

# Novel Approach and Analysis to Determine Absolute Heights Using a Single Long Aperture SAR Acquisition

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## Abstract

This draft paper presents a novel approach to determine absolute heights using a single long aperture SAR acquisition. The new approach is a pure auto-focus that exploits the efficiency of the TerraSAR-X Multi-Mode SAR Processor (TMSP). This approach is much more robust to outliers rejection, presents a better accuracy and it is oriented to process a whole scene. The theory and source of errors are reviewed on this paper. Results over different real test cases are presented to show the potential of this method.

## 1 Introduction

This draft paper presents an update on the research carried by the authors regarding the ability of deriving absolute heights using a single spaceborne long aperture SAR image. As it is explained in [1], the obtained accuracy depends mainly on Signal-Clutter Ratio (SCR) and the azimuth bandwidth. Notice that for a very high SAR resolution image, the integrated clutter power is much less compared to a nominal Stripmap (SM) acquisition. Thus, the same point target shows a much higher SCR in a ST acquisition than in a SM. The work has been developed using as examples SAR acquisitions from the German satellite TerraSAR-X in Staring Spotlight (ST) mode. This TerraSAR-X acquisition mode consists on steering the antenna to a fixed point on the ground during the whole acquisition time. The azimuth steering angles range from  $-2.2^\circ$  to  $2.2^\circ$ . The associated azimuth bandwidth is around  $38\text{ KHz}$  yielding an azimuth resolution below  $24\text{ cm}$ . For a set of point-like scatterers above a  $20\text{ dB}$  SCR is possible to determine their height within few meters accuracy. The previous work is based on analyzing the point target defocussing by means of the impulse response peak displacement for different azimuth sub-aperture images. It analyzes empirically the impact of the tropospheric delay and orbit inaccuracies. The present work presents a different approach, the processing is carried out without using sub-apertures. It is based on a pure directly auto-focus focusing the raw data several times assuming different fixed heights for the whole image. In this way, all the points are processed together. Moreover, the height accuracy is also slightly improved. This approach is in practice easy to implement and oriented to process a complete scene. This paper reviews the theory and analyzes more in deep the effects of tropospheric, ionospheric and orbit inaccuracies. Analytical expressions are derived for the mentioned effects. The paper is emphasized with three different results over real data. The first result refers to a real building facade located in Berlin. The second result is a set of height es-

timations over a controlled bright scatterer in Wettzell, Germany. Finally the third result of this draft paper is a preview of a first complete scene over the city of Oslo, Finland, compared with the heights retrieved by processing a stack of acquisitions by means of the PSI technique.

## 2 Theory Review

The FM rate is well-known defined by

$$FM = \frac{2V_{sat,eff}^2}{\lambda R_0}, \quad (1)$$

where  $\lambda$  is the wavelength,  $R_0$  is the range at closest approach and  $V_{sat,eff}$  the satellite's effective velocity. The azimuth FM rate mismatch due to assuming in focusing a different height than the real one can be approximated by [1]

$$\Delta FM_{rate} \approx \frac{2g_{H_s}}{\lambda R_0} \cdot \Delta h, \quad (2)$$

where  $g_{H_s}$  is acceleration related to the gravity at satellite's height and  $\Delta h$  is the height discrepancy. The impulse response at the output of the matched filter in azimuth frequency domain is given by:

$$S(f_{az}) = \text{rect}\left(\frac{f_{az}}{Bw_{az}}\right) \cdot e^{-j\pi \frac{\Delta FM f_{az}^2}{FM^2}} \cdot e^{-j2\pi f_{az} t_{az,ZD}}, \quad (3)$$

where  $f_{az}$  is the azimuth frequency,  $Bw_{az}$  is the total azimuth bandwidth and  $t_{az,ZD}$  is the zero Doppler azimuth time. The defocus is introduced by the first exponential term, which is dependent on the FM rate, FM rate mismatch and the azimuth bandwidth. The height accuracy Cramer-Rao Lower Bound (CRLB) derived in [1] is given by

$$\sigma_{h,N_{sub}} = \frac{2 \cdot V_{sat,eff}^4}{\lambda \cdot R_0 \pi \cdot Bw_{az}^2 \cdot g_{H_s}} \cdot \sqrt{\frac{18N_{sub}^4}{SCR \cdot (N_{sub}^2 - 1)}}, \quad (4)$$

