MEASUREMENTS OF MOON'S ROTATION BY CO-REGISTRATION OF LASER ALTIMETER PROFILES AND STEREO TERRAIN MODELS.

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Introduction: The Moon exhibits a complex rotational state, including precessions, nutations and librations. The "physical librations", known already from early Earth-based optical observations [1], are forced by changes in the tidal torque acting upon the Moon, associated with the orbit of the Moon about the Earth, the orbit of the Earth-Moon system around the Sun, among other. Furthermore, the analysis of data obtained through terrestrial laser ranging to retroreflectors on the Moon's confirmed the existence of free physical librations [2]. The amplitudes of forced and free librations are connected to the properties of the Moon's interior. They in turn have strong implications on models about the origin and the evolution of Earth's satellite.

Lunar Rotation: Two common reference frames for the Moon are the mean Earth/rotation axes (ME) and the principal axis (PA) reference frames. While the former gives the orientation of Moon's surface, the latter is connected to the Moon's principal moments of inertia. Both reference frames can be represented in closed form, as a harmonic series in the Euler angles, or by means of an interpolated time series connected to a lunar ephemeris. However, the closed form representation suffers from errors up to 150 m. Fig. 1 visualizes the differences in the Euler angles between the closed and the ephemeris form within a time frame of 4 years. Apart from higher terms of the forced libration series the differences also include oscillations due to free librations [2]. The free libration in longitude has an amplitude of 10.9 m (1.3 arc seconds) at the Moon's equator and a period of 2.9 years. The wobble mode of the spin axis has amplitudes of 27.8 m and 68.9 m (3.3 and 8.2 arc seconds) with periods of 74.6 years [2]. Since the free libration modes are usually damped by dissipative processes, the mechanism for their maintenance is still under study [3].

Co-registration technique: In a recent study [4] the co-registration technique was used to measure the libration amplitude of the planet Mercury. The application of this technique to the Moon, offers the possibility to track the rotation of the Moon. In fact, we purpose to use data obtained by the Lunar Reconnaissance Orbiter (LRO), which is equipped with a camera system, the LRO Camera (LROC), and a laser altimeter, the Lunar Orbiter Laser Altimeter (LOLA). Starting in 2009 both instruments acquired a wealth of high-precision data on the topography of the Moon [5, 6].

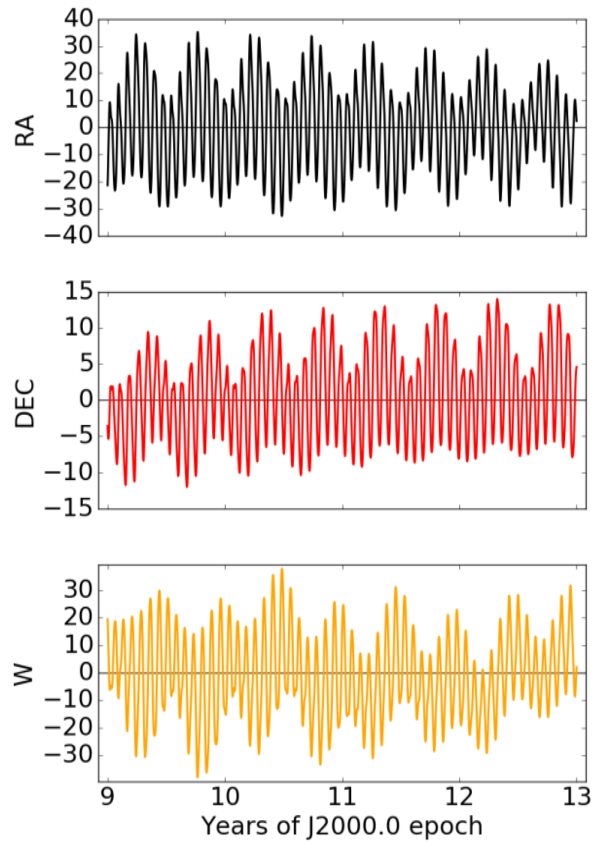

Figure 1: Euler angles differences (in arc seconds) between the closed and the ephemeris form representations of Moon's ME reference frame.

A previous study [7] demonstrated that laser profiles obtained by LOLA can be co-registered to stereo digital terrain models (stereo DTMs) derived from LROC images [8] with a very high vertical and lateral accuracy. Thereby the relative positioning of the two topographic data sets is determined precisely. The functional model g for the co-registration is given by

$$g^i(\boldsymbol{p}) = r^i_{\mathrm{DTM}}(\boldsymbol{p}) - \left|\boldsymbol{r}^i_{\mathrm{LA}}(\boldsymbol{p})\right|, \quad (1)$$

where $\boldsymbol{p}$ is a vector of co-registration parameters, r^i_{DTM} is the stereo DTM radius associated with the i-th laser altimeter measurement and $\boldsymbol{r}^i_{\mathrm{LA}}$ is the radius vector of the Moon obtained from the laser altimeter measurement. The stereo DTM $r_{\mathrm{DTM}} = r_{\mathrm{DTM}}(l, s)$ is represented by a structured grid of lines l and samples s obtained from spherical or Cartesian coordinates with the

