Towards Interactive Verification of Programmable Logic Controllers using Modal Kleene Algebra and KIV

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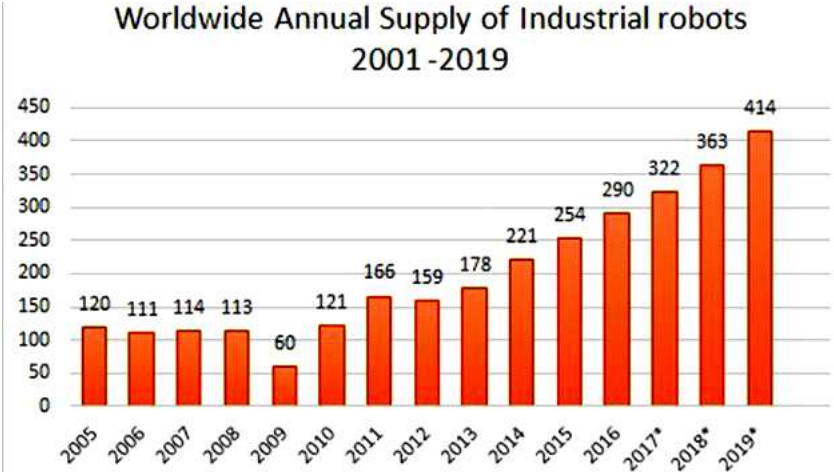
Outline

1. Motivation
2. PLC Crash Course
3. Modal Kleene Algebra and Linear Temporal Logic
4. Function Block Diagrams in Modal Kleene Algebra
5. Case Study: Mutual Exclusion
6. Conclusion and Outlook


Worldwide Annual Supply of Industrial robots
2001 -2019
450
400
350
300
250
200
150
100
50
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120
111
114
113
60
121
166
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363
414
2005
2006
2007
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2009
2010
2011
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2013
2014
2015
2016
2017*
2018*
2019*

robots are:

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- cost saving

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- reliable

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- reliable
- strong

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- strong
- very strong

robots are:

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- reliable
- strong
- very strong
- insensitive

robots are:

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- reliable
- strong
- very strong
- insensitive
- dangerous

⇒ careful control is indispensable

PLC - Purpose and Function

- Programmable Logic Controllers (PLCs) used for controlling various plants
- robots, pumps, valves, mechanical and automated devices, ...
- PLC works in cyclic way (1 - 150 ms):
  - reads input channels (sensors, switches, internal variables)
  - computes new values
  - writes new values to associated output channels/registers
  - input/output/internal variables

Data Types and Safety

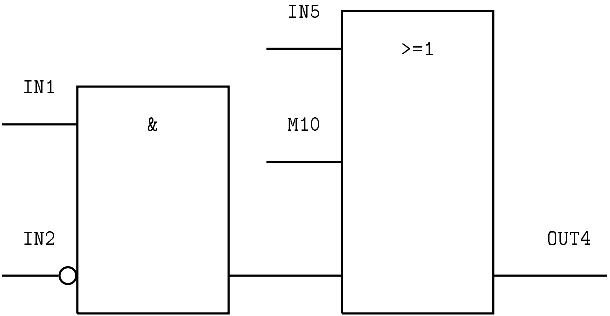
- possible data types: `bool`, `int`, `float`, `date`, ...
- with usual operations (numerical, comparison, ...)
- special part for safety critical operations with reduced instruction set
- from now on only Boolean data and operations

Programming Languages

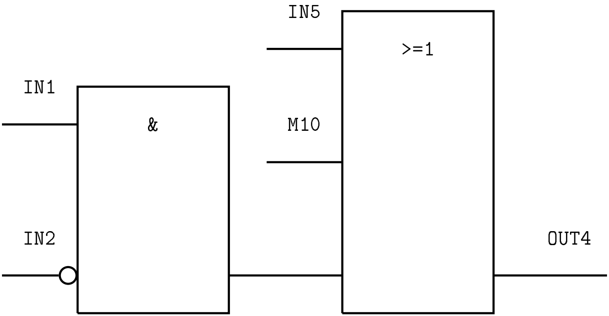
Programming done via:

- Instruction List (IL): assembly-like
- Ladder Diagram (LD): similar to circuit diagrams
- Sequential Function Chart (SFC): inspired by state diagrams
- Structured Text (ST): resembles C syntax
- Function Block Diagram (FBD): see next