being  $N_{sub}$  the number of independent azimuth sub-apertures. The new approach presented on this paper is oriented to process the whole scene at once. The process is relatively simple, firstly the raw data is focused at a reference fix height for the complete scene. Then, an easy point target candidate selection is carried out setting a SCR threshold and seeking for peaks. Next step is to reprocess several times the raw data with different fixed heights. The amplitude of each candidate is recorded for every height processing. Thus, at the end, there is a sampling of the amplitude function w.r.t. height processing for every point target candidate. So, the height for every point target is chosen as the one which give the maximum amplitude. Those candidates which present a non clear maximum amplitude are rejected. All the candidates all that are not a unique point target in the resolution cell or that present disturbances by nearby brighter scattering mechanism will be discarded. This new approach is a pure auto-focus where there is only one full band. In order to calculate the CRLB for this approach, the azimuth spectral phase signal model has to take into account two parameters, a phase offset and a curvature parameter. Since the processor can be azimuth variant, a delay in azimuth time, i.e. a phase ramp in the frequency domain, is not necessary. The signal model has the following format

$$\psi(f_{az}) = \phi_0 - \pi \cdot \frac{\Delta FM}{FM^2} \cdot f_{az}^2 \quad (5)$$

The Fisher information will be a matrix of 2x2 elements. The element (1,1) is related to the derivatives of  $\psi(f_{az})$  w.r.t.  $\phi_0$ ; the element (2,2) w.r.t.  $\Delta FM$ , and so on.

$$\mathbf{FIM}(f_{az}) = 2SCR \cdot \begin{pmatrix} 1 & \frac{\pi}{FM^2} f_{az}^2 \\ \frac{\pi}{FM^2} f_{az}^2 & \frac{\pi^2}{FM^4} f_{az}^4 \end{pmatrix} \quad (6)$$

The integrated Fisher information will be

$$\mathbf{FIM} = \frac{1}{Bw_{az}} \int_{-\frac{Bw_{az}}{2}}^{\frac{Bw_{az}}{2}} \mathbf{FIM}(f_{az}) df_{az} \quad (7)$$

So, the CRLB for the standard deviation estimation of  $\Delta FM$  becomes

$$\sigma_{\Delta FM, CRLB} = \frac{FM^2}{\pi \cdot Bw_{az}^2} \cdot \sqrt{\frac{90}{SCR}} \quad (8)$$

Finally, taking into account (8), (2) and (1), the height estimation CRLB for the new approach is defined as

$$\sigma_{h, autofoc} = \frac{2 \cdot V_{sat,eff}^4}{\lambda \cdot R_0 \pi \cdot Bw_{az}^2 \cdot g_{Hs}} \cdot \sqrt{\frac{90}{SCR}} \quad (9)$$

Notice that the new approach presents a significant accuracy improvement w.r.t. previous sub-aperture method. For two sub-apertures, the new method improves just by 3.3%. However, the previous method presents problems dealing with outliers when the number of sub-apertures

is reduced. In practice, as shown in [1], the best performance is shown when the number of sub-apertures is 3 to 5. In that case, the new approach presents an accuracy improvement of 42% and 128% for 3 and 5 sub-apertures respectively. Moreover, since the new approach process all the points at the same time, it is oriented to obtain the heights for the whole scene in a much more fast and efficient way. The height processing spacing can be easily determined by the aforementioned height accuracy and a priori range of reasonable heights of the scene. In that way, the method can take advantage of already developed precise and efficient focusing processors, such as TerraSAR-X Multi-Mode SAR Processor (TMSP) which is able to process a whole ST acquisition in 2-3 minutes.

## 2.1 Sources of error

The main source of error is due to the presence of clutter. As it has been mentioned, the SCR is directly link to the height accuracy that is possible to obtain. However, there are other kind of source that can introduce a bias in the height estimation. Here, we analyze the effect related to an unaccounted signal path delay due to troposphere and ionosphere and the effect of range orbit inaccuracies.

### 2.1.1 Tropospheric and ionospheric effects

There are two mechanisms that can induce a defocussing for the tropospheric case: the group delay will move the signal to a later range bin, for which the processor that ignores or partially ignores the troposphere will use a different chirp rate; and the phase delay will change during the aperture since the pulse will travel through variable sections of the troposphere. The two effects are equal and have the same sign. The FM rate group delay variation for a certain unaccounted slant-range delay is obtained by deriving (1) w.r.t. range, it yields:

$$\Delta FM_{group} = \frac{2 \cdot V_{sat,eff}^2}{\lambda \cdot R_0^2} \cdot SPD, \quad (10)$$

being  $SPD$  the one-way slant path delay at zero Doppler. The processing chirp rate will be less negative than what it should be. The effect of the phase delay during the aperture due to troposphere is already analyzed in [2]. Here, we analyze the complete effect and link the total FM rate error with an introduced bias in the proposed height estimation. The target will have the following extra phase

$$\psi(\theta) = -\frac{4\pi}{\lambda} \frac{SPD}{\cos(\theta)} \approx -\frac{4\pi}{\lambda} SPD \cdot \left(1 + \frac{\theta^2}{2}\right), \quad (11)$$

where  $\theta$  is the azimuth squint angle. The approximation is valid for small angles of  $\theta$ , which is in general true for Spaceborne SAR acquisitions. As example, the azimuth squint angle variation for TerraSAR-X ST acquisition mode goes from  $-2.2^\circ$  to  $2.2^\circ$ . Thus,  $\theta$  can be approximated as  $\theta = V_{sat,eff} \cdot t_{az} / R_0$ , being  $t_{az}$  the azimuth time. The quadratic phase variation w.r.t. azimuth

time yields

$$\psi(t_{az}) = -\frac{4\pi}{\lambda} \cdot SPD \cdot \frac{V_{sat,eff}^2}{2R_0^2} \cdot t_{az}^2. \quad (12)$$