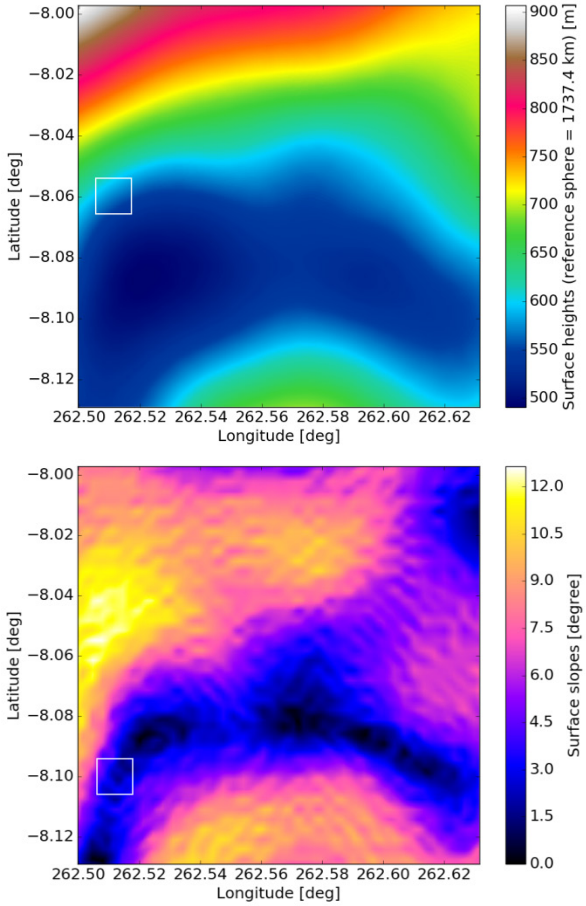

Figure 2: **Top**: Lunar topography near the northern rim of the Mare Orientale basin as part of the Global Lunar DTM "GLD100" with its 100 m per pixel lateral resolution [8]. **Bottom**: Surface slopes, i.e. angle between the surface gradients and the radius vector, of the stereo DTM shown above. The sub-pixel heights and slopes were computed using cubic B-spline interpolation of the gridded stereo DTM. The white squares denote the size of one DTM grid element (pixel), i.e. an area of 100 ×100 m.

help of a map projection Π. One important aspect of the co-registration method is the parametrization. An intuitive example is solving for a relative correction in line Δl, in sample Δs and height Δh, i.e. $\boldsymbol{p} = (\Delta l, \Delta s, \Delta h)$ [7]. As each laser footprint is associated to a certain measurement epoch the corrections of several profiles allow to track the rotation of the Moon. Neglecting for now a possible time dependency of the correction parameters the function model of Eq. 1 transforms to

$$g^i(\boldsymbol{p}) = r^i_{\mathrm{DTM}}\left(\Pi\left(\theta^i_{\mathrm{LA}}, \varphi^i_{\mathrm{LA}}\right) + (\Delta l, \Delta s)\right) + \Delta h - r^i_{\mathrm{LA}}\,, \quad (2)$$

where $\theta^i_{\mathrm{LA}}, \varphi^i_{\mathrm{LA}}, r^i_{\mathrm{LA}}$ are latitude, longitude and radius of $\boldsymbol{r}^i_{\mathrm{LA}}$, respectively. In order to estimate the co-registration parameters the stereo DTM radius $r^i_{\mathrm{DTM}}(\boldsymbol{p})$ and the partial derivatives $\partial r^i_{\mathrm{DTM}}(\boldsymbol{p})/\partial \boldsymbol{p}$ have to be computed. Sub-pixel accuracy in the co-registration can be achieved when interpolation is performed between the stereo DTM grid elements. In the case of Eq. 2 the partial derivatives in line and sample are actually the slopes of the terrain in the respective direction. Fig. 2 visualizes a stereo DTM and its slope using cubic B-spline interpolation.

Given the partial derivatives and stereo DTM radii at the location of the laser profile footprints the co-registration parameters $\boldsymbol{p}$ can be obtained by means of a non-linear least-squares estimation. Thereby the variance of the height residuals $\boldsymbol{g}(\boldsymbol{p})$ is minimized. Fig. 3 visualizes the residual field, i.e. the standard deviation of the components of the $\boldsymbol{g}(\Delta l, \Delta s, \Delta h)$ vector by variation of Δl and Δs. Note the clear minimum of the residuals; its position can be determined down to the sub-pixel level.

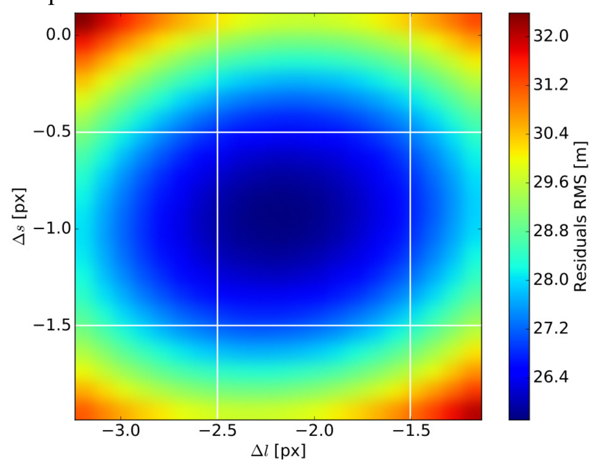

Figure 3: Residual field of the co-registration of a LOLA profile (13189 footprints) and a regional DTM extracted from the GLD100 (a larger part of the region shown in Fig. 2). The white lines delineate the boundaries of the DTM grid elements (px).

Outlook: The achievable accuracy of the co-registration suggests that the technique can significantly contribute to our knowledge on the rotational state of the Moon.

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