AND, OR and Negation in FBD



AND, OR and Negation in FBD

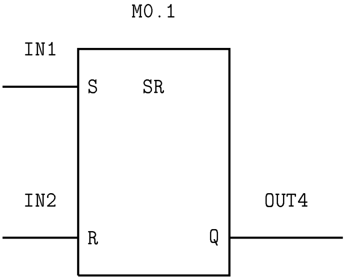


$$\mathtt{OUT4} \equiv (\mathtt{IN1} \land \neg \mathtt{IN2}) \lor \mathtt{IN5} \lor \mathtt{M10}$$

Flip-Flops (Purpose and Function)

- Flip-Flops show dynamic behavior
- two inputs, one marker, one output
- `TRUE`-signal on set input sets output and marker persistently to `TRUE`
- `TRUE`-signal on reset input resets output and marker persistently to `FALSE`
- (until next signal on set/reset input)
- set/reset dominant depending on winner at set/reset conflict
- storing/clearing depending on input signals

Flip-Flops (FBD)



Flip-Flops (Truth Table)

S_n	R_n	Q_{n+1}
TRUE	FALSE	TRUE
FALSE	TRUE	FALSE
FALSE	FALSE	Q_n
TRUE	TRUE	TRUE (set dominant)
TRUE	TRUE	FALSE (reset dominant)

Semirings

Definition

An idempotent *semiring* is a structure $(M, +, 0, \cdot, 1)$ with

- $x + y = y + x$
- $x + (y + z) = (x + y) + z$
- $x + x = x$
- $x + 0 = x$
- $x \cdot (y \cdot z) = (x \cdot y) \cdot z$
- $x \cdot 1 = x = 1 \cdot x$
- $x \cdot 0 = 0 = 0 \cdot x$
- $x(y + z) = xy + xz$ and $(x + y)z = xz + yz$

- $+$ can model choice, $\cdot$ composition
- natural order defined by $x \leq y \;\Leftrightarrow_{df}\; x + y = y$

Semiring examples

- powerset semiring: $(\mathcal{P}(M), \cup, \emptyset, \cap, M)$
- endorelations: $(\mathbf{Rel}(M), \cup, \emptyset, ;, \mathrm{id}_M)$
- monoid semiring: $(\mathcal{P}(M), \cup, \emptyset, \cdot, \{\varepsilon\})$
  for monoid $(M, \cdot, \varepsilon)$ and $\cdot$ lifted to sets
- max-min semiring $(\mathbb{R} \cup \pm\infty, \max, -\infty, \min, \infty)$
- tropical semiring $(\mathbb{R} \cup -\infty, \max, -\infty, +, 0)$
- matrix semiring: $(M^{n\times n}, +, 0^{n\times n}, \cdot, \mathbb{1})$
  defined analogously to conventional linear algebra

Kleene Algebra

Definition

A *Kleene algebra* is a structure $(M, +, 0, \cdot, 1, ^*)$ where $(M, +, 0, \cdot, 1)$ is an idempotent semiring and $^* : M \to M$ has the following properties:

$$1 + xx^* \leq x^* \qquad x + yz \leq z \Rightarrow y^*x \leq z$$
$$1 + x^*x \leq x^* \qquad x + yz \leq y \Rightarrow xz^* \leq y$$

- * models iteration
- $x^* = \sum_{n=0}^{\infty} x^n$ in case of existence
- $a^* = \mu_f = \mu_g$ for $f(x) = 1 + ax$ and $g(x) = 1 + xa$

Kleene Algebra Examples

- endorelations: $(\mathbf{Rel}(M), \cup, \emptyset, ;, \mathrm{id}_M, {}^*)$
- formal languages: $(\mathcal{P}(\Sigma^*), \cup, \emptyset, \cdot, \{\varepsilon\}, {}^*)$
- path algebra: $(\mathcal{P}(\mathbf{path}(A)), \cup, \emptyset, \bowtie, A, A \cup P \cup P \bowtie P \cup P \bowtie P \cup \cdots)$ with

  $$p_1 a \bowtie b p_2 = \begin{cases} p_1 a p_2 & \text{if } a = b \\ \text{undefined} & \text{otherwise,} \end{cases}$$

  lifted to sets
- matrix Kleene algebra: $(M^{n \times n}, +, 0^{n \times n}, \cdot, \mathbb{1}, {}^*)$

  with $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^* = \begin{pmatrix} f^* & f^* b d^* \\ d^* c f^* & d^* + d^* c f^* b d^* \end{pmatrix}$ and $f = a + b d^* c$

Tests

given an idempotent semiring $S = (M, +, 0, \cdot, 1)$ subsets of M can be modeled by tests:

Definition

Given an idempotent semiring $S = (M, +, 0, \cdot, 1)$ an element $p \in M$ is called a *test* if an element $\neg p$ (the *complement* of p) exists with the properties $p + \neg p = 1$ and $p \cdot \neg p = 0 = \neg p \cdot p$.