So, the azimuth chirp rate error is easily derived as

$$\Delta FM_{phase} = \frac{2 \cdot V_{sat,eff}^2}{\lambda \cdot R_0^2} \cdot SPD. \quad (13)$$

Also in this case the processing will use a FM rate which is less negative than it should be. Then, the total FM error is

$$\Delta FM_{total} = \frac{4 \cdot V_{sat,eff}^2}{\lambda \cdot R_0^2} \cdot SPD. \quad (14)$$

The height bias can be deduced putting (2) and (14) together as

$$\Delta h_{tropo,bias} = \frac{2 \cdot V_{sat,eff}^2}{g_{H_s} R_0} \cdot SPD \approx \frac{2 \cdot V_{sat,eff}^2}{g_{H_s} H_s} \cdot ZPD. \quad (15)$$

The height bias becomes independent of the incident angle when approximating  $R_0 = H_s / \cos(\theta_{inc})$ , being  $H_s$  the satellite height and  $\theta_{inc}$  the incidence angle, and the Zenith Path Delay (ZPD) by  $ZPD = SPD / \cos(\theta_{inc})$ .

Regarding the ionospheric effect, it presents also a group and phase delays. The group delay has the same effect like in the troposphere (10), it will make the phase history excessively curved for the range bin in which the signal will appear. However the phase will now advance, opposite to the group delay, by the same amount. This is because the ionosphere has the effect of advancing phases, while it delays wave packets. The final result would be that the target will be properly focused, though appearing in the wrong range bin.

The height bias introduced by the tropospheric delay effect can be removed if external information is available.

### 2.1.2 Orbit range inaccuracies

Here, it has to be highlighted that, without taking into account path delays, the range measured by the radar is correct. That means, that in case there is an orbit range error, what slightly changes between what the system assumes and the reality is the incidence angle. So, the introduced height bias is related to the range orbit error as

$$\Delta h_{rg-orb,bias} = \Delta R_{orb} \cdot \cos(\theta_{inc}). \quad (16)$$

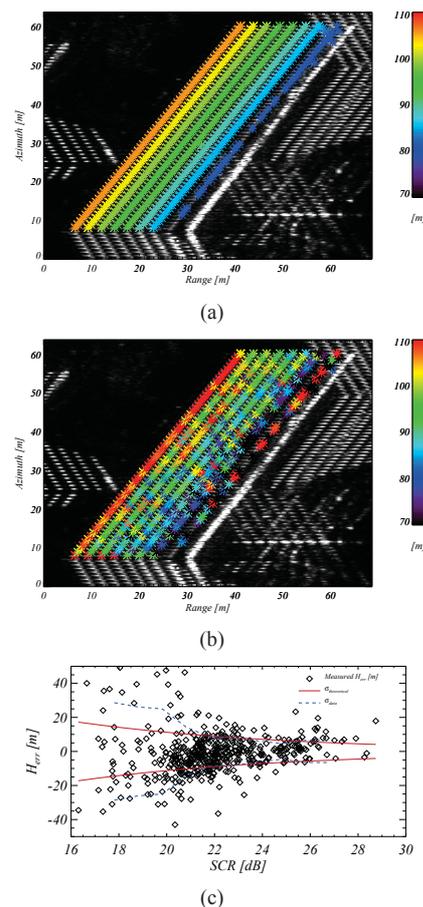
This means that a fix orbit error for the aperture introduces a bias in height in the same order of magnitude. Thus, for TerraSAR-X where the orbit error is in the order of few centimeters, the introduced height error can be assumed negligible. The bias due to orbit errors analyzed in [1] is erroneous, since it assumes that the measured range magnitude by the radar varies at the presence of orbit inaccuracies.

## 3 Results

In this section we present three different results over real data using the described method.

