poor. This reveals again the influence of slum classification methods on the results.

We have also shown that the distributions for morphological slums in Rio de Janeiro collected via remote sensing produce a similar type of distributions as the IBGE data. However, while we did not consider the time lag between both investigated data sets, this will be investigated in future research.

Nevertheless, a question arises in this context concerns the size of the elements per se. What is the definition of a slum? If, for example, a railway line or a road passes through a slum area, the question arises as to whether this is understood as separating element and we consider one or two settlements. Is a slum an object that is described by physical densities or must the social connections between the individual inhabitants be compulsorily considered (compare (Kohli, Stein, & Sliuzas, 2016))?

An other challenge is the fact, that the investigated information about slum areas provides no information about the population in that slum. Do the area and the population of a slum correlate? Rozenfeld, Rybski, Gabaix, & Makse, (2011) and Taubenböck et al., (2015b) show, that it is very hard to estimate population data from area data about slums. Taubenböck et al. show that the area and the population growth can differ from another and it is also hard to connect the data (Kuffer, Pfeffer, Sliuzas, Baud, & Maarseveen, 2017; Taubenböck, 2015).

Future work has to systematize what influences the classification has on the resulting distribution. How does the distribution change, when different slums count as one? Quantitative studies along the methodological path of Veneri are in demand (Veneri, 2016). He showed that Zipf's law fits inter-urban distributions better when using functional defined areas than administrative ones.

Despite all that, slum simulation models (compare the review of Roy, Lees, Palavalli, Pfeffer, & Sloot, (2014)) may consider the findings of this paper.

When looking at the data, the question is whether the log-normal distribution is an appropriate function to describe the distribution of

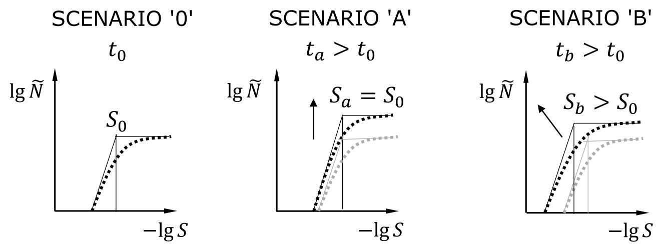

Fig. 11. Model of temporal behavior the rank size distribution of morphological slums within an urban system. Scenario '0' showns the rank size distribution at a starting point t_0. In scenario 'A' the typical slum size $S_a = S_0$ does not change over time, while it grows in scenario 'B' $S_b > S_0$.

the size of slums, since this question is also discussed for the discussion of inter-urban distributions (Giesen et al., 2010). We do not want to pursue this question further in this publication, since the aim of this paper is a basic empirical description of the size distribution of morphological slums in different cities. The question should, however, be considered in future investigations.

Combined with the question of the description of the distribution of the size of morphological slums is the question about the mechanisms of its development.

Assuming that the size of 10^{-2} km^2 is actually a universal size in terms of slums in cities, it would be possible to make statements about the growth behavior of slums. Therefore, we consider Fig. 11 and assume that the slum system at time t_0 has the rank size distribution shown.

If the total area of morphological slums grows, there are two possible scenarios. In scenario b, both the largest slum and the number of slums are growing. The typical size would thus shift to larger slum sizes. This would be a similar behavior like inter-urban distributions (Luckstead & Devadoss, 2014). However, assuming that the growth of each slum reaches saturation at a particular slum size, it could be assumed that only the number of slums in the system would change while the size of the largest elements with low rank stay nearly stable over time. This would keep the typical size of morphological slums constant over time (see scenario a). However, this model only considers the course of the size distributions over time and not the real processes leading to it, such as upgrading or slum eviction.

We are aware that in this study the amount of cities under investigation is still very small at the moment and the hypothesis should be examined in a larger sample and detailed analysis about the underlying mechanisms should be made.

6. Conclusion and outlook

While typical sizes of cities in different countries differ from another, the typical size of slums in different urban systems are stable near 10^{-2} km^2 and independent from country, continent or culture. Both distributions are very similar to the log-normal distribution. This information could be used for the development of supply infrastructures.

The results may provide the basis for further investigations to create examples of cities with generic but representative distributions of slum areas and their optimal supply structure. A greater amount of cities has to be investigated to support our thesis. However, all the results showed in this paper are empirical, mechanisms leading to this result are not analyzed in this regard. Are there special sociological factor leading to these typical size? Future work has to do investigations on the temporal behavior of slums and has to investigate the question if the typical size changes over time or if it is stable.

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