Discrete optimisation problems on an adiabatic quantum computer

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Outlook

- Introduction to Adiabatic Quantum Computing
- Hardware limitations
- Application: Clique Problem
What is an Adiabatic Quantum Computer?

- Solver for Quadratic Unconstrained Binary Optimisation Problems (QUBOs)

\[
E(s_1, s_2, \ldots, s_n) = \sum_i H_i s_i + \sum_{ij} J_{ij} s_i s_j \quad \text{with } s_i \in \{0,1\}
\]

- Device by company D-Wave Systems commercially available
- We have access to hardware simulator / programming interface

\[H_i \in \mathbb{R} \quad \text{On-site strength}\]
\[J_{ij} \in \mathbb{R} \quad \text{Coupling}\]

Source: D-Wave Sys.
Quantum physics

- Eigenvalue equation determines physical state of a system

\[ \hat{H} |\psi_i\rangle = E_i |\psi_i\rangle \]

where \( E_i \) is the energy of the state and \( |\psi_i\rangle \in \text{Hilbert space} \) and \( \hat{H} \) is the Hamilton operator.

- Discrete Eigenvalues lead to quantised energy levels.
Measurement in Quantum physics

• Let the system be in a superposition of energy Eigen states $|\psi_i\rangle$

$$|\chi\rangle = \sum_i a_i |\psi_i\rangle$$

• Measurement of energy $E_j$ changes state

$$|\chi\rangle \rightarrow |\psi_j\rangle \langle \psi_j | \chi \rangle = a_i |\psi_j\rangle$$

• Probability $P_j$ to measure state $|\psi_j\rangle$

$$P_j = |a_i|^2$$
Adiabatic Quantum Computer – How does it work?

- Encode objective function in ground state of quantum system
- Initial system groundstate simple and implementable
Adiabatic Quantum Computer – How does it work?

- Sufficiently slow (adiabatic) transition to final system
- Lowest energy gap $\Delta E$ determines runtime
Quantum Bits – Two Level Quantum Systems

Classical bits

• Voltage

Quantumbits (Qubits)

• State is Superposition of „0“ und „1“

\[ |\varphi\rangle = a|0\rangle + b|1\rangle \]

• Measurement changes the state

→ Observe „0“ with probability \( |a|^2 \)
  \[ \Rightarrow |\varphi\rangle = |0\rangle \]

→ Observe „1“ with probability \( |b|^2 \)
  \[ \Rightarrow |\varphi\rangle = |1\rangle \]

• Multiple Qubits

\[ |\chi\rangle_{12} = |\psi\rangle_1 \otimes |\varphi\rangle_2 \]

e.g. \[ |\chi\rangle_{12} = |0\rangle_1 \otimes |1\rangle_2 \]
Adiabatic Optimisation Procedure

All qubits in „mixed state“

\[
\begin{align*}
\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
\end{align*}
\]

Solution

Farhi et. al., Quantum Computation by Adiabatic Evolution (2001)
Why use an Adiabatic Quantum Computer?

- QUBO / Ising is NP-hard

- Quantum speed-up assumed but subject to discussion
  (Defining and detecting quantum speedup, Rønnow et. al., Science 345, 1695, (2014))

- Map other NP-hard problems to QUBO/Ising
  (Ising Formulations of many NP problems, A. Lucas, Frontiers in Physics (2014))
Hardware limitations on a D-Wave device

- *Unit cell* with 8 Qubits an 2 partitions

- 8x8 Unit cells on D-Wave On chip ⇒ 512 Qubits

- Range of strenghts $H_i$ and couplings $J_{ij}$ limited and subject to uncertainties
Hardware limitations on a D-Wave device

- Not all Qubits are connected
- How to realise complete graphs?
- Solution:
  - Connection via other qubits
  - Representation of a *logical* qubit with several physical qubits
Overcome connectivity limitation

• Rule for implementing a complete graph onto the D-Wave hardware:

Source: D-Wave Sys. Simulator Software Documentation
Maximum Clique problem - Example

- Find largest complete subgraph
Maximum Clique problem - Example
Clique problem as QUBO

- Set negative strength at all nodes, e.g. 
  \( H_i = -1 \)

- Penalise all edges of graph complement with positive couplings, e.g. \( J_{ij} = +10 \)

- Edges of the graph itself can be activated without penalty,

\[
E(s_1, s_2, ..., s_n) = \sum_i H_i s_i + \sum_{ij} J_{ij} s_i s_j
\]
Cliquen Problem on D-Wave Device

- 1 logical qubit → 4 physical qubits
Outlook

• What kind of problems can be converted to QUBO?
• Combine conventional algorithms with AQC (hybrid approach)
• Investigate scaling behaviour