- set of tests denoted by **test**(S)
- in relational context: subsets of identity
- restriction corresponds to px and xp, resp.

Boxes and Diamonds

(pre)image or (pre I post)condition modeled by diamond/box operators:

Definition

A *modal semiring* is a structure $S = (M, +, 0, \cdot, 1, |\cdot\rangle, \langle\cdot|)$ where $S' = (M, +, 0, \cdot, 1)$ is an idempotent semiring and $|\cdot\rangle$ and $\langle\cdot|$ are functions of the type $M \rightarrow (\textbf{test}(S') \rightarrow \textbf{test}(S'))$ with the properties $|x\rangle p \leq q \Leftrightarrow \neg qxp \leq 0 \Leftrightarrow \langle x|p \leq \neg q$, $|xy\rangle p = |x\rangle|y\rangle p$ and $\langle xy|p = \langle y|\langle x|p$ for all $x \in M$ and $p, q \in S'$.

- $|a\rangle p$: transition into p is possible
- $|a]p =_{df} \neg|a\rangle\neg p$: transition into p is inevitable

Modal Kleene Algebra

putting all together:

Definition

A *modal Kleene algebra* (*MKA* for short) is a structure $(M, +, 0, \cdot, 1, |\cdot\rangle, \langle\cdot|, ^*)$ where $(M, +, 0, \cdot, 1, |\cdot\rangle, \langle\cdot|)$ is a modal semiring and $(M, +, 0, \cdot, 1, ^*)$ is a Kleene algebra.

concrete example: $(\mathbf{Rel}(M), \cup, \emptyset, ;, \mathrm{id}_M, \mathrm{preim}, \mathrm{im}, ^*)$

Modal Kleene Algebra and Linear Temporal Logic

work by Möller, Höfner and Struth (2006):

- model transition system by a general MKA element a
- transforming sets of states into sets of successors
- left total function modeled by $|a\rangle p = |a]p$ for all tests p
- formulae in linear temporal logic (LTL) correspond to expressions in MKA
- LTL formula is valid iff corresponding MKA expression evaluates to 1

Explicit Correspondence

$$
\begin{array}{lcl}
\llbracket \bot \rrbracket & = & 0 \\
\llbracket \neg \psi \rrbracket & = & \neg \llbracket \psi \rrbracket \\
\llbracket \psi_1 \wedge \psi_2 \rrbracket & = & \llbracket \psi_1 \rrbracket \cdot \llbracket \psi_2 \rrbracket \\
\llbracket \psi_1 \vee \psi_2 \rrbracket & = & \llbracket \psi_1 \rrbracket + \llbracket \psi_2 \rrbracket \\
\llbracket \psi_1 \rightarrow \psi_2 \rrbracket & = & \llbracket \psi_1 \rrbracket \rightarrow \llbracket \psi_2 \rrbracket \qquad (p \rightarrow q =_{df} \neg p + q) \\
\llbracket \Box\, \psi \rrbracket & = & |a^*] \llbracket \psi \rrbracket \\
\llbracket \Diamond\, \psi \rrbracket & = & |a^* \rangle \llbracket \psi \rrbracket \\
\llbracket \circ \psi \rrbracket & = & |a \rangle \llbracket \psi \rrbracket \qquad (\text{recall } |a\rangle = |a]) \\
\llbracket \psi_1 \, \mathsf{U} \, \psi_2 \rrbracket & = & |(\llbracket \psi_1 \rrbracket \cdot a)^* \rangle \llbracket \psi_2 \rrbracket
\end{array}
$$

Variables and Overall Behavior

FBDs in MKA:

Variables and Overall Behavior

FBDs in MKA:

- inputs/outputs/internal variables correspond to tests
- for every signal/variable $\mathtt{p}$ introduce two tests $\mathtt{p\_0}$ and $\mathtt{p\_1}$
- indicating a value of $\mathtt{FALSE}$ and $\mathtt{TRUE}$, resp.
- clearly $\neg\mathtt{p\_0} = \mathtt{p\_1}$ and $\neg\mathtt{p\_1} = \mathtt{p\_0}$
- characterize behavior of elementary gates ($\mathtt{OR}$, $\mathtt{AND}$, Flip-Flops, ...)
- elementary gates do not change noninvolved signals/variables
- remember left total functionality
- write overall behavior *a* as product of elementary gates