### 3.1 Building Facade

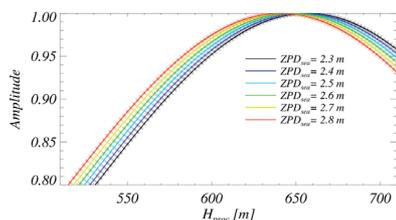
The first result refers to the heights obtained over a building facade in the Berlin city, Germany. The height process is performed over a TerraSAR-X ST acquisition on 9th October 2014. The incidence angle is  $36.1^\circ$  and the azimuth bandwidth is  $38.3 \text{ KHz}$ . **Figure 1** shows the obtained results. **Figure 1(a)** illustrates the heights derived from a Lidar DEM while **Figure 1(b)** shows the obtained heights with the single image auto-focus method. Here, the tropospheric delay has been estimated using weather models [3] and the corresponding height bias has been removed accordingly. **Figure 1(c)** pictures the statistics of the discrepancies between the obtained heights and Lidar. The results taking into account the provided tropospheric delay present a bias of  $-1.4 \text{ m}$ . However, the obtained accuracy depending on the SCR, blue dotted line in **Figure 1(c)**, shows a good agreement with the theoretical accuracy derived from (9), solid red line.



**Figure 1:** Results over a facade in Berlin, Germany. (a) refers to the Lidar ground truth heights while (b) are the obtained heights using the described auto-focus method. (c) illustrates the statistics of the discrepancies between both.

### 3.2 Wettzell controlled Corner Reflector

The second result is over a controlled corner reflector deployed in Wettzell, Germany. This test scenario presents very accurate ground data. It is located next to a dedicated Geodetic Observatory station. So, accurate values of tropospheric and ionospheric delays are available for the image acquisition times as well as precise GPS control of the phase center. Moreover, the corner presents a SCR of 50 dB, the corresponding height accuracy according to (9) is just 15 cm. Several images have been acquired in ST mode over this test case. The acquisition raw data has been processed assuming different fixed heights and tropospheric delays at sea level. **Figure 2** shows the recorded amplitude for every processing in ZPD and height processing. The amplitude function is displaced for the different assumed ZPD at the ratio given by (15)



**Figure 2:** Amplitude function for different assumed ZPD in processing.

The analyzed set comprises a total of five ST images. The acquisition dates, incidence angles and differences w.r.t. on ground GPS are depicted in **Table 1**.

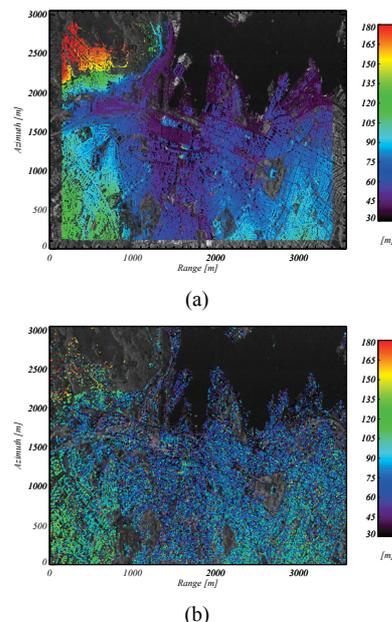
Date	$\theta_{inc}$	$h_{found} - h_{GPS}$
2015 - 02 - 07	33.2°	-0.86 m
2015 - 02 - 08	54.1°	-3.54 m
2015 - 05 - 12	45.1°	3.40 m
2015 - 05 - 18	54.1°	1.44 m
2015 - 05 - 29	54.1°	2.92 m

**Table 1:** Discrepancies between the derived auto-focus height and the ground truth GPS.

As it can be seen on **Table 1**, the differences are significantly above the expected CRLB accuracy. This kind of random error behavior may be related to a non homogeneous ionosphere. Just as an example, a curve variation during the approx. 60 Km of aperture in TECU units of -0.2, 0 and -0.2 would derive in a bias of around 5 m. The authors would investigate and characterize more in deep this effect in the final version of the paper.

### 3.3 Whole scene: single image vs PSI

Finally, the last result is a processing of complete scene over Oslo, Finland. The result is presented in **Figure 3**. A Permanent Scatterer Interferometry (PSI) processing has been carried out over a stack of 49 images to be used as a reference. **Figure 3(a)** shows the PSI result which shows a good agreement with the obtained auto-focus height estimation using just one image **Figure 3(b)**. The final version of the paper will show a more in deep comparison.



**Figure 3:** Height obtained by PSI processing (a) and by a single image autofocus method (b).

## 4 Summary & Conclusions

This paper presents a new approach to derive absolute heights from a single SAR long aperture acquisition. This approach presents a better accuracy than the sub-band approach defined in previous works, it is more robust to out-lier rejection and it is oriented to process a complete scene. It has been observed that the impact of the tropospheric delay is significant and it is needed and estimation on the tropospheric delay to obtain the absolute heights. A constant ionospheric delay and range orbit inaccuracies have practically no impact. The results have demonstrated that is possible to process a complete scene with height accuracies in the few meters level.

## References

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