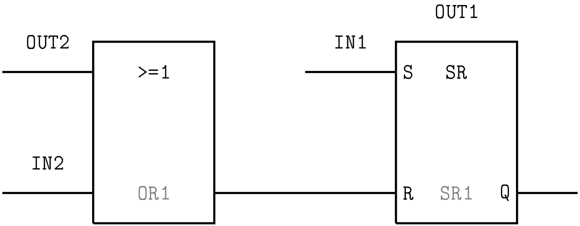
Elementary Gates

- $\texttt{AND}$-gate $\texttt{ANDk}$ with inputs $\texttt{in1},\ \texttt{in2}\ \ldots, \texttt{inn}$:
  - $\texttt{in1\_1} \cdot \texttt{in2\_1} \cdot \cdots \cdot \texttt{inn\_1} \leq |\texttt{andk}\rangle \texttt{andk\_1}$
  - $\texttt{in1\_0} + \texttt{in2\_0} + \cdots + \texttt{inn\_0} \leq |\texttt{andk}\rangle \texttt{andk\_0}$.
- $\texttt{OR}$-gate $\texttt{ORk}$ with inputs $\texttt{in1},\ \texttt{in2}\ \ldots, \texttt{inn}$:
  - $\texttt{in1\_1} + \texttt{in2\_1} + \cdots + \texttt{inn\_1} \leq |\texttt{ork}\rangle \texttt{ork\_1}$
  - $\texttt{in1\_0} \cdot \texttt{in2\_0} \cdot \cdots \cdot \texttt{inn\_0} \leq |\texttt{ork}\rangle \texttt{ork\_0}$.
- negation of $\texttt{sk}$: switch $\texttt{sk\_1}$ and $\texttt{sk\_0}$

Flip-Flops

- set dominant flip-flop RSk with set input s, reset input r, output q and internal marker m :
  - $\texttt{s\_1} + \texttt{m\_1} \cdot \texttt{r\_0} \leq |\texttt{rsk}\rangle \texttt{q\_1}$
  - $\texttt{s\_1} + \texttt{m\_1} \cdot \texttt{r\_0} \leq |\texttt{rsk}\rangle \texttt{m\_1}$
  - $\texttt{s\_0} \cdot \texttt{r\_1} + \texttt{m\_0} \cdot \texttt{s\_0} \leq |\texttt{rsk}\rangle \texttt{q\_0}$
  - $\texttt{s\_0} \cdot \texttt{r\_1} + \texttt{m\_0} \cdot \texttt{s\_0} \leq |\texttt{rsk}\rangle \texttt{m\_0}$

Example Construction (not Complete!)



$$\mathtt{out2\_1} + \mathtt{in2\_1} \leq |\mathtt{or1}\rangle\mathtt{or1\_1}$$

$$\mathtt{out2\_0} \cdot \mathtt{in2\_0} \leq |\mathtt{or1}\rangle\mathtt{or1\_0}$$

$$\mathtt{in1\_0} \leq |\mathtt{or1}\rangle\mathtt{in1\_0}$$

$$\mathtt{in1\_1} \leq |\mathtt{or1}\rangle\mathtt{in1\_1}$$

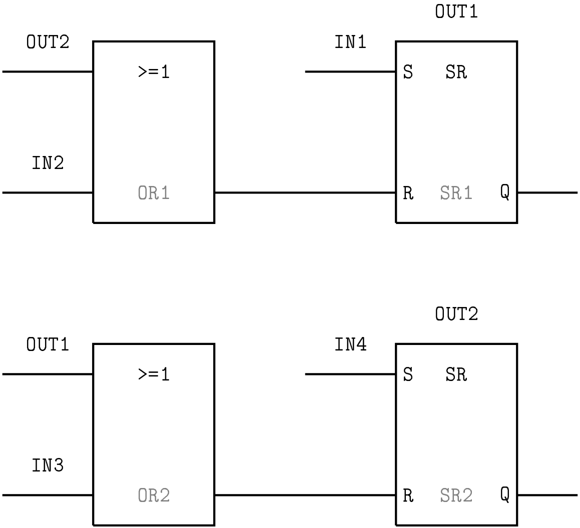
$$|\mathtt{or1}\rangle\mathtt{p} = |\mathtt{or1}]\mathtt{p}$$

$$\mathtt{or1\_1} + \mathtt{out1\_0} \cdot \mathtt{in1\_0} \leq |\mathtt{sr1}\rangle\mathtt{out1\_0}$$

$$\mathtt{in1\_1} \cdot \mathtt{or1\_0} + \mathtt{out1\_1} \cdot \mathtt{or1\_0} \leq |\mathtt{sr1}\rangle\mathtt{out1\_1}$$

$$\mathtt{cycle} = \mathtt{or1} \cdot \mathtt{sr1}$$

Mutual Exclusion



Behavior and Desired Properties

- behavior given by $\texttt{cycle} = \texttt{or1} \cdot \texttt{sr1} \cdot \texttt{or2} \cdot \texttt{sr2}$

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- desired properties in LTL:
  - $\mathtt{out1\_0} \land \mathtt{out2\_0} \rightarrow \Box(\mathtt{out1\_1} \rightarrow \mathtt{out2\_0})$
  - $\mathtt{out1\_0} \land \mathtt{out2\_0} \rightarrow \Box(\mathtt{out2\_1} \rightarrow \mathtt{out1\_0})$

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  - $\mathtt{out1\_0} \wedge \mathtt{out2\_0} \rightarrow \Box(\mathtt{out2\_1} \rightarrow \mathtt{out1\_0})$
- in MKA:
  - $\mathtt{out1\_0} \cdot \mathtt{out2\_0} \rightarrow |\mathtt{cycle}^*](\mathtt{out1\_1} \rightarrow \mathtt{out2\_0}) = 1$
  - $\mathtt{out1\_0} \cdot \mathtt{out2\_0} \rightarrow |\mathtt{cycle}^*](\mathtt{out2\_1} \rightarrow \mathtt{out1\_0}) = 1$

Proof Sketch

to show: $\mathtt{out1\_0} \cdot \mathtt{out2\_0} \to |\mathtt{cycle}^*](\mathtt{out1\_1} \to \mathtt{out2\_0}) = 1$

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- MKA: $\texttt{out1\_0} \cdot \texttt{out2\_0} + \texttt{out1\_0} \cdot \texttt{out2\_1} + \texttt{out1\_1} \cdot \texttt{out2\_0}$ is an invariant of $\texttt{cycle}^*$

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- MKA: $\mathtt{out1\_0} \cdot \mathtt{out2\_0} + \mathtt{out1\_0} \cdot \mathtt{out2\_1} + \mathtt{out1\_1} \cdot \mathtt{out2\_0}$ is an invariant of $\mathtt{cycle}^*$
- MKA: $p \leq q \wedge qx\neg q = 0 \wedge q \leq r \Rightarrow p \to |x]r = 1$

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- MKA: $p \leq q \wedge qx\neg q = 0 \wedge q \leq r \Rightarrow p \rightarrow |x]r = 1$
- finish:
  - $\texttt{out1\_0} \cdot \texttt{out2\_0} \leq \texttt{out1\_0} \cdot \texttt{out2\_0} + \texttt{out1\_0} \cdot \texttt{out2\_1} + \texttt{out1\_1} \cdot \texttt{out2\_0}$
  - $\texttt{out1\_0} \cdot \texttt{out2\_0} + \texttt{out1\_0} \cdot \texttt{out2\_1} + \texttt{out1\_1} \cdot \texttt{out2\_0} \leq \texttt{out1\_1} \rightarrow \texttt{out2\_0}$

DLR

Proof Sketch

to show: $\mathtt{out1\_0} \cdot \mathtt{out2\_0} \to [\mathtt{cycle}^*](\mathtt{out1\_1} \to \mathtt{out2\_0}) = 1$

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  - $\mathtt{out1\_0} \cdot \mathtt{out2\_0} + \mathtt{out1\_0} \cdot \mathtt{out2\_1} + \mathtt{out1\_1} \cdot \mathtt{out2\_0} \leq \mathtt{out1\_1} \to \mathtt{out2\_0}$
- proof done interactively in KIV

Conclusion

We saw:

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- Programmable Logic Controllers

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We saw:

- Programmable Logic Controllers
- Modal Kleene Algebra
- Linear Temporal Logic
- interactive proving with KIV
- and all working together

Outlook

Done:

- formalization of timers
- verification of until-properties

Todo:

- embracing numerical operations
- automated construction of input files
- handling larger systems

References

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